

**EXISTENCIA DE SOLUCIONES PARA  
ALGUNOS PROBLEMAS ESTACIONARIOS CON  
DIFUSIÓN NO LOCAL**

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ABSTRACT. Consideraremos el problema no local estacionario  $\int_{\mathbb{R}^N} J(x-y)u(y)dy - u(x) = -u(x)^p$  en un dominio  $\Omega$ , con la condición “de contorno”  $u = 0$  en  $\mathbb{R}^N \setminus \Omega$ . Demostraremos la existencia de una solución positiva para todo  $p > 0$ , probando en particular la no existencia de exponente crítico para esta clase de problemas.

## 1. INTRODUCTION

$$(1.1) \quad \begin{cases} \int_{\mathbb{R}^N} J(x-y)u(y)dy - u(x) = -u(x)^p & x \in \Omega \\ u = 0 & x \in \mathbb{R}^N \setminus \Omega. \end{cases}$$

## 2. THE INITIAL VALUE PROBLEM

3. THE CASE  $p < 1$ 4. THE CASE  $p > 1$ 

## NOTAS

1.

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