Mathematical Methods II. Test of chapter 3. December 15th, 2021

Name: **ID card number:**

In the following exercise, substitute α for the number of units on your ID card. If the number is zero, use the tens number (and so on).

Given the differential equation

$$2xy'' + (1-x)y' + \alpha y = 0\,,$$

1. Is $x_0 = 0$ an ordinary or a singular point? Is it regular? Reason your answer.
2. Find the general solution by power series about $x_0 = 0$.
3. Are there polynomial solutions? What degree are they?
4. Are all real solutions defined on the whole real line $x \in (-\infty, \infty)$?
5. What is the radius of convergence? Compute it using the ratio test.

Solution: x_0 is a singular regular point.
$y(x) = c_1 y_1(x) + c_2 y_2(x) = c_1\ {}_1F_1(-\alpha, \frac{1}{2}, \frac{x}{2}) + c_2\sqrt{x}\ {}_1F_1(\frac{1}{2} - \alpha, \frac{3}{2}, \frac{x}{2})$
$y_1(x)$ is a polynomial of degree α.
$y_2(x)$ is defined on $x \in [0, \infty)$, because of the square root.
$R = \infty$.