NORMS TRANSFORMING ALGEBRAIC ORTHOGONALITY INTO GEOMETRIC ORTHOGONALITY ON C*-ALGEBRAS

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Abstract. Let $(A, \|.\|_0)$ be a C*-algebra. Two elements a, b in A are said to be *(algebraically) orthogonal* (denoted by $a \perp b$) whenever $ab^* = b^*a = 0$. It is well known that orthogonal elements in A are *(geometrically) M-orthogonal* in the underlying Banach space, that is, $\|a \pm b\|_0 = \max\{\|a\|_0, \|b\|_0\}$ whenever $a \perp b$. In other words, the C^*-norm behaves as the maximum norm on every couple of orthogonal elements in A. A norm $\|.\|_1$ on A is said to be an *M-norm* (resp., a *semi-M-norm*) if for every a, b in A with $a \perp b$ we have $\|a \pm b\|_1 = \max\{\|a\|_1, \|b\|_1\}$ (resp., $\|a \pm b\|_1 \geq \max\{\|a\|_1, \|b\|_1\}$). We have already commented that the original C*-norm, $\|.\|_0$, is an M-norm on A. On the other hand, it is well known that there exists (complete) M-norms on A which do not satisfy the Gelfand-Naimark axiom. We shall present in this talk the latest advance in the study of the following problem:

Problem. Is every complete semi-M-norm on a C*-algebra automatically continuous with respect to the original C*-norm?

In collaboration with T. Oikhberg and A. Peralta we establish that every complete (semi-) M-norm on a von Neumann algebra or on a compact C*-algebra, A is equivalent to the original C*-norm of A

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