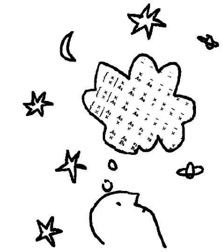


# NS-NS gauge invariance of non-Abelian D-brane actions

**Bert Janssen**

**Universidad de Granada & CAFPE**

In collaboration with: J. Adam (KUL) and I.A. Illán (UGR)

References: JHEP 10 (2005) 022, hep-th/0507198 and hep-th/0511191.

# Motivation

A lot has been learned about the dynamics of multiple D-branes in the last past years:

- $U(1)^N \rightarrow U(N)$ symmetry enhancement
- effective actions describing the non-Abelian dynamics
- many applications of non-Abelian effects in modern string theory

# Motivation

A lot has been learned about the dynamics of multiple D-branes in the last past years:

- $U(1)^N \rightarrow U(N)$ symmetry enhancement
- effective actions describing the non-Abelian dynamics
- many applications of non-Abelian effects in modern string theory

Question: Is the action (dynamics) gauge invariant?

# Motivation

A lot has been learned about the dynamics of multiple D-branes in the last past years:

- $U(1)^N \rightarrow U(N)$ symmetry enhancement
- effective actions describing the non-Abelian dynamics
- many applications of non-Abelian effects in modern string theory

Question: Is the action (dynamics) gauge invariant?

$\longrightarrow$ $U(N)$ invariance is straight forward.
$\longrightarrow$ Background gauge invariance is less obvious.

## Motivation

A lot has been learned about the dynamics of multiple D-branes in the last past years:

- $U(1)^N \to U(N)$ symmetry enhancement
- effective actions describing the non-Abelian dynamics
- many applications of non-Abelian effects in modern string theory

Question: Is the action (dynamics) gauge invariant?

$\longrightarrow U(N)$ invariance is straight forward.
$\longrightarrow$ Background gauge invariance is less obvious.

Answer: Yes!

# Motivation

A lot has been learned about the dynamics of multiple D-branes in the last past years:

- $U(1)^N \rightarrow U(N)$ symmetry enhancement
- effective actions describing the non-Abelian dynamics
- many applications of non-Abelian effects in modern string theory

Question: Is the action (dynamics) gauge invariant?

$\longrightarrow U(N)$ invariance is straight forward.
$\longrightarrow$ Background gauge invariance is less obvious.

Answer: Yes!

Price/Reward: New symmetry $\delta X \sim [X, X]$

# Outlook

1.Introduction:

$\rightarrow$ Non-Abelian physics of multiple D-branes

2. Problems with background gauge invariance

$\rightarrow$ solution for R-R gauge transformations

3. Problems with NS-NS gauge invariance

$\rightarrow$ derivation of NS-NS transformations of fields

$\rightarrow$ invariance of action

# 1. Non-Abelian physics of multiple parallel branes

The physics of $N$ separated parallel D$p$-branes is very different from physics of $N$ coinciding D$p$-branes.

- separated: $\rightarrow$ Abelian theory
- coinciding: $\rightarrow$ non-Abelian theory

# 1. Non-Abelian physics of multiple parallel branes

The physics of $N$ separated parallel D$p$-branes is very different from physics of $N$ coinciding D$p$-branes.

- separated: $\rightarrow$ Abelian theory
- coinciding: $\rightarrow$ non-Abelian theory

This non-Abelian character has numerous manifestations in modern string theory:

- Dielectric effect [Myers]
- Gravity duals of (confining) gauge theories [Polchinsky, Strassler]
- Enhançons [Johnson]
- Matrix models in non-trivial backgrounds [Berenstein, Maldacena, Nastase]
- Microscopic description of giant gravitons [B.J., Lozano, Rodríguez]
- ...

## Difference in degrees of freedom

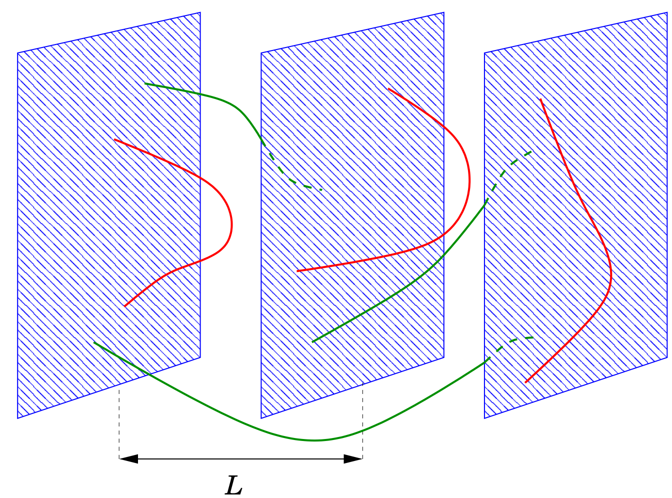
- separated: $N\ U(1)$ vector fields $V_a^I$, $(9-p)N$ scalars $X^{iI}$
  $\rightarrow$ Abelian worldvolume action
- coinciding: $1\ U(N)$ Yang-Mills vector $V_a^I$, $N$ adjoint scalars $X^{iI}$
  $\rightarrow$ non-Abelian worldvolume action

## Difference in degrees of freedom

- separated: $N\ U(1)$ vector fields $V_a^I$, $(9-p)N$ scalars $X^{iI}$
  $\rightarrow$ Abelian worldvolume action

- coinciding: $1\ U(N)$ Yang-Mills vector $V_a^I$, $N$ adjoint scalars $X^{iI}$
  $\rightarrow$ non-Abelian worldvolume action

Extra degrees of freedom come from massless strings stretched between coinciding strings: [Witten]

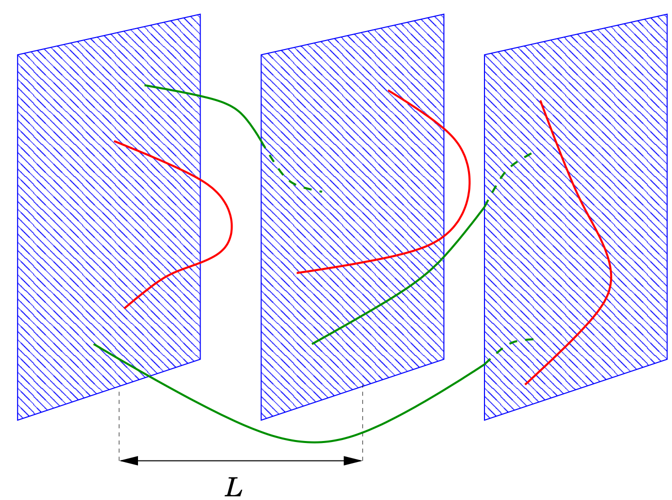



strings between same brane: $m \sim 0$
strings between different branes: $m \sim L$
As $L \rightarrow 0$:
$\Longrightarrow N + N(N-1) = N^2$ degrees of freedom
$\Longrightarrow U(1)^N \rightarrow U(N)$ gauge enhancement

## Difference in degrees of freedom

- separated: $N\ U(1)$ vector fields $V_a^I$, $(9-p)N$ scalars $X^{iI}$
  $\rightarrow$ Abelian worldvolume action
- coinciding: $1\ U(N)$ Yang-Mills vector $V_a^I$, $N$ adjoint scalars $X^{iI}$
  $\rightarrow$ non-Abelian worldvolume action

Extra degrees of freedom come from massless strings stretched between coinciding strings: [Witten]




strings between same brane: $m \sim 0$
strings between different branes: $m \sim L$
As $L \rightarrow 0$:
$\Longrightarrow N + N(N-1) = N^2$ degrees of freedom
$\Longrightarrow U(1)^N \rightarrow U(N)$ gauge enhancement

Different degrees of freedom and different symmetries
$\Longrightarrow$ different dynamics
$\Longrightarrow$ different worldvolume action

## Difference in worldvolume actions

- $X^\mu$: Abelian scalars $\longrightarrow$ Scalars in adjoint of $U(N)$

  $$\Rightarrow [X^\mu, X^\nu] \neq 0$$

## Difference in worldvolume actions

- $X^\mu$: Abelian scalars $\longrightarrow$ Scalars in adjoint of $U(N)$

$$\Rightarrow [X^\mu, X^\nu] \neq 0$$

- $V_a$: $N$ Born-Infeld vectors $\longrightarrow$ 1 Yang-Mills vector

$$F_{ab} = 2\partial_{[a} V_{b]} \longrightarrow F_{ab} = 2\partial_{[a} V_{b]} + i[V_a, V_b]$$

## Difference in worldvolume actions

- $X^\mu$: Abelian scalars $\longrightarrow$ Scalars in adjoint of $U(N)$

  $\Rightarrow [X^\mu, X^\nu] \neq 0$

- $V_a$: $N$ Born-Infeld vectors $\longrightarrow$ 1 Yang-Mills vector

  $F_{ab} = 2\partial_{[a} V_{b]} \longrightarrow F_{ab} = 2\partial_{[a} V_{b]} + i[V_a, V_b]$

- Background fields: $\Phi = \Phi(x) \longrightarrow \Phi = \Phi(X)$

  $\longrightarrow \Phi(X^\lambda) = \sum_n \frac{1}{n!} \partial_{\mu_1} ... \partial_{\mu_n} \Phi(x^\lambda)|_{x^\lambda=0} X^{\mu_1} ... X^{\mu_n}$ [Douglas][Garousi, Myers]

  $\longrightarrow$ Symmetrized trace prescription [Tseytlin]

## Difference in worldvolume actions

- $X^\mu$: Abelian scalars $\longrightarrow$ Scalars in adjoint of $U(N)$

  $\Rightarrow [X^\mu, X^\nu] \neq 0$

- $V_a$: $N$ Born-Infeld vectors $\longrightarrow$ 1 Yang-Mills vector

  $F_{ab} = 2\partial_{[a}V_{b]} \longrightarrow F_{ab} = 2\partial_{[a}V_{b]} + i[V_a, V_b]$

- Background fields: $\Phi = \Phi(x) \longrightarrow \Phi = \Phi(X)$

  $\longrightarrow \Phi(X^\lambda) = \sum_n \frac{1}{n!}\partial_{\mu_1}...\partial_{\mu_n}\Phi(x^\lambda)|_{x^\lambda=0}X^{\mu_1}...X^{\mu_n}$ [Douglas][Garousi, Myers]

  $\longrightarrow$ Symmetrized trace prescription [Tseytlin]

- $U(N)$ covariant pullbacks: $\partial_a X^\mu \longrightarrow D_a X^\mu = \partial_a X^\mu + i[V_a, X^\mu]$

  $\longrightarrow \delta V_a = D_a\chi, \qquad \delta X^i = i[\chi, X^i], \qquad \delta D_a X^i = i[\chi, D_a X^i]$ [Dorn][Hull]

## Difference in worldvolume actions

- $X^\mu$: Abelian scalars $\longrightarrow$ Scalars in adjoint of $U(N)$

  $$\Rightarrow [X^\mu, X^\nu] \neq 0$$

- $V_a$: $N$ Born-Infeld vectors $\longrightarrow$ 1 Yang-Mills vector

  $$F_{ab} = 2\partial_{[a}V_{b]} \longrightarrow F_{ab} = 2\partial_{[a}V_{b]} + i[V_a, V_b]$$

- Background fields: $\Phi = \Phi(x) \longrightarrow \Phi = \Phi(X)$

  $\longrightarrow$ $\Phi(X^\lambda) = \sum_n \frac{1}{n!}\partial_{\mu_1}...\partial_{\mu_n}\Phi(x^\lambda)|_{x^\lambda=0}X^{\mu_1}...X^{\mu_n}$ [Douglas][Garousi, Myers]

  $\longrightarrow$ Symmetrized trace prescription [Tseytlin]

- $U(N)$ covariant pullbacks: $\partial_a X^\mu \longrightarrow D_a X^\mu = \partial_a X^\mu + i[V_a, X^\mu]$

  $\longrightarrow$ $\delta V_a = D_a\chi, \qquad \delta X^i = i[\chi, X^i], \qquad \delta D_a X^i = i[\chi, D_a X^i]$ [Dorn][Hull]

- Non-Abelian couplings, proportional to $[X^\mu, X^\nu]$:

  $$\longrightarrow \mathcal{L}_{D1} \sim \mathrm{STr}\Big\{C_{\mu\nu}D_aX^\mu D_bX^\nu + i[X^\mu, X^\nu]C_{\mu\nu\rho\lambda}D_aX^\rho D_bX^\lambda\Big\}$$

  $$= \mathrm{STr}\Big\{P[C_2] + \tfrac{1}{2}iP[(\mathbf{i}_X\mathbf{i}_X)C_4]\Big\}$$

  [Taylor, van Raamsdonk][Myers]

## T-duality origin of non-Abelian couplings

T-duality in worlvolume direction $\underline{x}$:

$$\text{D}p\text{-brane} \quad \longrightarrow \quad \text{D}(p-1)\text{-brane}$$
$$(\hat{V}_{\hat{a}}, Y^i) \quad \longrightarrow \quad (V_a, X^{\hat{\imath}})$$

## T-duality origin of non-Abelian couplings

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

Matching of degrees of freedom: [Bergshoeff, de Roo]

$$\hat{V}_a \longrightarrow V_a, \qquad Y^i \longrightarrow X^i,$$

$$\hat{V}_x \longrightarrow X^{\underline{x}}, \qquad Y^{\underline{x}} = \sigma^x.$$

## T-duality origin of non-Abelian couplings

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \mathrm{D}p\text{-brane} & \longrightarrow & \mathrm{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{i}}) \end{array}$$

Matching of degrees of freedom: [Bergshoeff, de Roo]

$$\hat{V}_a \longrightarrow V_a, \qquad Y^i \longrightarrow X^i,$$

$$\hat{V}_x \longrightarrow X^{\underline{x}}, \qquad Y^{\underline{x}} = \sigma^x.$$

Field strengths, Abelian case:

$$\partial_a Y^i \longrightarrow \partial_a X^i, \qquad \partial_x Y^i = 0, \qquad \partial_{\hat{a}} Y^{\underline{x}} = \delta_{\hat{a}}^{\underline{x}},$$

$$\hat{F}_{ax} \longrightarrow \partial_a X^{\underline{x}}, \qquad \hat{F}_{ab} \longrightarrow F_{ab}.$$

## T-duality origin of non-Abelian couplings

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

Matching of degrees of freedom: [Bergshoeff, de Roo]

$$\hat{V}_a \longrightarrow V_a, \qquad Y^i \longrightarrow X^i,$$

$$\hat{V}_x \longrightarrow X^{\underline{x}}, \qquad Y^{\underline{x}} = \sigma^x.$$

Field strengths, Abelian case:

$$\partial_a Y^i \longrightarrow \partial_a X^i, \qquad \partial_x Y^i = 0, \qquad \partial_{\hat{a}} Y^{\underline{x}} = \delta_{\hat{a}}^{\underline{x}},$$

$$\hat{F}_{ax} \longrightarrow \partial_a X^{\underline{x}}, \qquad \hat{F}_{ab} \longrightarrow F_{ab}.$$

Field strengths, non-Abelian case: [Myers]

$$\hat{D}_a Y^i \longrightarrow D_a X^i, \qquad \hat{D}_x Y^i = i[V_x, Y^i] \longrightarrow i[X^{\underline{x}}, X^i], \qquad \partial_{\hat{a}} Y^{\underline{x}} = \delta_{\hat{a}}^{\underline{x}},$$

$$\hat{F}_{ax} \longrightarrow D_a X^{\underline{x}}, \qquad \hat{F}_{ab} \longrightarrow F_{ab}.$$

Dielectric terms:

$$\begin{aligned} C_{\hat{\mu}\hat{\nu}} \hat{D}_a Y^{\hat{\mu}} \hat{D}_x Y^{\hat{\nu}} &= C_{\mu\underline{x}} \hat{D}_a Y^{\mu} \hat{D}_x Y^{\underline{x}} + C_{\mu\nu} \hat{D}_a Y^{\mu} \hat{D}_x Y^{\nu} \\ &\rightarrow C_{\hat{\mu}} D_a X^{\hat{\mu}} + i[X^{\underline{x}}, X^{\nu}] C_{\mu\nu\underline{x}} D_a X^{\hat{\mu}} \end{aligned}$$

Dielectric terms:

$$C_{\hat\mu\hat\nu}\hat D_a Y^{\hat\mu}\hat D_x Y^{\hat\nu} = C_{\mu\underline{x}}\hat D_a Y^{\mu}\hat D_x Y^{\underline{x}} + C_{\mu\nu}\hat D_a Y^{\mu}\hat D_x Y^{\nu}$$
$$\rightarrow C_{\hat\mu}D_a X^{\hat\mu} + i[X^{\underline{x}}, X^{\nu}]C_{\mu\nu\underline{x}}D_a X^{\hat\mu}$$

Non-Abelian action:

- Born-Infeld part : highly non-trivial problem
- Chern-Simons part: Concentrate here

$$S_{\mathrm{D}p} = T_p\int \mathrm{STr}\Big\{P\Big[e^{(\mathrm{i}_X\mathrm{i}_X)}(Ce^B)\Big]\,e^F\Big\}$$
$$= T_p\int \mathrm{STr}\Big\{P\Big[C_p + C_{p-2}(F+B) + \ldots\Big]$$
$$+P\Big[(\mathrm{i}_X\mathrm{i}_X)\Big(C_{p+2} + C_p B + \ldots\Big)\Big]$$
$$+P\Big[(\mathrm{i}_X\mathrm{i}_X)\Big(C_p + C_{p-2} B + \ldots\Big)\Big]F$$
$$+P\Big[(\mathrm{i}_X\mathrm{i}_X)^2\Big(C_{p+4} + C_{p+2} B + \ldots\Big)\Big] + \ldots\Big\}$$

## 2. Problems with background gauge invariance

R-R gauge transformation: $\delta C_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]}$

$$\delta C_{\mu\nu\rho\lambda} = \partial_{[\mu}\Lambda_{\nu\rho\lambda]}$$

$$\delta\Big\{C_{\mu\nu}D_aX^\mu D_bX^\nu\Big\} = \Big\{\partial_{[\mu}\Lambda_{\nu]}D_aX^\mu D_bX^\nu\Big\}$$

$$\neq \ \partial_{[a}\Big\{\Lambda_{\nu]}D_bX^\nu\Big\}$$

$$\delta\Big\{[X^\nu, X^\mu]C_{\mu\nu\rho\lambda}D_aX^\rho D_bX^\lambda\Big\} = \Big\{[X^\nu, X^\mu]\partial_{[\mu}\Lambda_{\nu\rho\lambda]}D_aX^\mu D_bX^\nu\Big\}$$

$$\neq \ \partial_{[a}\Big\{[X^\nu, X^\mu]\Lambda_{\nu\rho\lambda]}D_bX^\nu\Big\}$$

# 2. Problems with background gauge invariance

R-R gauge transformation: $\delta C_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]}$

$$\delta C_{\mu\nu\rho\lambda} = \partial_{[\mu}\Lambda_{\nu\rho\lambda]}$$

$$\delta\Big\{C_{\mu\nu}D_aX^\mu D_bX^\nu\Big\} = \Big\{\partial_{[\mu}\Lambda_{\nu]}D_aX^\mu D_bX^\nu\Big\}$$

$$\neq \;\partial_{[a}\Big\{\Lambda_{\nu]}D_bX^\nu\Big\}$$

$$\delta\Big\{[X^\nu, X^\mu]C_{\mu\nu\rho\lambda}D_aX^\rho D_bX^\lambda\Big\} = \Big\{[X^\nu, X^\mu]\partial_{[\mu}\Lambda_{\nu\rho\lambda]}D_aX^\mu D_bX^\nu\Big\}$$

$$\neq \;\partial_{[a}\Big\{[X^\nu, X^\mu]\Lambda_{\nu\rho\lambda]}D_bX^\nu\Big\}$$

NS-NS gauge transformations: $\delta B_{\mu\nu} = \partial_{[\mu}\Sigma_{\nu]}$

$$\delta V_a = -\Sigma_\mu D_aX^\mu$$

$$\delta\mathcal{F}_{ab} = \delta\Big(F_{ab} + P[B]_{ab}\Big) = 0, \qquad \delta([X^\mu, X^\nu]B_{\mu\nu}) = [X^\mu, X^\nu]\partial_{[\mu}\Sigma_{\nu]}$$

## 2. Problems with background gauge invariance

R-R gauge transformation:

$$\delta C_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]}$$
$$\delta C_{\mu\nu\rho\lambda} = \partial_{[\mu}\Lambda_{\nu\rho\lambda]}$$

$$\delta\Big\{C_{\mu\nu}D_aX^\mu D_bX^\nu\Big\} = \Big\{\partial_{[\mu}\Lambda_{\nu]}D_aX^\mu D_bX^\nu\Big\}$$
$$\neq \ \partial_{[a}\Big\{\Lambda_{\nu]}D_bX^\nu\Big\}$$
$$\delta\Big\{[X^\nu, X^\mu]C_{\mu\nu\rho\lambda}D_aX^\rho D_bX^\lambda\Big\} = \Big\{[X^\nu, X^\mu]\partial_{[\mu}\Lambda_{\nu\rho\lambda]}D_aX^\mu D_bX^\nu\Big\}$$
$$\neq \ \partial_{[a}\Big\{[X^\nu, X^\mu]\Lambda_{\nu\rho\lambda]}D_bX^\nu\Big\}$$

NS-NS gauge transformations:

$$\delta B_{\mu\nu} = \partial_{[\mu}\Sigma_{\nu]}$$
$$\delta V_a = -\Sigma_\mu D_aX^\mu$$

$$\delta\mathcal{F}_{ab} = \delta\Big(F_{ab} + P[B]_{ab}\Big) = 0, \qquad \delta([X^\mu, X^\nu]B_{\mu\nu}) = [X^\mu, X^\nu]\partial_{[\mu}\Sigma_{\nu]}$$

$\Longrightarrow$ Naive substitution does not give a total derivative !

## R-R gauge transformations

First addressed by [Ciocarlie] in 2001.

$\longrightarrow$ Very complex, incomplete

Systematic appraoch by [Adam, Gheerardyn, B.J., Lozano]:

## R-R gauge transformations

First addressed by [Ciocarlie] in 2001.

$\longrightarrow$ Very complex, incomplete

Systematic appraoch by [Adam, Gheerardyn, B.J., Lozano]:

$P[C_p]$ is gauge invariant under $\delta C_p = \partial\Lambda_{p-1}$ iff

- total derivative: $\delta\Big\{P[C_p]\Big\} = \partial_a\Big\{...\Big\}$
- $\delta\Big\{P[C_p]\Big\}$ is a scalar under $U(N)$
- for $[X, X] = 0$: $\delta\Big\{P[C_p]\Big\} = \Big\{P[\partial\Lambda_{p-1}]\Big\}$

## R-R gauge transformations

First addressed by [Ciocarlie] in 2001.

$\longrightarrow$ Very complex, incomplete

Systematic appraoch by [Adam, Gheerardyn, B.J., Lozano]:

$P[C_p]$ is gauge invariant under $\delta C_p = \partial\Lambda_{p-1}$ iff

- total derivative: $\delta\Big\{P[C_p]\Big\} = \partial_a\Big\{...\Big\}$
- $\delta\Big\{P[C_p]\Big\}$ is a scalar under $U(N)$
- for $[X,X]=0$: $\delta\Big\{P[C_p]\Big\} = \Big\{P[\partial\Lambda_{p-1}]\Big\}$

$\longrightarrow$ Modify way to implement gauge transformations in non-Abelian action

Definition:

- $$\delta\Big\{P[C_p]\Big\} \equiv \partial\Big\{P[\Lambda_{p-1}]\Big\} = \Big\{DP[\Lambda_{p-1}]\Big\}$$
$$= \Big\{P[\partial\Lambda_{p-1}] + P[\Lambda_{p-1}]\tfrac{i}{2}[F, X]\Big\}$$

Definition:

- $\delta\Big\{P[C_p]\Big\} \equiv \partial\Big\{P[\Lambda_{p-1}]\Big\} = \Big\{DP[\Lambda_{p-1}]\Big\}$
  $= \Big\{P[\partial\Lambda_{p-1}] + P[\Lambda_{p-1}]\tfrac{i}{2}[F, X]\Big\}$

- $\Big\{\delta P[(\mathbf{i}_X\mathbf{i}_X)^r C_p]\Big\} = \Big\{DP[(\mathbf{i}_X\mathbf{i}_X)^r\Lambda_{p-1}]\Big\}$

Definition:

- $\delta\Big\{P[C_p]\Big\} \equiv \partial\Big\{P[\Lambda_{p-1}]\Big\} = \Big\{DP[\Lambda_{p-1}]\Big\}$
$$= \Big\{P[\partial\Lambda_{p-1}] + P[\Lambda_{p-1}]\tfrac{i}{2}[F, X]\Big\}$$

- $\Big\{\delta P[(\mathrm{i}_X\mathrm{i}_X)^r C_p]\Big\} = \Big\{DP[(\mathrm{i}_X\mathrm{i}_X)^r\Lambda_{p-1}]\Big\}$

- general case $\delta C_p = \partial\Lambda e^B$:

$$\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)C]\Big\} = \Big\{DP[\mathrm{i}_X\mathrm{i}_X\Lambda]P[e^B] + DP[\mathrm{i}_X\Lambda]P[\mathrm{i}_X e^B] + DP[\Lambda]P[\mathrm{i}_X\mathrm{i}_X e^B]\Big\}$$

$\longrightarrow$ $Ce^B$ is invariant under $\delta C_p = \partial\Lambda e^B$ (Abelian case)

Definition:

- $\delta\Big\{P[C_p]\Big\} \equiv \partial\Big\{P[\Lambda_{p-1}]\Big\} = \Big\{DP[\Lambda_{p-1}]\Big\}$
$$= \Big\{P[\partial\Lambda_{p-1}] + P[\Lambda_{p-1}]\tfrac{i}{2}[F, X]\Big\}$$
- $\Big\{\delta P[(\mathrm{i}_X\mathrm{i}_X)^r C_p]\Big\} = \Big\{DP[(\mathrm{i}_X\mathrm{i}_X)^r\Lambda_{p-1}]\Big\}$
- general case $\delta C_p = \partial\Lambda e^B$:

$$\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)C]\Big\} = \Big\{DP[\mathrm{i}_X\mathrm{i}_X\Lambda]P[e^B] + DP[\mathrm{i}_X\Lambda]P[\mathrm{i}_X e^B] + DP[\Lambda]P[\mathrm{i}_X\mathrm{i}_X e^B]\Big\}$$

$\longrightarrow$ $Ce^B$ is invariant under $\delta C_p = \partial\Lambda e^B$ (Abelian case)

$\longrightarrow$ $P[(\mathrm{i}_X\mathrm{i}_X)^r(Ce^B)]$ is invariant under $\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)^t C]\Big\}$

Definition:

- $\delta\Big\{P[C_p]\Big\} \equiv \partial\Big\{P[\Lambda_{p-1}]\Big\} = \Big\{DP[\Lambda_{p-1}]\Big\}$
$$= \Big\{P[\partial\Lambda_{p-1}] + P[\Lambda_{p-1}]\tfrac{i}{2}[F,X]\Big\}$$

- $\Big\{\delta P[(\mathrm{i}_X\mathrm{i}_X)^r C_p]\Big\} = \Big\{DP[(\mathrm{i}_X\mathrm{i}_X)^r\Lambda_{p-1}]\Big\}$

- general case $\delta C_p = \partial\Lambda e^B$:

$$\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)C]\Big\} = \Big\{DP[\mathrm{i}_X\mathrm{i}_X\Lambda]P[e^B] + DP[\mathrm{i}_X\Lambda]P[\mathrm{i}_X e^B] + DP[\Lambda]P[\mathrm{i}_X\mathrm{i}_X e^B]\Big\}$$

$\longrightarrow$ $Ce^B$ is invariant under $\delta C_p = \partial\Lambda e^B$ (Abelian case)

$\longrightarrow$ $P[(\mathrm{i}_X\mathrm{i}_X)^r(Ce^B)]$ is invariant under $\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)^t C]\Big\}$

$\longrightarrow$ $S = T_p\int\Big\{P\Big[e^{(\mathrm{i}_X\mathrm{i}_X)}(Ce^B)\Big]\ e^F\Big\}$ is invariant under $\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)^s C]\Big\}$

# 3. Problems with NS-NS gauge transformations

$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]}, \qquad \delta V_a = -\Sigma_\mu D_a X^\mu$

$\Longrightarrow \delta P[B] = 2DP[\Sigma], \qquad \delta F_{ab} = -2DP[\Sigma]$

$\Longrightarrow \delta \mathcal{F} = \delta\Big(F + P[B]\Big) = 0.$

# 3. Problems with NS-NS gauge transformations

$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]}$, $\qquad$ $\delta V_a = -\Sigma_\mu D_a X^\mu$

$\Longrightarrow \delta P[B] = 2DP[\Sigma]$, $\qquad$ $\delta F_{ab} = -2DP[\Sigma]$

$\Longrightarrow \delta \mathcal{F} = \delta\Big(F + P[B]\Big) = 0.$

Rewrite D$p$-brane action: [Adam, Gheerardyn, B.J., Lozano]

$$(\mathrm{i}_X \mathrm{i}_X)\Big(Ce^B\Big) = (\mathrm{i}_X\mathrm{i}_X C)e^B + (\mathrm{i}_X C)(\mathrm{i}_X B)e^B + C(\mathrm{i}_X B)(\mathrm{i}_X B)e^B + C(\mathrm{i}_X\mathrm{i}_X B)e^B$$

# 3. Problems with NS-NS gauge transformations

$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]}, \qquad \delta V_a = -\Sigma_\mu D_a X^\mu$

$\Longrightarrow \delta P[B] = 2DP[\Sigma], \qquad \delta F_{ab} = -2DP[\Sigma]$

$\Longrightarrow \delta\mathcal{F} = \delta\Big(F + P[B]\Big) = 0.$

Rewrite D$p$-brane action: [Adam, Gheerardyn, B.J., Lozano]

$$
\begin{aligned}
(\mathrm{i}_X \mathrm{i}_X)\Big(Ce^B\Big) &= (\mathrm{i}_X\mathrm{i}_X C)e^B + (\mathrm{i}_X C)(\mathrm{i}_X B)e^B \\
&\quad + C(\mathrm{i}_X B)(\mathrm{i}_X B)e^B + C(\mathrm{i}_X\mathrm{i}_X B)e^B
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow S &= T_p \int \Big\{ P\Big[(\mathrm{i}_X\mathrm{i}_X C)e^{\mathcal{F}} + (\mathrm{i}_X C)(\mathrm{i}_X B)e^{\mathcal{F}} \\
&\qquad + C(\mathrm{i}_X B)(\mathrm{i}_X B)e^{\mathcal{F}} + C(\mathrm{i}_X\mathrm{i}_X B)e^{\mathcal{F}}\Big]\Big\}
\end{aligned}
$$

What happens to terms of the type $(\mathrm{i}_X B)$ and $(\mathrm{i}_X\mathrm{i}_X B)$?

$\longrightarrow$ Extra non-Abelian variations which have not been taken in account?

## T-duality on gauge transformations

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_{a}, X^{\hat{i}}) \end{array}$$

- Matching of degrees of freedom:

$$\hat{V}_{a} \longrightarrow V_{a}, \qquad\qquad Y^{i} \longrightarrow X^{i},$$

$$\hat{V}_{x} \longrightarrow X^{\underline{x}}, \qquad\qquad Y^{\underline{x}} = \sigma^{x}.$$

## T-duality on gauge transformations

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

- Matching of degrees of freedom:

$$\hat{V}_a \longrightarrow V_a, \qquad Y^i \longrightarrow X^i,$$

$$\hat{V}_x \longrightarrow X^{\underline{x}}, \qquad Y^{\underline{x}} = \sigma^x.$$

- Role played by fields in D$p$-brane action is the same as role of fields in D$(p-1)$-brane action.

  Fields in both actions transform in the same way under:

  - Worldvolume general coordinate transformations $\zeta^a$
  - $U(1)$ or $U(N)$ transformations $\chi$
  - NS-NS transformations $\Sigma$
  - Target space general coordinate transformations

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} \, + \, \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} \, + \, \partial_{\hat{a}} \hat{\chi} \, - \, \Sigma_{\hat{\mu}} \partial_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \mathrm{D}p\text{-brane} & \longrightarrow & \mathrm{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_{a}, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} \; + \; \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} \; + \; \partial_{\hat{a}} \hat{\chi} \; - \; \Sigma_{\hat{\mu}} \partial_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_{a} \longrightarrow V_{a}$$

$$\Longrightarrow \delta \hat{V}_{a} \; = \; \hat{\zeta}^{b} \partial_{b} \hat{V}_{a} \; + \; \partial_{a} \hat{\zeta}^{b} \hat{V}_{b} \; + \; \partial_{a} \hat{\zeta}^{x} \hat{V}_{x} \; + \; \partial_{a} \hat{\chi} \; - \; \Sigma_{\hat{\mu}} \partial_{a} Y^{\hat{\mu}}$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \mathrm{D}p\text{-brane} & \longrightarrow & \mathrm{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} \ + \ \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} \ + \ \partial_{\hat{a}} \hat{\chi} \ - \ \Sigma_{\hat{\mu}} \partial_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

$$\Longrightarrow \delta \hat{V}_a \ = \ \hat{\zeta}^b \partial_b \hat{V}_a \ + \ \partial_a \hat{\zeta}^b \hat{V}_b \ + \ \partial_a \hat{\zeta}^x \hat{V}_x \ + \ \partial_a \hat{\chi} \ - \ \Sigma_{\hat{\mu}} \partial_a Y^{\hat{\mu}}$$

$$\longrightarrow \ \zeta^b \partial_b V_a \ + \ \partial_a \zeta^b V_b \ + \ \partial_a \Sigma_{\underline{x}} X_{\underline{x}} \ + \ \partial_a \tilde{\chi} \ - \ \Sigma_{\mu} \partial_a X^{\mu}$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} + \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} + \partial_{\hat{a}}\hat{\chi} - \Sigma_{\hat{\mu}}\partial_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

$$\Longrightarrow \delta\hat{V}_a = \hat{\zeta}^b\partial_b\hat{V}_a + \partial_a\hat{\zeta}^b\hat{V}_b + \partial_a\hat{\zeta}^x\hat{V}_x + \partial_a\hat{\chi} - \Sigma_{\hat{\mu}}\partial_a Y^{\hat{\mu}}$$

$$\longrightarrow \zeta^b\partial_b V_a + \partial_a\zeta^b V_b + \partial_a\Sigma_{\underline{x}}X_{\underline{x}} + \partial_a\tilde{\chi} - \Sigma_\mu\partial_a X^\mu$$

Field redefinition: $\tilde{\chi} = \chi - \Sigma_{\underline{x}}X^{\underline{x}}$

$$= \zeta^b\partial_b V_a + \partial_a\zeta^b V_b + \partial_a\chi - \Sigma_{\hat{\mu}}\partial_a X^{\hat{\mu}}$$

$$= \delta V_a$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_{a}, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} + \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} + \partial_{\hat{a}} \hat{\chi} - \Sigma_{\hat{\mu}} \partial_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_{x} \rightarrow X^{\underline{x}}$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\text{D}p\text{-brane} \longrightarrow \text{D}(p-1)\text{-brane}$$

$$(\hat{V}_{\hat{a}}, Y^{i}) \longrightarrow (V_a, X^{\hat{i}})$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} + \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} + \partial_{\hat{a}}\hat{\chi} - \Sigma_{\hat{\mu}}\partial_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_x \rightarrow X^{\underline{x}}$$

$$\Longrightarrow \delta\hat{V}_x = \hat{\zeta}^{b}\partial_b\hat{V}_x - \Sigma_{\underline{x}}\partial_x Y^{\underline{x}}$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_{a}, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} + \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} + \partial_{\hat{a}} \hat{\chi} - \Sigma_{\hat{\mu}} \partial_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_x \rightarrow X^{\underline{x}}$$

$$\Longrightarrow \delta \hat{V}_x = \hat{\zeta}^b \partial_b \hat{V}_x - \Sigma_{\underline{x}} \partial_x Y^{\underline{x}}$$

$$\longrightarrow \zeta^b \partial_b X^{\underline{x}} - \xi^{\underline{x}}$$

$$= \delta X^{\underline{x}}$$

$\longrightarrow$ $X^{\underline{x}}$ behaves as a scalar in worldvolume and coordinate in target space.

$\longrightarrow$ $\delta X^{\hat{\mu}} = \zeta^b \partial_b X^{\hat{\mu}} - \xi^{\hat{\mu}}$?

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\text{D}p\text{-brane} \longrightarrow \text{D}(p-1)\text{-brane}$$
$$(\hat{V}_{\hat{a}}, Y^i) \longrightarrow (V_a, X^{\hat{\imath}})$$

T-dual variation of embedding scalars:

$$Y^{\hat{\mu}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} Y^{\hat{\mu}} \; - \; \xi^{\hat{\mu}}$$

$$Y^{\mu} \longrightarrow X^{\mu}$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \mathrm{D}p\text{-brane} & \longrightarrow & \mathrm{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_{a}, X^{\hat{\imath}}) \end{array}$$

T-dual variation of embedding scalars:

$$Y^{\hat{\mu}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} Y^{\hat{\mu}} \; - \; \xi^{\hat{\mu}}$$

$$Y^{\mu} \longrightarrow X^{\mu}$$

$$\Longrightarrow \delta Y^{\mu} \; = \; \hat{\zeta}^{b} \partial_{b} Y^{\mu} \; - \; \xi^{\mu}$$

## T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of embedding scalars:

$$Y^{\hat{\mu}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} Y^{\hat{\mu}} - \xi^{\hat{\mu}}$$

$$Y^{\mu} \longrightarrow X^{\mu}$$

$$\Longrightarrow \delta Y^{\mu} = \hat{\zeta}^{b} \partial_{b} Y^{\mu} - \xi^{\mu}$$

$$\longrightarrow \zeta^{b} \partial_{b} X^{\mu} - \xi^{\mu}$$

$$= \delta X^{\mu}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} + \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} + \hat{D}_{\hat{a}}\hat{\chi} - \Sigma_{\hat{\mu}}\hat{D}_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} + \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} + \hat{D}_{\hat{a}}\hat{\chi} - \Sigma_{\hat{\mu}}\hat{D}_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

$$\Longrightarrow \delta\hat{V}_a = \hat{\zeta}^b\partial_b\hat{V}_a + \partial_a\hat{\zeta}^b\hat{V}_b + \partial_a\hat{\zeta}^x\hat{V}_x + \hat{D}_a\hat{\chi} - \Sigma_{\hat{\mu}}\hat{D}_aY^{\hat{\mu}}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_a, X^{\hat{i}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} \; + \; \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} \; + \; \hat{D}_{\hat{a}} \hat{\chi} \; - \; \Sigma_{\hat{\mu}} \hat{D}_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

$$\Longrightarrow \delta \hat{V}_a \; = \; \hat{\zeta}^b \partial_b \hat{V}_a \; + \; \partial_a \hat{\zeta}^b \hat{V}_b \; + \; \partial_a \hat{\zeta}^x \hat{V}_x \; + \; \hat{D}_a \hat{\chi} \; - \; \Sigma_{\hat{\mu}} \hat{D}_a Y^{\hat{\mu}}$$

$$\longrightarrow \; \zeta^b \partial_b V_a \; + \; \partial_a \zeta^b V_b \; + \; \partial_a \Sigma_{\underline{x}} X_{\underline{x}} \; + \; D_a \tilde{\chi} \; - \; \Sigma_{\mu} D_a X^{\mu}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} + \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} + \hat{D}_{\hat{a}}\hat{\chi} - \Sigma_{\hat{\mu}}\hat{D}_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_a \longrightarrow V_a$$

$$\Longrightarrow \delta\hat{V}_a = \hat{\zeta}^b\partial_b\hat{V}_a + \partial_a\hat{\zeta}^b\hat{V}_b + \partial_a\hat{\zeta}^x\hat{V}_x + \hat{D}_a\hat{\chi} - \Sigma_{\hat{\mu}}\hat{D}_aY^{\hat{\mu}}$$

$$\longrightarrow \zeta^b\partial_bV_a + \partial_a\zeta^bV_b + \partial_a\Sigma_{\underline{x}}X_{\underline{x}} + D_a\tilde{\chi} - \Sigma_\mu D_aX^\mu$$

Field redefinition: $\tilde{\chi} = \chi - \Sigma_{\underline{x}}X^{\underline{x}}$

$$= \zeta^b\partial_bV_a + \partial_a\zeta^bV_b + D_a\tilde{\chi} - \Sigma_{\hat{\mu}}D_aX^{\hat{\mu}}$$

$$= \delta V_a$$

$\longrightarrow$ $V - a$ behaves as a vector in worldvolume, $U(N)$ Yang-Mills vector and shift under NS-NS transf .

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta \hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}} \partial_{\hat{b}} \hat{V}_{\hat{a}} \ + \ \partial_{\hat{a}} \hat{\zeta}^{\hat{b}} \hat{V}_{\hat{b}} \ + \ \hat{D}_{\hat{a}} \hat{\chi} \ - \ \Sigma_{\hat{\mu}} \hat{D}_{\hat{a}} Y^{\hat{\mu}}$$

$$\hat{V}_x \to X^{\underline{x}}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \mathrm{D}p\text{-brane} & \longrightarrow & \mathrm{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} \; + \; \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} \; + \; \hat{D}_{\hat{a}}\hat{\chi} \; - \; \Sigma_{\hat{\mu}}\hat{D}_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_x \rightarrow X^{\underline{x}}$$

$$\Longrightarrow \delta\hat{V}_x \; = \; \hat{\zeta}^{b}\partial_b\hat{V}_x \; + \; i[V_x, \chi] \; - \; \Sigma_\mu\hat{D}_x Y^{\mu} \; - \; \Sigma_{\underline{x}} D_x Y^{\underline{x}}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of BI vector:

$$\delta\hat{V}_{\hat{a}} = \hat{\zeta}^{\hat{b}}\partial_{\hat{b}}\hat{V}_{\hat{a}} + \partial_{\hat{a}}\hat{\zeta}^{\hat{b}}\hat{V}_{\hat{b}} + \hat{D}_{\hat{a}}\hat{\chi} - \Sigma_{\hat{\mu}}\hat{D}_{\hat{a}}Y^{\hat{\mu}}$$

$$\hat{V}_x \rightarrow X^{\underline{x}}$$

$$\Longrightarrow \delta\hat{V}_x = \hat{\zeta}^b\partial_b\hat{V}_x + i[V_x, \chi] - \Sigma_\mu\hat{D}_x Y^\mu - \Sigma_{\underline{x}} D_x Y^{\underline{x}}$$

$$\longrightarrow \zeta^b\partial_b\hat{V}_x + i[X^{\underline{x}}, \chi] - i\Sigma_\mu[X^{\underline{x}}, X^\mu] - \xi^{\underline{x}}$$

$$= \delta X^{\underline{x}}$$

$\longrightarrow$ $X^{\underline{x}}$ behaves as a scalar in worldvolume, adjoint scalar under $U(N)$, coordinate in target space and has extra $\Sigma$ symmetry.

$$\longrightarrow \delta X^{\hat{\mu}} = \zeta^b\partial_b X^{\hat{\mu}} + i[X^\mu, \chi] - \Sigma_\rho[X^\mu, X^\rho] - \xi^{\hat{\mu}}?$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of embedding scalars:

$$\delta Y^{\hat{\mu}} = \hat{\zeta}^b \partial_b Y^{\hat{\mu}} + i[Y^{\hat{\mu}}, \chi] - i\Sigma_{\hat{\rho}}[Y^{\hat{\mu}}, Y^{\hat{\rho}}] - \xi^{\hat{\mu}}$$

$$Y^{\mu} \longrightarrow X^{\mu}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{\imath}}) \end{array}$$

T-dual variation of embedding scalars:

$$\delta Y^{\hat{\mu}} = \hat{\zeta}^b \partial_b Y^{\hat{\mu}} + i[Y^{\hat{\mu}}, \chi] - i\Sigma_{\hat{\rho}}[Y^{\hat{\mu}}, Y^{\hat{\rho}}] - \xi^{\hat{\mu}}$$

$$Y^{\mu} \longrightarrow X^{\mu}$$

$$\Longrightarrow \delta Y^{\mu} = \hat{\zeta}^b \partial_b Y^{\mu} + i[Y^{\mu}, \chi] - i\Sigma_{\hat{\rho}}[Y^{\mu}, Y^{\hat{\rho}}] - \xi^{\mu}$$

## T-duality on gauge transformations: non-Abelian case

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \mathrm{D}p\text{-brane} & \longrightarrow & \mathrm{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^{i}) & \longrightarrow & (V_{a}, X^{\hat{\imath}}) \end{array}$$

T-dual variation of embedding scalars:

$$\delta Y^{\hat{\mu}} = \hat{\zeta}^{b}\partial_{b}Y^{\hat{\mu}} + \ i[Y^{\hat{\mu}}, \chi] \ - \ i\Sigma_{\hat{\rho}}[Y^{\hat{\mu}}, Y^{\hat{\rho}}] \ - \ \xi^{\hat{\mu}}$$

$$Y^{\mu} \longrightarrow X^{\mu}$$

$$\Longrightarrow \delta Y^{\mu} = \hat{\zeta}^{b}\partial_{b}Y^{\mu} + \ i[Y^{\mu}, \chi] \ - \ i\Sigma_{\hat{\rho}}[Y^{\mu}, Y^{\hat{\rho}}] \ - \ \xi^{\mu}$$

$$\longrightarrow \ \zeta^{b}\partial_{b}X^{\mu} + \ i[X^{\mu}, \chi] \ - \ i\Sigma_{\hat{\rho}}[X^{\mu}, X^{\hat{\rho}}] \ - \ \xi^{\mu}$$

$$= \ \delta X^{\mu}$$

$\longrightarrow X^{\mu}$ has extra non-Abelian $\Sigma$ variations

$$\delta X^{\mu} = i\Sigma_{\rho}[X^{\rho}, X^{\mu}]$$

## NS-NS transformations of the fields

NS-NS gauge invariance much more complicated,
as everything starts transforming:

$$
\begin{aligned}
\delta F_{ab} &= i[X^\rho, \Sigma_\rho F_{ab}] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]F_{ab} - 2\partial_{[\sigma}\Sigma_{\rho]}D_{[a}X^\sigma D_{b]}X^\rho, \\
\delta D_a X^\mu &= i[X^\rho, \Sigma_\rho D_a X^\mu] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]D_a X^\mu \\
&\qquad + 2i\partial_{[\sigma}\Sigma_{\rho]}D_a X^\sigma [X^\rho, X^\mu], \\
\delta[X^\mu, X^\nu] &= i[X^\rho, \Sigma_\rho [X^\mu, X^\nu]] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho][X^\mu, X^\nu] \\
&\qquad - 2i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\mu][X^\rho, X^\nu], \\
\delta\Phi(X) &= i[X^\rho, \Sigma_\rho \Phi(X)] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]\Phi(X).
\end{aligned}
$$

## NS-NS transformations of the fields

NS-NS gauge invariance much more complicated,
as everything starts transforming:

$$\begin{aligned}
\delta F_{ab} &= i[X^\rho, \Sigma_\rho F_{ab}] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]F_{ab} - 2\partial_{[\sigma}\Sigma_{\rho]}D_{[a}X^\sigma D_{b]}X^\rho, \\
\delta D_a X^\mu &= i[X^\rho, \Sigma_\rho D_a X^\mu] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]D_a X^\mu \\
&\quad + 2i\partial_{[\sigma}\Sigma_{\rho]}D_a X^\sigma[X^\rho, X^\mu], \\
\delta[X^\mu, X^\nu] &= i[X^\rho, \Sigma_\rho[X^\mu, X^\nu]] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho][X^\mu, X^\nu] \\
&\quad - 2i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\mu][X^\rho, X^\nu], \\
\delta\Phi(X) &= i[X^\rho, \Sigma_\rho\Phi(X)] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]\Phi(X).
\end{aligned}$$

General rule:

$$\delta Z = i[X^\rho, \Sigma_\rho Z] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]Z + \text{possible correction terms}$$

## NS-NS transformations of the fields

NS-NS gauge invariance much more complicated,
as everything starts transforming:

$$
\begin{aligned}
\delta F_{ab} &= i[X^\rho, \Sigma_\rho F_{ab}] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]F_{ab} - 2\partial_{[\sigma}\Sigma_{\rho]}D_{[a}X^\sigma D_{b]}X^\rho, \\
\delta D_a X^\mu &= i[X^\rho, \Sigma_\rho D_a X^\mu] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]D_a X^\mu \\
&\qquad + 2i\partial_{[\sigma}\Sigma_{\rho]}D_a X^\sigma[X^\rho, X^\mu], \\
\delta[X^\mu, X^\nu] &= i[X^\rho, \Sigma_\rho[X^\mu, X^\nu]] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho][X^\mu, X^\nu] \\
&\qquad - 2i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\mu][X^\rho, X^\nu], \\
\delta\Phi(X) &= i[X^\rho, \Sigma_\rho\Phi(X)] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]\Phi(X).
\end{aligned}
$$

General rule:

$$
\delta Z = i[X^\rho, \Sigma_\rho Z] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho]Z + \text{possible correction terms}
$$

$$
\begin{aligned}
\delta(Z_1 Z_2) &= i[X^\rho, \Sigma_\rho(Z_1 Z_2)] + i\partial_{[\sigma}\Sigma_{\rho]}[X^\sigma, X^\rho](Z_1 Z_2) \\
&\qquad + \text{possible } Z_1 \text{ correction terms} + \text{possible } Z_2 \text{ correction terms}
\end{aligned}
$$

## Invariance of the action

Typical term: $\mathcal{L} \sim \Big\{ P\Big[ (\mathrm{i}_X \mathrm{i}_X)^r \Big( C e^{B} \Big) \Big] e^{F} \Big\}$

## Invariance of the action

Typical term: $\mathcal{L} \; \sim \; \Big\{ P\Big[ (\mathrm{i}_X \mathrm{i}_X)^r \Big( C e^B \Big) \Big] e^F \Big\}$

$$\Longrightarrow \delta\mathcal{L} \sim \Big\{ i[X^\rho, \Sigma_\rho \mathcal{L}] \; + \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^r \Big( C e^B \partial\Sigma \Big) \Big] e^F$$
$$- \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^r \Big( C e^B \Big) \partial\Sigma \Big] e^F \; - \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^{r+1} \Big( C e^B \partial\Sigma \Big) \Big] e^F$$
$$+ \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^{r+1} \Big( C e^B \Big) \partial\Sigma \Big] e^F \Big\}.$$

## Invariance of the action

Typical term: $\mathcal{L} \sim \left\{ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^r \Big( C e^B \Big) \Big] e^F \right\}$

$$\Longrightarrow \delta\mathcal{L} \sim \Big\{ i[X^\rho, \Sigma_\rho \mathcal{L}] \ + \ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^r \Big( C e^B \partial\Sigma \Big) \Big] e^F$$
$$- \ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^r \Big( C e^B \Big) \partial\Sigma \Big] e^F \ - \ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^{r+1} \Big( C e^B \partial\Sigma \Big) \Big] e^F$$
$$+ \ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^{r+1} \Big( C e^B \Big) \partial\Sigma \Big] e^F \Big\}.$$

Total action: $\mathcal{L} \ = \ \left\{ P\Big[ e^{(\mathbf{i}_X \mathbf{i}_X)} \Big( C e^B \Big) \Big] e^F \right\}$

In the variation, all terms cancel, except the ones with maximum value $r = R$:

$$\delta\mathcal{L} \sim \Big\{ i[X^\rho, \Sigma_\rho \mathcal{L}] + \ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^{R+1} \Big( C e^B \partial\Sigma \Big) \Big] e^F \ - \ P\Big[ (\mathbf{i}_X \mathbf{i}_X)^{R+1} \Big( C e^B \Big) \partial\Sigma \Big] e^F \Big\}$$

## Invariance of the action

Typical term: $\mathcal{L} \sim \left\{ P\left[ (\mathrm{i}_X \mathrm{i}_X)^r \left( C e^B \right) \right] e^F \right\}$

$$\Longrightarrow \delta\mathcal{L} \sim \Big\{ i[X^\rho, \Sigma_\rho \mathcal{L}] \; + \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^r \Big( C e^B \partial\Sigma \Big) \Big] e^F \\ - P\Big[ (\mathrm{i}_X \mathrm{i}_X)^r \Big( C e^B \Big) \partial\Sigma \Big] e^F \; - \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^{r+1} \Big( C e^B \partial\Sigma \Big) \Big] e^F \\ + P\Big[ (\mathrm{i}_X \mathrm{i}_X)^{r+1} \Big( C e^B \Big) \partial\Sigma \Big] e^F \Big\}.$$

Total action: $\mathcal{L} \; = \; \left\{ P\left[ e^{(\mathrm{i}_X \mathrm{i}_X)} \left( C e^B \right) \right] e^F \right\}$

In the variation, all terms cancel, except the ones with maximum value $r = R$:

$$\delta\mathcal{L} \sim \Big\{ i[X^\rho, \Sigma_\rho \mathcal{L}] + \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^{R+1} \Big( C e^B \partial\Sigma \Big) \Big] e^F \; - \; P\Big[ (\mathrm{i}_X \mathrm{i}_X)^{R+1} \Big( C e^B \Big) \partial\Sigma \Big] e^F \Big\}$$

$$= 0$$

as $\left\{ i[X^\rho, \Sigma_\rho \mathcal{L}] \right\}$ is a trace over a single commutator

as $(\mathrm{i}_X \mathrm{i}_X)^{R+1} \left( C e^B \partial\Sigma \right)$ involves more $X^\mu$ as transverse directions to D-brane

## Different prescription for R-R and NS-NS transformations?

R-R: modified gauge variations

NS-NS: substitution of different variations

$$\begin{aligned} \delta P[C_2] &= 2P[\partial\Lambda_1] - i\Lambda_1[X,F] = DP[\Lambda] \\ \delta P[B_2] &= 2P[\partial\Sigma] - 2iP[(\mathrm{i}_X\mathrm{i}_X\partial\Sigma)B] - 8iP[(\mathrm{i}_X\partial\Sigma)(\mathrm{i}_X B)] \end{aligned}$$

$\longrightarrow$ What about S-duality?

## Different prescription for R-R and NS-NS transformations?

R-R: modified gauge variations

NS-NS: substitution of different variations

$$\begin{aligned} \delta P[C_2] &= 2P[\partial\Lambda_1] - i\Lambda_1[X,F] = DP[\Lambda] \\ \delta P[B_2] &= 2P[\partial\Sigma] - 2iP[(\mathrm{i}_X\mathrm{i}_X\partial\Sigma)B] - 8iP[(\mathrm{i}_X\partial\Sigma)(\mathrm{i}_X B)] \end{aligned}$$

$\longrightarrow$ What about S-duality?

There exists an alternative description of R-R variations: [Adam]

- $\mathcal{L} \sim \Big\{ P[C_2] \Big\}$ is consistent truncation with $C_p = B = V = 0$

  $\Longrightarrow \delta\mathcal{L} = \Big\{ DP[\Lambda_1] \Big\} = \Big\{ P[\partial\Lambda_1] \Big\}$ since $F = 0$

- For $F \neq 0 \Longrightarrow \mathcal{L} \sim \Big\{ P[C_2] + P[(\mathrm{i}_X\mathrm{i}_X)C_2]F \Big\}$

## Different prescription for R-R and NS-NS transformations?

R-R: modified gauge variations

NS-NS: substitution of different variations

$$\delta P[C_2] \;=\; 2P[\partial\Lambda_1] \;-\; i\Lambda_1[X,F] = DP[\Lambda]$$
$$\delta P[B_2] \;=\; 2P[\partial\Sigma] \;-\; 2iP[(\mathrm{i}_X\mathrm{i}_X\partial\Sigma)B] \;-\; 8iP[(\mathrm{i}_X\partial\Sigma)(\mathrm{i}_X B)]$$

$\longrightarrow$ What about S-duality?

There exists an alternative description of R-R variations: [Adam]

- $\mathcal{L} \sim \Big\{ P[C_2] \Big\}$ is consistent truncation with $C_p = B = V = 0$

  $\Longrightarrow \delta\mathcal{L} = \Big\{ DP[\Lambda_1] \Big\} = \Big\{ P[\partial\Lambda_1] \Big\}$ since $F = 0$

- For $F \neq 0 \Longrightarrow \mathcal{L} \sim \Big\{ P[C_2] + P[(\mathrm{i}_X\mathrm{i}_X)C_2]F \Big\}$

  $\Longrightarrow$ Naive substitution yields results similar to NS-NS case

  $\Longrightarrow$ R-R gauge invariance also possible through naive sustitution

## Conclusions

- R-R background gauge invariance $\delta C = \partial\Lambda$ can be achieved through the implementation $\delta\Big\{P[(\mathrm{i}_X\mathrm{i}_X)C]\Big\} = \Big\{DP[(\mathrm{i}_X\mathrm{i}_X)\Lambda]\Big\}$
- Demanding that fields play same role before and after T-duality $\Longrightarrow$ new symmetry $\delta X^\mu = \Sigma_\nu[X^\nu, X^\mu]$
- taking in account this new symmetry: D-brane action invariant
- $\delta X^\mu = \Sigma_\nu[X^\nu, X^\mu]$ usefull in construction of non-Abelian Born-Infeld actions or in covariant formulation of non-Abelian actions?