Zero Order Perturbations to Fully Nonlinear equations: Comparison, existence and uniqueness

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Abstract

We study existence and non-existence of nontrivial viscosity solutions to the following fully nonlinear equations with *power-like* right hand side,

$$(P1)\begin{cases} F(\nabla u_\lambda, D^2 u_\lambda) = \lambda\, u_\lambda^q, \\ u_\lambda > 0 \quad \text{in } \Omega, \\ u_\lambda = 0 \quad \text{on } \partial\Omega, \end{cases} \qquad (P2)\begin{cases} F(\nabla u_\lambda, D^2 u_\lambda) = \lambda\, u_\lambda^q + u_\lambda^r, \\ u_\lambda > 0 \quad \text{in } \Omega, \\ u_\lambda = 0 \quad \text{on } \partial\Omega, \end{cases}$$

where F is elliptic and homogeneous of degree m, and $0 < q < m < r$, with $\lambda > 0$.

We will prove existence and uniqueness of solutions to $(P1)$ via a comparison result up to the boundary. For problem $(P2)$, it will be shown that there exists $\Lambda > 0$ such that $(P2)$ has at least one positive solutions for every $\lambda \in (0, \Lambda)$, and no positive viscosity solution for $\lambda > \Lambda$.

In order to prove existence some further structure on F, under which the Harnack inequality holds, is required. Several examples, including uniformly elliptic operators and equations of Monge-Ampere type, the p-laplacian and infinity laplacian (the latter, with both normalizations) will be considered.

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