Program Z_EQUIV.EXE

The program provides the asymptotical unconditional p-value (obtained with and without a continuity correction) for the comparison of two independent proportions (p_2 and p_1) by means of the z-pooled statistic $Z=(\hat{d}-\delta\pm c)/s$ with:

- x_i ~ two independent binomial (n_i, p_i);
- Sample 1 (2): standard (new) treatment;
- $\hat{d}=\hat{p}_2-\hat{p}_1$ and $\hat{p}_i=x_i/n_i$;
- δ = the value of $d = p_2-p_1$ under null hypotheses H;
- c = the continuity correction (c.c. in the following) (see the Table);
- $s=\sqrt{\tilde{p}(1-\tilde{p})/n_1+(\tilde{p}+\delta)(1-\tilde{p}-\delta)/n_2}$ =E.E.= the standard error of $\hat{d}$ under H;
- $\tilde{p}$ = the estimator of maximum likelihood of $p=p_1$ under H;
- E = minimum expected quantity = $\{n_1\tilde{p}, n_1(1-\tilde{p}), n_2(\tilde{p}+\delta), n_2(1-\tilde{p}-\delta)\}$

The null (H) and alternative (K) hypotheses are:

Hypotheses:	**Null**	**Alternative**
For SG:	$d \leq \delta$	$d > \delta$
For IG:	$d \leq \delta$	$d < \delta$
For SDG:	$\delta_1 \leq d \leq \delta_2$	$d < \delta_1$ or $d > \delta_2$
For SD:	$\lvert d \rvert \leq \Delta$	$\lvert d \rvert > \Delta$
For SG2:	$d = \delta$	$d \neq \delta$
For PEG:	$d \leq \delta_1$ or $d \geq \delta_2$	$\delta_1 < d < \delta_2$
For PE:	$\lvert d \rvert \geq \Delta$	$\lvert d \rvert < \Delta$

To calculate the p-value, see the Table. For further details, see the paper:

Martín Andrés, A. and Herranz Tejedor, I. (2004). Asymptotical tests on the equivalence, substantial difference an non-inferiority problems with two proportions. *Biometrical Journal 46 (3)*, 305-319.

Table

Case	p-value	c = Variable c.c. (×2 if $n_1 = n_2$)
SG (IG)	$P = F\left\{\frac{-\hat{d}+\delta+c}{s}\right\} \quad \left(F\left\{\frac{\hat{d}-\delta+c}{s}\right\}\right)$	$[N(1-\delta)]^{-1}$ if $\delta \geq 0$ ($\delta \leq 0$) $(1-\delta)[N\{2-(1+\delta)^2\}]^{-1}$ if $\delta<0$ ($\delta>0$)
SDG	$P=F\left\{\frac{-\lvert\hat{d}-\delta_1\rvert+c}{s_o}\right\}+F\left\{\frac{-\lvert\hat{d}-\delta_2\rvert+c}{s_o}\right\}$ if $\hat{d} \in \delta_1+\delta_2 \pm 1$ $P=\max_{i=1,2} F\left\{\frac{-\lvert\hat{d}-\delta_i\rvert+c}{s_i}\right\}$ if $\hat{d} \notin \delta_1+\delta_2 \pm 1$	$\frac{(1+\delta_1)+(1-\delta_2)}{N\{(1+\delta_1)^2+(1-\delta_2)^2\}}$ if $\delta_1 \leq 0 \leq \delta_2$ $\frac{2-\lvert\delta_2-\delta_1\rvert}{N\{2-\lvert(1-\lvert\delta_2\rvert)^2-(1-\lvert\delta_1\rvert)^2\rvert\}}$ other case
SD	$P=F\left(\frac{-\lvert\hat{d}+\Delta\rvert+c}{s_o}\right)+F\left(\frac{-\lvert\hat{d}-\Delta\rvert+c}{s_o}\right)$	$\frac{1}{N(1-\Delta)}$
SG2	$P=2\times F\left\{\frac{-\lvert\hat{d}-\delta\rvert+c}{s}\right\}$ if $\hat{d} \in 2\delta \pm 1$ $P=F\left\{\frac{-\lvert\hat{d}-\delta\rvert+c}{s}\right\}$ if $\hat{d} \notin 2\delta \pm 1$	$\frac{1}{N}$
PEG	$P=\max_{i=1,2}\left[F\left\{\frac{\delta_r+\lvert\hat{d}-\delta_o\rvert+c}{s_i}\right\}-F\left\{\frac{\delta_r-\lvert\hat{d}-\delta_o\rvert-c}{s_i}\right\}\right]$ where $\delta_o=(\delta_1+\delta_2)/2$ and $\delta_r=(\delta_2-\delta_1)/2$	$\frac{\delta_2-\delta_1}{N\{2-(1+\delta_1)^2-(1-\delta_2)^2\}}$ if $\delta_1<0<\delta_2$ $[N\{2-\lvert\delta_1\rvert-\lvert\delta_2\rvert\}]^{-1}$ other case
PE	$P=\max_{i=1,2}\left[F\left\{\frac{\Delta+\lvert\hat{d}\rvert+c}{s_i}\right\}-F\left\{\frac{\Delta-\lvert\hat{d}\rvert-c}{s_i}\right\}\right]$	$\frac{1}{N(2-\Delta)}$

NOTES

(1) s is the value of $\sqrt{\tilde{p}(1-\tilde{p})/n_1+(\tilde{p}+\delta)(1-\tilde{p}-\delta)/n_2}$ in the observed point and for the δ-value of each case; when there are two value of δ implied (δ_1 y δ_2; $-\Delta$ y $+\Delta$), the δ-values are respectively s_1 and s_2, whose maximum is s_o;
(2) F($\cdot$) is the distribution function of a normal typical distribution;
(3) $N = (n_1+1)\times(n_2+1)$.
(4) A continuity correction valid for all the cases is $c = 1/N$ ($2/N$ if $n_1 = n_2$): **Constant c.c.**
(5) The p-value **without** continuity correction is obtained for c=0.