On a critical problem for Heat equation with Hardy Term

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Bounded domain.

We will consider the problem:

$$\begin{cases} u_t - \Delta u &= \lambda \dfrac{u}{|x|^2} + u^p + f \text{ in } \Omega_T \equiv \Omega \times (0,T), \\ u(x,t) &> 0 \quad \text{in} \quad \Omega_T, \\ u(x,t) &= 0 \quad \text{on} \quad \partial\Omega \times (0,T), \\ u(x,0) &= u_0(x) \text{ if } x \in \Omega, \end{cases}$$

where Ω bounded, $\Omega \subset I\!R^N$, $N \geq 3$, $0 \in \Omega$, $\lambda > 0$, $p > 1$.
f, u_0 are non negative measurable functions.

Related problems.

- Heat equation, $\lambda = 0$:

$$\begin{cases} u_t - \Delta u = u^p + f \text{ in } \Omega_T \equiv \Omega \times (0,T), \\ u(x,t) > 0 \quad \text{in} \quad \Omega_T, u(x,t) = 0 \quad \text{on} \quad \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) \text{ if } x \in \Omega. \end{cases}$$

Well known results.

- Associated elliptic problem, $\lambda > 0$:

$$-\Delta u = \lambda \frac{u}{|x|^2} + u^p + f \text{ in } \Omega,\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega.$$

(BDT) No distributional solution for $p \geq p_+(\lambda)$.

(BDT) H. Brezis, L. Dupaigne, A. Tesei, *On a semilinear elliptic equation with inverse-square potential.* Selecta Math. 11 (2005), no. 1.

Definition of solution: the weakest possible.

$u \in \mathcal{C}((0,T); L^1_{loc}(\Omega))$ is a very weak supersolution (subsolution) if $\frac{u}{|x|^2} \in L^1_{loc}(\Omega_T)$, $u^p \in L^1_{loc}(\Omega_T)$, $f \in L^1_{loc}(\Omega_T)$ and for all $\forall \phi \in \mathcal{C}_0^\infty(\Omega \times (0,T))$ such that $\phi \geq 0$,

$$\int_0^T \int_\Omega (-\phi_t - \Delta\phi) u \, dx dt \geq \ (\leq) \int_0^T \int_\Omega (\lambda \frac{u}{|x|^2} + u^p + f)\phi \, dx dt.$$

If u is a very weak super and subsolution, then we say that u is a very weak solution.

If u is a very weak supersolution (subsolution), then $u \in \mathcal{C}((0,T); L^1_{loc}(\Omega)) \cap L^p((0,T); L^p_{loc}(\Omega))$.

Behavior of the very weak supersolutions

Notation: The radial elliptic problem, $\lambda < \Lambda_N$,

$$-\Delta w - \lambda \frac{w}{|x|^2} = 0.$$

$|x|^{-\alpha_1}, |x|^{-\alpha_2}$ are the radial solutions with

$\alpha_1 = \frac{N-2}{2} - \sqrt{\left(\frac{N-2}{2}\right)^2 - \lambda}$, $\alpha_2 = \frac{N-2}{2} + \sqrt{\left(\frac{N-2}{2}\right)^2 - \lambda}$,

roots of $\alpha^2 - (N-2)\alpha + \lambda = 0$.

Hardy's inequality

$$\int_\Omega |\nabla \phi|^2 \, dx \geq \Lambda_N \int_\Omega \frac{|\phi|^2}{|x|^2} \, dx, \;\; \Lambda_N = \left(\frac{N-2}{2}\right)^2.$$

Singularity in a neighborhood of the origin

- u is a nonnegative function in Ω, $u \not\equiv 0$,
- $u \in L^1_{loc}(\Omega_T)$ and $\dfrac{u}{|x|^2} \in L^1_{loc}(\Omega_T)$,
- u satisfies $u_t - \Delta u - \lambda \dfrac{u}{|x|^2} \geq 0$ in $\mathcal{D}'(\Omega_T)$ with $\lambda \leq \Lambda_N$.

Fixed $0 < t_1 < t_2 \leq T$, there exists a constant $C(N, r, t_1, t_2)$ such that $u \geq C|x|^{-\alpha_1}$ in $B_r(0) \times (t_1, t_2)$.

Behavior of the very weak supersolutions

- If u is a very weak supersolution to problem $u_t - \Delta u - \lambda \dfrac{u}{|x|^2} \geq g$, then g must satisfy $\displaystyle\int_0^T \int_{B_r(0)} |x|^{-\alpha_1} g \, dx < \infty$. (Approximated problems,test function $(\varphi_n)_t - \Delta \varphi_n - \lambda \dfrac{\varphi_n}{|x|^2 + \frac{1}{n}} = 1$, pass to the limit).
- If u is a very weak supersolution, then there exists $r > 0$,

$$\int_{B_r(0)} |x|^{-\alpha_1} u_0(x) \, dx < \infty.$$

(Approximated problems, test function $-\Delta \varphi_n - \frac{\lambda \varphi_n}{|x|^2 + \frac{1}{n}} = c\varphi_n$, EDO, contradiction).

From a very weak supersolution, get a minimal solution

$\bar{u} \in \mathcal{C}((0,T); L^1_{loc}(\Omega))$ is a very weak supersolution, $\lambda \leq \Lambda_N$, then there exists a minimal solution in $B_r(0) \times (t_1, t_2) \subset\subset \Omega_T$ obtained by approximation.

Idea: Sub and super solutions method (aproximation and comparison).

$$\begin{cases} (v_0)_t - \Delta v_0 &= f \quad \text{in} \quad B_r(0) \times (t_1, t_2), \\ (v_n)_t - \Delta v_n &= \lambda \dfrac{v_{n-1}}{|x|^2 + \frac{1}{n}} + v_{n-1}^p + f \quad \text{in} \quad B_r(0) \times (t_1, t_2), \\ v_n(x, t_1) &= T_n(\bar{u}(x, t_1)) \text{ if } x \in B_r(0), \\ v_n(x, t) &= 0 \text{ on } \partial B_r(0) \times (t_1, t_2). \end{cases}$$

$v_{nt} - \Delta v_n \in L^1(B_r(0) \times (t_1, t_2))$.

$v_0 \leq \cdots \leq v_{n-1} \leq v_n \leq \bar{u}$, $v = \limsup v_n$, $v \leq \bar{u}$.

Critical exponents

Studying the elliptic radial case:

$$-u_{rr} - \frac{(N-1)}{r} u_r - \lambda \frac{u}{r^2} = u^p \quad \text{in} \quad B_r(0).$$

$u = Ar^{-\beta}$, with $\beta = \dfrac{2}{p-1}$, $A^{p-1} = -\beta^2 + (N-2)\beta - \lambda$.

We search $u > 0$: $-\beta^2 + (N-2)\beta - \lambda > 0$.

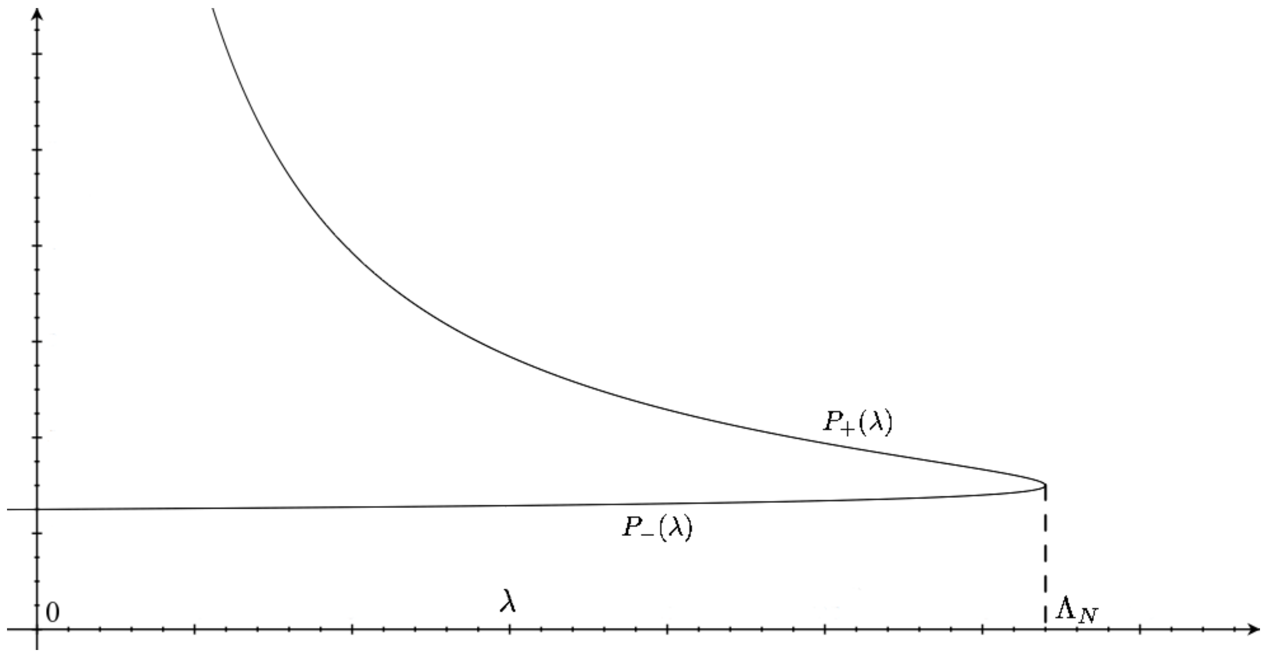
$$\alpha_1 < \beta < \alpha_2 \Leftrightarrow p_-(\lambda) < p < p_+(\lambda)$$

$$p_+(\lambda) = 1 + \frac{2}{\alpha_1}, \; p_-(\lambda) = 1 + \frac{2}{\alpha_2}$$

Critical exponents

$$p_+(\lambda) \to 2^* - 1 = \frac{N+2}{N-2} \text{ as } \lambda \to \lambda_N, \qquad p_+(\lambda) \to \infty \text{ as } \lambda \to 0,$$
$$p_-(\lambda) \to 2^* - 1 = \frac{N+2}{N-2} \text{ as } \lambda \to \lambda_N, \quad p_-(\lambda) \to \tfrac{N}{N-2} \text{ as } \lambda \to 0,$$

$p_+(\lambda)$ decreasing, $p_-(\lambda)$ increasing, $p_-(\lambda) \leq 2^* - 1 \leq p_+(\lambda)$.



Strong nononexistence result

If $p \geq p_+(\lambda)$, then the problem has no positive very weak supersolution. If $f \equiv 0$, the unique nonnegative is $u \equiv 0$.

Idea of the proof: Contradiction with Hardy inequality

Case 1: $\lambda > \Lambda_N$. Immediate.

Case 2: $\lambda < \Lambda_N,\ p > p_+(\lambda)$.

- Approximated problems (Truncated Hardy potential).
- $\frac{|\phi|^2}{u_n}$, $\phi \in \mathcal{C}_0^\infty(B_r(0))$, Picone, Holder, Sobolev inequalities.
- $$\int_0^T \int_{B_r(0)} |\nabla \phi|^2 \, dx dt - \lambda \int_0^T \int_{B_r(0)} \frac{\phi^2}{|x|^2} \, dx dt \geq C \int_0^T \int_{B_r(0)} \frac{\phi^2}{|x|^{(p-1)\alpha_1}}.$$
  As $p > p_+(\lambda)$, then $(p-1)\alpha_1 > 2$.

Strong nononexistence result

Case 3: $p = p_+(\lambda), \lambda < \Lambda_N$: $(p-1)\alpha_1 = 2.$

- Behavior: $u(x) \geq \frac{c_0}{|x|^{\alpha_1}}$ in $B_\eta(0) \times (t_1, t_2) \subset\subset \Omega_T$.
- Function test: $w(x,t) = |x|^{-\alpha_1}((t-t_1)^2(\log(\frac{1}{|x|}))^\beta + 1)$.
- Comparison argument: $cu \geq w$ in $B_\eta(0) \times (t_1, t_2)$.
- $\phi \in \mathcal{C}_0^\infty(B_\eta(0))$, Picone, Sobolev, Holder inequalities.
- Contradiction with Hardy's inequality,
  $c\int_{B_r(0)} \frac{|\phi|^2}{|x|^2}\Big(\log(\frac{1}{|x|})\Big)^\beta dx \leq \int_{B_r(0)} |\nabla\phi|^2 dx,\ r << \eta.$

Case 4: $p = p_+(\lambda), \lambda = \Lambda_N$: $\alpha_1(p+1) = N.$

$$\int_{t_1}^{t_2}\int_{B_r(0)} |x|^{-\alpha_1} u^p\, dx \geq C^p(t_2 - t_1)\int_{B_r(0)} |x|^{-\alpha_1(p+1)}\, dx = \infty.$$

Instantaneous and Complete blow up

- Punctual blow up in problems with approximated Hardy potential.

$u_n \in \mathcal{C}((0,T);L^1(\Omega)) \cap L^p_{loc}((0,T);L^p_{loc}(\Omega))$ very weak solution to,

$$\begin{cases} u_{nt} - \Delta u_n &= \dfrac{u_n^p}{1+\frac{1}{n}u_n^p} + \lambda a_n(x) u_n + c\,f \quad \text{in} \quad \Omega_T, \\ u_n(x,t) &= 0 \quad \text{on} \quad \partial\Omega \times (0,T), \\ u_n(x,0) &= 0 \text{ if } x \in \Omega, \end{cases}$$

with $f \not\equiv$, $a_n(x) = \dfrac{1}{|x|^2 + \frac{1}{n}}$, and $p \geq p_+(\lambda)$. Then

$$\boxed{u_n(x_0,t_0) \to \infty, \forall (x_0,t_0) \in \Omega \times (0,T).}$$

Instantaneous and Complete blow up

- Punctual blow up in problems with a sequence tending to the critical exponent.

Let $p_n(\lambda) = 1 + \dfrac{2}{\alpha_1 + \frac{1}{n}}$, a positive $f \in L^\infty(\Omega_T)$ and $u_n \in \mathcal{C}((0,T); L^1_{loc}(\Omega))$ a very weak supersolution to

$$\begin{cases} u_{nt} - \Delta u_n & \geq & \lambda \dfrac{u_n}{|x|^2} + u_n^{p_n} + f \quad \text{in} \quad \Omega_T, \\ u_n(x,t) & = & 0 \quad \text{on} \quad \partial\Omega \times (0,T), \\ u_n(x,0) & = & 0 \quad \text{in} \quad \Omega. \end{cases}$$

Then

$$\boxed{u_n(x_0,t_0) \to \infty, \forall (x_0,t_0) \in \Omega \times (0,T).}$$

Sketch of the proofs

- From the very weak supersolution we get a minimal solution obtained by approximation.
- By contradiction, we suppose $u_n(x_0, t_0) \to C < \infty$.
- $\displaystyle\int_{B_r(0)\times(t_1,t_2)} u_n(x,t)\,dx\,dt \le C,$ Harnack's inequality.
- $\displaystyle\int_{B_r(0)\times(t_1,t_2)} g_n\phi\,dx\,dt \le \int_{B_r(0)\times(t_1,t_2)} u_n(x,t)\,dx\,dt \le C.$
- In the case 1, MONOTONICITY. At the limit (Monotone Convergence), a very weak supersolution .
- In the case 2, NO MONOTONICITY. Another test function: $T_k(u_n)\phi$. At the limit (Fatou's Lemma), a very weak supersolution .

Difference with the Heat Equation

- HEAT EQUATION, $\lambda = 0$:

$$u_t - \Delta u = u^p + f \text{ in } \Omega_T,\ u > 0, u = 0 \text{ on } \partial\Omega \times (0,T),\ u_0(x).$$

Existence of local solution (regular initial data).

$$\boxed{\lambda = 0 \Rightarrow \alpha_1 = 0 \Rightarrow p_+(0) = \infty}$$

- HEAT EQUATION WITH HARDY TERM, $\lambda > 0$:

$$u_t - \Delta u = \lambda \frac{u}{|x|^2} + u^p + f \text{ in } \Omega_T,\ u > 0, u = 0 \text{ on } \partial\Omega \times (0,T),\ u_0(x$$

Non existence of very weak solution for

$$\boxed{p \geq p_+(\lambda)}$$

Existence of solutions: $p < p_+(\lambda)$.

- $f \equiv 0$: For $\lambda < \Lambda_N$, $1 < p < p_+(\lambda)$ and suitable $u_0(x)$. The problem

$$u_t - \Delta u = \lambda \frac{u}{|x|^2} + u^p \text{ in } \Omega_T,\ u > 0, u = 0 \text{ on } \partial\Omega \times (0,T),\ u_0(x),$$

  has a solution.

- $f \gneqq 0$: If $f(x) \leq \frac{c_0}{|x|^2}$ with c_0 small, we get the existence of a minimal solution for all $p < p_+(\lambda)$.

Idea of the proof: If $p < 2^* - 1$, variational solution.

If $2^* - 1 < p < p_+(\lambda)$, construction of the elliptic radial solution $u = Ar^{-\beta}$, $\beta = \frac{2}{p-1}$ and small initial datum .

CAUCHY PROBLEM: HEAT EQUATION WITH HARDY TERM

We want to study the global existence in time when local existence is assumed:

$$\begin{cases} u_t - \Delta u = \lambda \dfrac{u}{|x|^2} + u^p & \text{in} \quad \mathbb{R}^N, \ t > 0, \\ u(x,0) = u_0(x) \geq 0 & \text{in} \quad \mathbb{R}^N, \end{cases}$$

H. Fujita, *On the blowing up of solutions of the Cauchy problem for* $u_t = -\Delta u + u^{1+\alpha}$. J. Fac. Sci. Univ. Tokyo Sect. I 13 1966 (1966)

T. Kawanago *Existence and behaviour of solutions for* $u_t = \Delta(u^m) + u^l$ Adv. Math. Sci. Appl. 7 (1997), no. 1.

H. Levine *The role of critical exponents in blowup theorems.* SIAM Rev. 32 (1990), no. 2 .

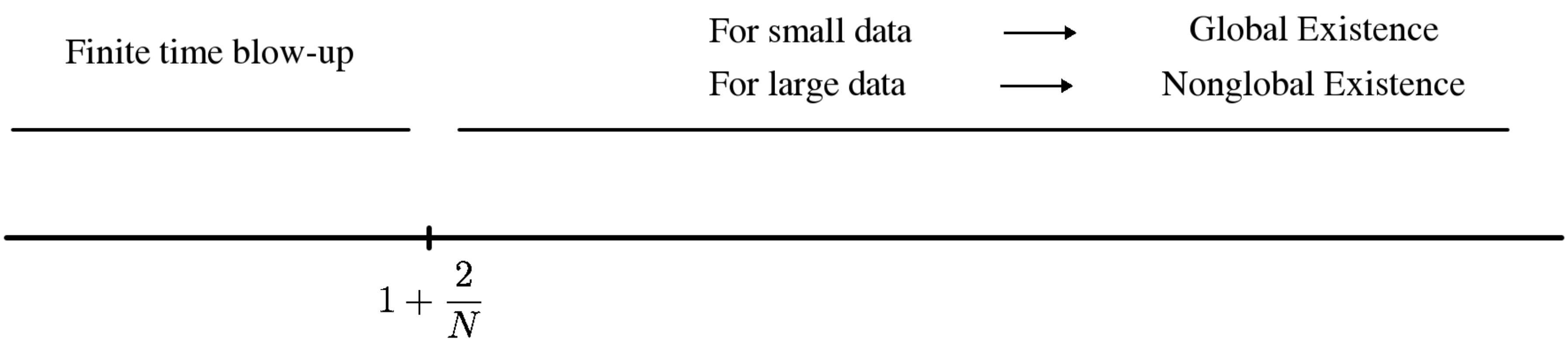
CAUCHY PROBLEM: HEAT EQUATION

Heat equation:

$$\begin{cases} u_t - \Delta u = u^p & \text{in } \mathbb{R}^N,\ t > 0, \\ u(x,0) = u_0(x) \geq 0 & \text{in } \mathbb{R}^N, \end{cases}$$

Fujita exponent: $1 + \frac{2}{N}$

Definition: Blow-up in finite time, $||u(\cdot, t_n)||_\infty \to \infty$, $t_n \to T^*$.

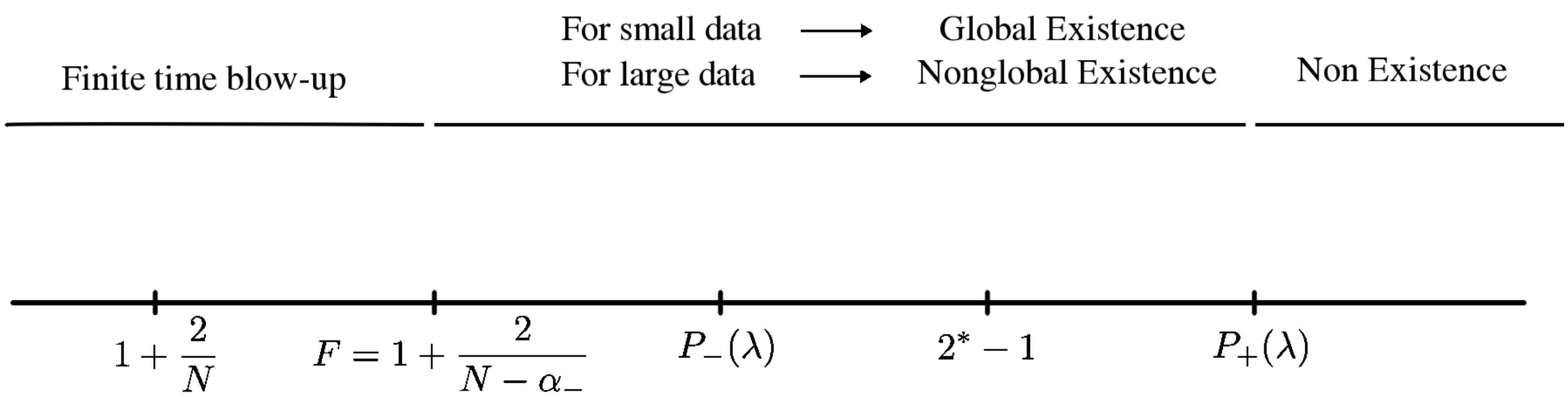


FUJITA TYPE EXPONENT WITH HARDY POTENTIAL

Fujita type exponent: $1+\frac{2}{N-\alpha_1}$, $\lambda=0,\ \alpha_1=0 \Rightarrow 1+\frac{2}{N}$.

Definition: Blow-up in finite time. Unbounded solutions.

There exists $T^* < \infty$, $\displaystyle\lim_{t\to T^*}\int_{B_r(0)} |x|^{-\alpha_1} u(x,t)\,dx = \infty.$



Assume that v is the solution to the equation

$$u_t - \Delta u - \lambda \frac{u}{|x|^2} = 0 \text{ in } \mathbb{R}^N,$$

then $v(r,t) = t^{-\frac{N}{2}+\alpha_1} r^{-\alpha_1} \exp^{(\frac{-1}{4}\frac{r^2}{t})}$ and satisfies $\int_{\mathbb{R}^N} r^{-\alpha_1} v(r,t)\, dx = C$.

For $\lambda = 0 \Rightarrow \alpha_1 = 0$, $v(r,t) = t^{-\frac{N}{2}} \exp^{(\frac{-1}{4}\frac{r^2}{t})}$ the FUNDAMENTAL SOLUTION to Heat equation, with $\int_{\mathbb{R}^N} v(r,t)\, dx = C'$.

FUJITA TYPE EXPONENT WITH HARDY POTENTIAL

$p < 1 + \dfrac{2}{N-\alpha_1}$, then u blows-up in finite time.

- We look for a family of subsolutions:

$$w(r,t,T) = (T-t)^{-\theta} f\left(\frac{r}{(T-t)^{\beta}}\right), \theta = -\tfrac{1}{p-1}, \beta = \tfrac{1}{2}, s = \frac{r}{(T-t)^{\beta}}$$

$$f(s) = A\phi(s), \phi(s) = s^{-\alpha_1} e^{-\frac{s^2}{4}}, \ A >> 0, w_t - w_{rr} - \frac{(N-1)}{r} w_r - \lambda \frac{w}{r^2} \leq w^p.$$

- w(r,t,T) has finite blow-up,

$$\int_{B_r(0)} |x|^{-\alpha_1} w(x,t,T)\, dx = C(T-t)^{-\frac{1}{p-1}+\frac{N}{2}-\frac{\alpha_1}{2}} \int_0^{\frac{r}{(T-t)^{\frac{1}{2}}}} \phi(s) s^{N-\alpha_1-1}\, ds = \infty.$$

$$(p < 1 + \frac{2}{N-\alpha_1} \Rightarrow -\frac{1}{p-1} + \frac{N}{2} - \frac{\alpha_1}{2} < 0)$$

FUJITA TYPE EXPONENT WITH HARDY POTENTIAL

- **COMPARISON ON INITIAL DATUM:**
  $\overline{u}(x,t)$, a time translation of a solution $\overline{u}(x,t) = u(x,t+T)$, is a supersolution to the homogenous equation with the same initial values. It is sufficient $v(x,T) \geq w(r,0,T)$, to get $\overline{u}(x,0) \geq w(r,0,T)$.

$$p < F(\lambda) = 1 + \frac{2}{N-\alpha_1}, \text{ for } T >> 1,\ T^{-\frac{N-\alpha_1}{2}} >> AT^{-\frac{1}{p+1}}.$$

- **COMPARISON PRINCIPLE:** $\overline{u}(x,t) \geq w(x,t), \forall t < T$.
  - $h(x,t) = w(x,t) - \overline{u}(x,t)$, $h_+ \in L^2(0,T,D^{1,2}(\mathbb{R}^N))$, satisfying $h_t - \Delta h \leq \lambda \dfrac{h}{|x|^2} + w^p - \overline{u}^p$.
  - Kato's inequality, $h_+(x,0) = 0$, (see (O)) .

$$h_t^+ - \Delta h_+ \leq \lambda \frac{h_+}{|x|^2} + p w^{p-1} h_+ \quad \text{in} \quad \mathbb{R}^N,\ t \in (0,T_1),\ T_1 < T.$$

  - NO BOUNDED SOLUTIONS, $p < 1 + \frac{2}{N-\alpha_1}$, $\exists C(T,T_1)$, $\forall \epsilon > 0, w^{p-1} \leq \epsilon \frac{1}{|x|^2} + C(T,T_1)$.
  - Gronwall's inequality, $h_+ = 0$, so $\overline{u}(x,t) \geq w(x,t), \forall t < T$.

(O) L. Oswald, *Isolated positive singularities for a non linear heat equation*, Houston Journal of Mathematics.

CRITICAL FUJITA TYPE EXPONENT $p = 1 + \frac{2}{N-\alpha_1}$.

$p = 1 + \dfrac{2}{N - \alpha_1}$, then u blows-up in finite time.

Ideas of the proof: Suppose

$$\int_{B_r(0)} |x|^{-\alpha_1} u(x,t)\, dx < \infty \text{ for all } t > 0.$$

With the change of variables, $v(x,t) = |x|^{\alpha_1} u(x,t)$,

$$|x|^{-2\alpha_1} v_t - \text{div}(|x|^{-2\alpha_1} \nabla v) = |x|^{-\alpha_1(p+1)} v^p,$$

with v satisfying $\int_\Omega |x|^{-2\alpha_1} v(x,t) dx < \infty$ for all $t > 0$. Modification of known arguments are followed.

(WZ), C. Wang, S. Zheng, *Critical Fujita exponents of degenerate and singular parabolic equations.* Proc. Roy. Soc. Edinburgh Sect. A 136 (2006), no. 2

$$F(\lambda) < p < p_+(\lambda).$$

GLOBAL EXISTENCE.

We look for a family of supersolutions

$$w(r,t,T) = (T+t)^{-\theta} g\Big(\frac{r}{(T+t)^{\beta}}\Big),\ \theta = \frac{1}{p-1}\ \beta = \frac{1}{2}.$$

$$w_t - w_{rr} - \frac{(N-1)}{r} w_r - \lambda \frac{w}{r^2} \geq w^p.$$

$g(s) = A\phi(cs)$, with $\phi(s) = s^{-\gamma} e^{-\frac{s^2}{4}}$, $\alpha_1 < \gamma < \dfrac{2}{p-1}$, $A > 0$, $c > 0$.It is sufficient to choose $c < 1$ and A small enough.

For suitable initial data we can construct a global solution.

L^2 FINITE TIME BLOW UP

We give a sufficient condition on the initial datum to get a blow-up behavior of the solution in a suitable norm different from the blow-up behavior obtained in previous theorems.
$1 < p < p_+(\lambda)$. If u is a positive solution , $u_0(x) \geq h(x)$ where $0 \leq h \in L^{p+1}(\mathbb{R}^N) \cap \mathcal{D}^{1,2}(\mathbb{R}^N)$ satisfies

$$\frac{1}{p+1}\int_{\mathbb{R}^N_+} h^{p+1}dx > \frac{1}{2}\int_{\mathbb{R}^N_+}\left(|\nabla h|^2 - \lambda\frac{h^2}{|x|^2}\right)dx,$$

then u blows-up in finite time. The sense: there exists $T^* < \infty$ such that

$$\int_{B_R(0)} u^2(x,t)dx \to \infty \text{ as } t \to T^*.$$

L^{p+1} INFINITE TIME BLOW UP

$F(\lambda) < p < 2^* - 1$. u is a global solution, then

$$\int_{\mathbb{R}^N} u^{p+1}(x,t)\,dx \to \infty \text{ as } t \to \infty. \text{ Infinite time.}$$

Idea of the proof: $\exists\, \overline{T} > 0, \; \sup_{t \in [\overline{T}, \infty]} \int_{\mathbb{R}^N} u^{p+1}(x,t)\,dx < \infty$.

Approximated problems. We have estimates that allow us to pass to the limit and to get a solution to the elliptic problem

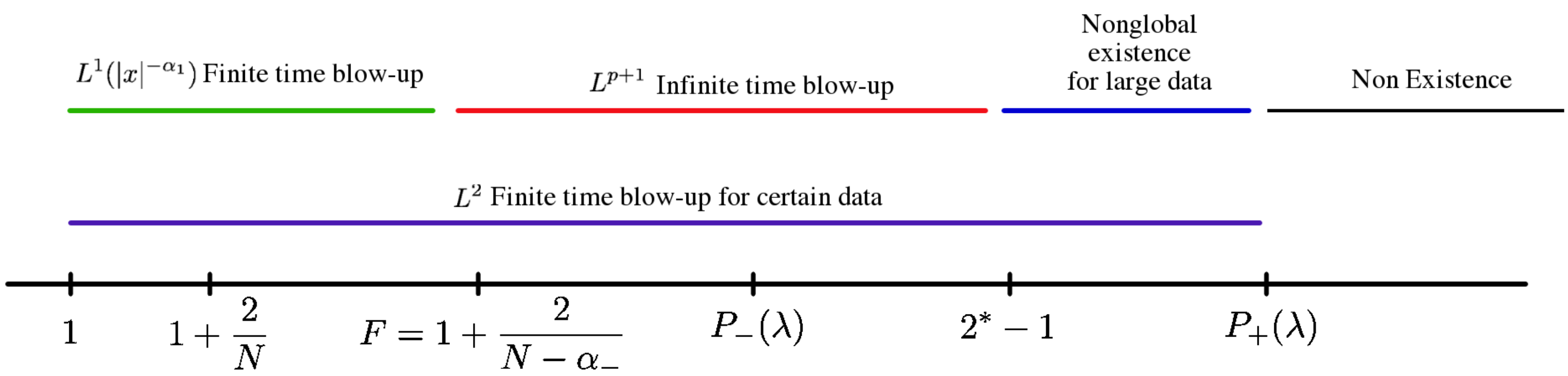
$$-\Delta u - \lambda \frac{u}{|x|^2} = u^p \in \mathbb{R}^N,$$

but $p < 2^* - 1$, a contradiction with (T).

(T) S. Terracini, *On positive entire solutions to a class of equations with a singular coefficient and critical exponent*, Adv. Differential Equations **1** (1996), no. 2.

SUMMARY CAUCHY PROBLEM

- $p \leq F(\lambda) = 1 + \frac{2}{N-\alpha_1}$. Blow-up in finite time:
$$\exists\, T^* < \infty, \lim_{t \to T^*} \int_{B_r(0)} |x|^{-\alpha_1} u(x,t)\, dx = \infty.$$
- $F(\lambda) < p < p_+(\lambda)$. Global existence for small data. Non global existence for large data.
- $1 < p < p_+(\lambda)$. u a solution, u_0 initial datum with sufficient property. Blow-up in finite time:
$$\exists\, T^* < \infty, \int_{B_R(0)} u^2(x,t) dx \to \infty \text{ as } t \to T^*.$$
- $F(\lambda) < p < 2^* - 1$. Blow-up in infinite time:
$$\int_{\mathbb{R}^N} u^{p+1}(x,t)\, dx \to \infty \text{ as } t \to \infty.$$


$L^1(|x|^{-\alpha_1})$ Finite time blow-up
L^{p+1} Infinite time blow-up
Nonglobal existence for large data
Non Existence
L^2 Finite time blow-up for certain data
1
$1+\dfrac{2}{N}$
$F=1+\dfrac{2}{N-\alpha_-}$
$P_-(\lambda)$
2^*-1
$P_+(\lambda)$


FIN
Cabo De Gata