Mathematical Methods II. Test of chapter 2. December 2nd, 2021

Name: **ID card number:**

In the following exercise, substitute α for the number of units on your ID card. If the number is zero, use the tens number (and so on).

Given the differential equation

$$\ddot{y} - 2\alpha\dot{y} + n\alpha^2 y = e^{\alpha t} A\cos(\omega t) + 4e^{-\alpha t} B\cos(\omega t)$$

1. **(5pt)** Find the general solution $y(t)$ for: $n = 1, A = B = 1, \omega = 0$.
   Solution: $y(t) = y_h(t) + y_p(t),\ y_h(t) = c_1 e^{\alpha t} + c_2 t e^{\alpha t},\ y_p(t) = y_{p_1}(t) + y_{p_2}(t),$
   $y_{p_1}(t) = \frac{1}{2}t^2 e^{\alpha t},\ y_{p_2}(t) = \frac{e^{-\alpha t}}{\alpha^2}$

2. **(5pt)** Find the general solution of the homogeneous equation $y_h(t)$ for $n = 2$. Using the method of undetermined coefficients, *propose* a particular solution $y_p(t)$ for $A = 1, B = t, \omega = \alpha$ (it is not necessary to determine the coefficients in this case).
   Solution: $y(t) = y_h(t) + y_p(t),\ y_h(t) = c_1 e^{\alpha t}\cos(\alpha t) + c_2 e^{\alpha t}\sin(\alpha t),$
   $y_p(t) = y_{p_1}(t) + y_{p_2}(t),$
   $y_{p_1} = t e^{\alpha t}(a\cos(\alpha t) + b\sin(\alpha t)),$
   $y_{p_2} = e^{-\alpha t}((a_1 t + a_2)\cos(\alpha t) + (b_1 t + b_2)\sin(\alpha t))$
   a, b, a_1, a_2, b_1, b_2 undetermined coefficients.