

# A PRIORI BOUNDS FOR STABLE SOLUTIONS OF NONLINEAR ELLIPTIC EQUATIONS IN LOW DIMENSIONS

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ABSTRACT. We consider the class of semi-stable solutions of semilinear equations  $-\Delta u = f(u)$  in a bounded smooth domain of  $\mathbb{R}^n$  (sometimes convex). This class includes all local minimizers, minimal, and extremal solutions. In dimensions  $n \leq 4$ , we establish a priori  $L^\infty$  bounds which hold for every semi-stable solution and every nonlinearity  $f$ . This extends previous work of G. Nedev for dimensions  $n \leq 3$ . In relation with these results, A. Capella and the author have studied the particular case of radial solutions, for which boundedness holds up to dimension  $n \leq 9$  for every  $f$ . Some quasilinear analogues have also been established, both in the radial and nonradial cases, by A. Capella, M. Sanchón, and the author.

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