THE DAUGAVETIAN INDEX OF A BANACH SPACE

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ABSTRACT. Given an infinite-dimensional Banach space X, we introduce the daugavetian index of X, $\operatorname{daug}(X)$, as the greatest constant $m \geqslant 0$ such that

$$\|Id + T\| \geqslant 1 + m\|T\|$$

for all $T \in K(X)$. We give two characterizations of this index and we estimate it in some examples. We show that the daugavetian index of a c_0-, l_1- or l_∞-sum of Banach spaces is the infimum index of the summands. Finally, we calculate the daugavetian index of some vector-valued function spaces: $\operatorname{daug}\big(C(K,X)\big)$ —resp. $\operatorname{daug}\big(L_1(\mu,X)\big)$, $\operatorname{daug}\big(L_\infty(\mu,X)\big)$— is the maximum of $\operatorname{daug}(X)$ and $\operatorname{daug}(C(K))$ —resp. $\operatorname{daug}(L_1(\mu))$, $\operatorname{daug}(L_\infty(\mu))$—.

1. INTRODUCTION

Given a Banach space X, we write X^* for the dual space and $L(X)$ —resp. $K(X)$— for the Banach algebra of bounded —resp. compact— linear operator on X. From now on, we deal with real Banach spaces. Since our results only depend on the underlying real structure, they trivially extend to complex spaces.

Let X be a Banach space. If X is infinite-dimensional, the compact operators on X are not inversible, so $\|Id + T\| \geqslant 1$ for every $T \in K(X)$. This allow us to define the *daugavetian index* of X as

$$\operatorname{daug}(X) = \max\left\{ m \geqslant 0 \ : \ \|Id + T\| \geqslant 1 + m\|T\| \quad \text{for all } T \in K(X) \right\}.$$

It is clear that $0 \leqslant \operatorname{daug}(X) \leqslant 1$. The extreme value $\operatorname{daug}(X) = 1$ means that X has the so-called Daugavet property. A Banach space X has the *Daugavet property* [7] if

$$\|Id + T\| = 1 + \|T\|$$

for all rank-one operator $T \in L(X)$. In this case, all weakly compact operators on X also satisfies the above equation (see [7, Theorem 2.3]). The Daugavet property has been deeply studied in the last decade. The state of the art on this topic can be found in [7, 17, 19].

In [13], the following weaker version of the Daugavet property is introduced. A Banach space X has the *pseudo-Daugavet property* if there exists an strictly increasing function $\psi : [0, +\infty) \longrightarrow [1, +\infty)$ such that

$$\|Id + T\| \geqslant \psi(\|T\|)$$

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for all $T \in K(X)$. In such a case, it is clear that $\psi(0) = 1$. In [2, 12, 13, 14] some results on this property are given. See also [15] for some related questions.

It is clear that $\operatorname{daug}(X) > 0$ implies that X has the pseudo-Daugavet property for the function $t \longmapsto \operatorname{daug}(X)t$. We shall prove a somehow converse: if X has the pseudo-Daugavet property for a function ψ which is derivable at 0, then $\operatorname{daug}(X) \geqslant \psi'(0)$.

We can give another approach to the daugavetian index of a Banach space which is related to numerical range of operators. Let us recall the relevant definitions. Given an operator $T \in L(X)$, the *numerical range*, $V(T)$, and the *numerical radius* $v(T)$ of T are defined by

$$V(T) = \{x^*(Tx) \;:\; x \in X,\ x^* \in X^*,\ \|x\| = \|x^*\| = x^*(x) = 1\},$$
$$v(T) = \sup\{|\lambda| \;:\; \lambda \in V(T)\}.$$

The *numerical index* of X is the number

$$n(X) = \inf\{v(T) \;:\; T \in L(X),\ \|T\| = 1\}$$

or, equivalently, the greatest constant $m \geqslant 0$ such that $v(T) \geqslant m\|T\|$ for all $T \in L(X)$. The interested reader can found more information in [3, 4, 9] and the references therein.

If $T \in L(X)$, we write $\omega(T) = \sup V(T)$, which is a sublinear functional on $L(X)$. It is a well-known result by F. Bauer [1] and G. Lumer [8] (see [3, §9]) that

$$\omega(T) = \lim_{\alpha \to 0^+} \frac{\|Id + \alpha T\| - 1}{\alpha}. \tag{1}$$

It follows that $\omega(T) \geqslant 0$ whenever $T \in K(X)$. We shall prove that

$$\operatorname{daug}(X) = \inf\{\omega(T) \;:\; T \in K(X),\ \|T\| = 1\}.$$

This sort of parallelism between the two indices allows us to translate some ideas from papers on numerical index to our proofs. This is the case of the results on numerical index of sums and vector-valued function spaces given in [10].

The outline of the paper is as follows.

We start section 2 by proving two characterizations of the daugavetian index. One related to $\omega(\cdot)$ and the other one related to the pseudo-Daugavet property. Next, we present several examples of Banach spaces whose daugavetian index can be estimated. Finally, we generalize the fact that spaces with the Daugavet property do not have unconditional bases, estimating the daugavetian index in terms of the unconditional basis constant.

In §3 we study the stability of the daugavetian index. First, we prove that the daugavetian index of a c_0-, l_1-, or l_∞-sum of Banach spaces is the infimum of the numerical indices of the summands. As a consequence of this result we obtain that every Banach can be equivalently renormed to have daugavetian index 0. Our main result deals with spaces of vector-valued functions. We prove that the daugavetian index of $C(K,X)$ —resp. $L_1(\mu,X)$, $L_\infty(\mu,X)$— is the maximum of $\operatorname{daug}(X)$ and $\operatorname{daug}(C(K))$ —resp. $\operatorname{daug}(L_1(\mu))$, $\operatorname{daug}(L_\infty(\mu))$—.

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2. Characterization and examples

Our first aim is to prove the result cited in the introduction which relates the daugavetian index and the numerical range of operators. We will use it later to get some stability properties of the daugavetian index.

Proposition 1. *Let X be an infinite-dimensional Banach space. Then*

$$\operatorname{daug}(X) = \inf\{\,\omega(T)\ :\ T \in K(X),\ \|T\| = 1\}$$

or, equivalently, $\operatorname{daug}(X)$ *is the greatest constant $m \geqslant 0$ such that $\omega(T) \geqslant m\|T\|$ for all $T \in K(X)$.*

Proof. Let m be the greatest nonnegative constant such that $\omega(T) \geqslant m\|T\|$ for all $T \in K(X)$. For $T \in K(X)$ and $x \in S_X$, $x^* \in S_{X^*}$ with $x^*(x) = 1$, we have

$$\|Id + T\| \geqslant x^*(x + Tx) = 1 + x^*(Tx).$$

Taking the supremum over all $x \in S_X$, $x^* \in S_{X^*}$ with $x^*(x) = 1$, we get

$$\|Id + T\| \geqslant 1 + \omega(T) \geqslant 1 + m\|T\|.$$

This implies that $\operatorname{daug}(X) \geqslant m$. To get the reverse inequality, we fix $T \in K(X)$ and observe that

$$\|Id + \alpha T\| \geqslant 1 + \alpha \operatorname{daug}(X)\|T\|$$

for every $\alpha > 0$. By (1), we get $\omega(T) \geqslant \operatorname{daug}(X)\|T\|$. Therefore, $\operatorname{daug}(X) \leqslant m$. □

We can use the above result to get another characterization of $\operatorname{daug}(X)$. As we have mention in the introduction, the daugavetian index is related to the pseudo-Daugavet property. Indeed, $\operatorname{daug}(X) > 0$ clearly implies that X has the pseudo-Daugavet property for the function $t \mapsto \operatorname{daug}(X)\,t$. The following proposition gives us a somehow converse of this result.

Proposition 2. *Let X be an infinite-dimensional Banach space having the pseudo-Daugavet property for a function $\psi : [0, +\infty) \to [1, +\infty)$. If ψ is differentiable at 0, then* $\operatorname{daug}(X) \geqslant \psi'(0)$. *As a consequence,*

$$\operatorname{daug}(X) = \max\{\psi'(0)\ :\ \psi \in \mathcal{F}\},$$

where $\mathcal{F}$ is the set of all increasing and differentiable at 0 functions $\psi : [0, +\infty) \to [1, +\infty)$ such that $\|Id + T\| \geqslant \psi(\|T\|)$ for all $T \in K(X)$.

Proof. Since ψ is differentiable at 0, for every $\rho \in (0, 1)$ there exists $t_\rho > 0$ such that

$$\psi(t) \geqslant 1 + \rho\,\psi'(0)t \qquad (0 < t < t_\rho).$$

For $T \in K(X)$ with $\|T\| = 1$ and $0 < \alpha < t_\rho$, we have

$$\|Id + \alpha T\| \geqslant \psi(\alpha) \geqslant 1 + \rho\,\psi'(0)\alpha.$$

It follows from (1) that

$$\omega(T) = \lim_{\alpha\to 0} \frac{\|Id + \alpha T\| - 1}{\alpha} \geqslant \rho\,\psi'(0).$$

Then, $\omega(T) \geqslant \psi'(0)$ for every $T \in K(X)$ with $\|T\| = 1$. Hence, Proposition 1 gives us $\operatorname{daug}(X) \geqslant \psi'(0)$.

For the second part of the proposition, we first observe that $\mathcal{F}$ is non-empty because it contains the function $\psi(t) = 1$ for all $t \in [0,+\infty)$. Now, let $m = \max\{\psi'(0) : \psi \in \mathcal{F}\}$. By the above argument, we have $m \leqslant \operatorname{daug}(X)$. The reverse inequality follows from the fact that the function $\psi(t) = 1 + \operatorname{daug}(X)\,t$ for all $t \in [0,+\infty)$ belongs to $\mathcal{F}$. □

We present now some examples of Banach spaces whose daugavetian index can be estimated.

Example 3. As we commented in the introduction, Banach spaces with the Daugavet property have daugavetian index 1. This is the case of the vector-valued function spaces $C(K,X)$ and $L_1(\mu,X)$ when the compact K is perfect and the measure μ is atomless (see [7]) for any Banach space X. If μ is also σ-finite, then $L_\infty(\mu,X)$ also has the Daugavet property (see [11]). More examples of spaces with the Daugavet property, and hence with daugavetian index 1, are atomless C^*-algebras [12], the disk algebra $A(\mathbb{D})$ and the algebra of bounded analytic functions H^∞ [18].

Example 4. One clearly has $\operatorname{daug}(X) = 0$ whenever X is an infinite-dimensional Banach space with a bicontractive projection with finite-rank, that is, a finite-rank projection P such that $\|P\| = \|Id - P\| = 1$. This is the case of the spaces c_0, c, l_p with $1 \leqslant p \leqslant \infty$, $C(K)$ for non-perfect K, and $L_1(\mu)$, $L_\infty(\mu)$ when μ has atoms.

We now intend to quantify the fact given in [6, Corollary 2.3] that spaces with the Daugavet property do not admit an unconditional basis. Let us recall some notation. Let X be a Banach space with unconditional basis $\{(e_n, e_n^*)\}$. For every finite subset A of $\mathbb{N}$, we define a finite-rank operator $P_A \in L(X)$ by

$$P_A(x) = \sum_{n\in A} e_n^*(x) e_n \qquad (x \in X).$$

The *unconditional basis constant* is the number

$$K = \sup\{\|P_A\| \ : \ A \subset \mathbb{N} \text{ finite } \} < +\infty.$$

We say that X admits a K-unconditional basis if X has an unconditional basis whose unconditional basis constant is less or equal than K.

A Banach space admitting an 1-unconditional basis has a lot of bicontractive projection with finite-rank. Hence, by Example 4, it has daugavetian index 0. This fact can be quantify in terms of the basis constant.

Proposition 5. *Let X be an infinite-dimensional Banach space admitting a K-unconditional basis. Then*

$$\operatorname{daug}(X) \leqslant \frac{K-1}{K}.$$

Proof. We will follow ideas from the proof of [6, Corollary 2.3]. Given $\varepsilon > 0$, we get a finite set $A_0 \subset \mathbb{N}$ such that $\|P_{A_0}\| \geqslant K - \varepsilon$. Since P_{A_0} has finite-rank, we have

$$\|Id - P_{A_0}\| \geqslant 1 + \operatorname{daug}(X)\,(K - \varepsilon).$$

On the other hand, since the basis is unconditional, we have

$$\|Id - P_{A_0}\| \leqslant \sup\big\{\|P_A\| \;:\; A \subset \mathbb{N} \setminus A_0 \text{ finite } \big\} \leqslant K.$$

Letting $\varepsilon \downarrow 0$, we get $1 + \operatorname{daug}(X)K \leqslant K$. □

Finally, let us show some examples of Banach spaces whose daugavetian index is greater than 0 and less than 1.

Example 6. *Let X a subspace of $L(l_2)$ containing $K(l_2)$. Then*

$$1/(8\sqrt{2}) \leqslant \operatorname{daug}(X) < 1.$$

For $X = L(l_2)$, we actually have

$$1/(8\sqrt{2}) \leqslant \operatorname{daug}\big(L(l_2)\big) \leqslant 2/\sqrt{5}.$$

Indeed, it is proved in [13] that $\|Id + \Phi\| \geqslant 1 + \|\Phi\|/(8\sqrt{2})$ for every $\Phi \in K(X)$, hence $\operatorname{daug}(X) \geqslant 1/(8\sqrt{2})$. It is known that, in our assumption, X^* has strongly exposed points (see [16, Corollary 1.4]). Then, X does not have the Daugavet property by [7, Lemma 2.1] and hence, $\operatorname{daug}(X) < 1$. The refinement for $L(l_2)$ is also proved in [13].

3. Stability properties

The aim of this section is to compute the daugavetian index of sums and some vector-valued function spaces.

We start by working with sums of spaces. Let us recall some definitions. Given an arbitrary family $\{X_\lambda\}_{\lambda\in\Lambda}$ of Banach spaces, we denote by $[\oplus_{\lambda\in\Lambda} X_\lambda]_{c_0}$ —resp. $[\oplus_{\lambda\in\Lambda} X_\lambda]_{l_1}$, $[\oplus_{\lambda\in\Lambda} X_\lambda]_{l_\infty}$— the c_0-sum —resp. l_1-sum, l_∞-sum— of the family. The sum of two spaces X and Y is denoted by the simpler notation $X \oplus_\infty Y$, $X \oplus_1 Y$. For infinite countable sums of copies of a space X we write $c_0(X)$, $l_1(X)$ or $l_\infty(X)$.

Proposition 7. *Let $\{X_\lambda\}_{\lambda\in\Lambda}$ be a family of infinite-dimensional Banach spaces and let Z be the c_0-, l_1- or l_∞-sum of the family. Then*

$$\operatorname{daug}(Z) = \inf\{\operatorname{daug}(X_\lambda) \;:\; \lambda \in \Lambda\}.$$

To prove this proposition, we can adapt the proof of [10, Proposition 1], using the parallelism between the daugavetian index and the numerical index given in Proposition 1. It is enough to change the numerical radius by the supremum of the numerical range, $\omega(\cdot)$, and to observe that, when starting with compact operators, all the operator involved are also compact.

Remark 8. Let $\{X_\lambda\}_{\lambda\in\Lambda}$ be an arbitrary family of Banach spaces and let Z be the c_0-, or l_1-, or l_∞-sum of the family. If one of the summands is finite-dimensional, then Z has a bicontractive projection with finite-rank. Hence, $\operatorname{daug}(Z) = 0$. Defining $\operatorname{daug}(X) = 0$

for every finite-dimensional space, Proposition 7 is also true for arbitrary families of Banach spaces.

As a corollary of the above remark we get the following isomorphic result.

Corollary 9. *Let X be an infinite-dimensional Banach space. Then there exists a Banach space Y isomorphic to X with* $\operatorname{daug}(Y) = 0$.

Proof. Let us write $X = \mathbb{R} \oplus Z$ for a suitable subspace Z. Then, X is isomorphic to $Y = \mathbb{R} \oplus_\infty Z$ and the above remark gives $\operatorname{daug}(Y) = 0$. □

Another consequence of Proposition 7 is that

$$\operatorname{daug}\big(c_0(X)\big) = \operatorname{daug}\big(l_1(X)\big) = \operatorname{daug}\big(l_\infty(X)\big) = \operatorname{daug}(X)$$

for every Banach space X. One may wonder whether this result is also true for arbitrary vector-valued function spaces, but it is easy to see that this is not the case. Indeed, $\operatorname{daug}(l_2) = 0$ in spite of the fact that $C([0,1], l_2)$, $L_1([0,1], l_2)$ and $L_\infty([0,1], l_2)$ have daugavetian index 1 (see Example 3). Let us recall some notation. Given a compact Hausdorff space K and a Banach space X, we write $C(K,X)$ for the Banach space of all continuous functions from K into X, endowed with the supremum norm. If (Ω, Σ, μ) is a positive measure space, $L_1(\mu, X)$ is the Banach space of all Bochner-integrable functions $f : \Omega \to X$ with the usual norm. If μ is σ-finite, then $L_\infty(\mu, X)$ stands for the space of all essentially bounded Bochner-measurable functions f from Ω into X, endowed with the essential supremum norm. We refer to [5] for background.

The following result extends those given in [10, Remarks 6 and 9] and [11, Theorem 5] for the Daugavet property.

Theorem 10. *Let X be an infinite-dimensional Banach space. Then:*

(i) *If K is a compact Hausdorff space, then*

$$\operatorname{daug}\big(C(K,X)\big) = \max\big\{\operatorname{daug}\big(C(K)\big), \operatorname{daug}(X)\big\}.$$

(ii) *If μ is a positive measure, then*

$$\operatorname{daug}\big(L_1(\mu,X)\big) = \max\big\{\operatorname{daug}\big(L_1(\mu)\big), \operatorname{daug}(X)\big\}.$$

(iii) *If μ is a σ-finite measure, then*

$$\operatorname{daug}\big(L_\infty(\mu,X)\big) = \max\big\{\operatorname{daug}\big(L_\infty(\mu)\big), \operatorname{daug}(X)\big\}.$$

Proof. (i). We start by proving that $\operatorname{daug}\big(C(K,X)\big) \geqslant \operatorname{daug}(X)$. To this end, we fix $T \in K(C(K,X))$ and prove that

$$\|Id + T\| \geqslant 1 + \operatorname{daug}(X)\,\|T\|.$$

For every $\varepsilon > 0$, we may find $f_0 \in C(K,X)$ with $\|f_0\| = 1$ and $t_0 \in K$ such that

$$\big\|[T f_0](t_0)\big\| > \|T\| - \varepsilon. \tag{2}$$

We find a continuous function $\varphi : K \longrightarrow [0,1]$ such that $\varphi(t_0) = 1$ and $\varphi(t) = 0$ if $\| f_0(t) - f_0(t_0)\| \geqslant \varepsilon$, write $f_0(t_0) = \lambda x_1 + (1-\lambda)x_2$ with $0 \leqslant \lambda \leqslant 1$, $x_1, x_2 \in S_X$, and consider the functions

$$f_j = (1-\varphi) f_0 + \varphi x_j \in C(K,X) \qquad (j = 1,2)$$

and $g = \lambda f_1 + (1-\lambda) f_2$. Since $g(t) - f_0(t) = \varphi(t)\big(f_0(t_0) - f_0(t)\big)$, we have $\|g - f_0\| < \varepsilon$ and therefore, by (2), we have

$$\|[T f_1](t_0)\| > \|T\| - 2\varepsilon \qquad \text{or} \qquad \|[T f_2](t_0)\| > \|T\| - 2\varepsilon. \tag{3}$$

We make the right choice of $x_0 = x_1$ or $x_0 = x_2$ to get $x_0 \in S_X$ such that

$$\Big\| \big[T\big((1-\varphi) f_0 + \varphi x_0\big)\big]\,(t_0)\Big\| > \|T\| - 2\varepsilon. \tag{4}$$

Next we fix $x_0^* \in S_{X^*}$ with $x_0^*(x_0) = 1$, denote

$$\Phi(x) = x_0^*(x)(1-\varphi) f_0 + \varphi x \in C(K,X) \qquad (x \in X),$$

and consider the operator $S \in L(X)$ given by

$$Sx = [T(\Phi(x))](t_0) \qquad (x \in X).$$

We observe that $S \in K(X)$ and, by (4), that

$$\|S\| \geqslant \|Sx_0\| > \|T\| - 2\varepsilon.$$

Then, find $x \in S_X$ such that

$$\|x + Sx\| \geqslant 1 + \operatorname{daug}(X)\big(\|T\| \ - 2\varepsilon\big),$$

define $g \in S_{C(K,X)}$ by $g = \Phi(x)$ for this x, note that

$$\|Id + T\| \geqslant \Big\| \big[(Id + T)(g)\big](t_0)\Big\| = \|x + Sx\| \geqslant 1 + \operatorname{daug}(X)\big(\|T\| \ - 2\varepsilon\big),$$

and let $\varepsilon \downarrow 0$. It should be pointed out that the above argument is based on the one given in [10, Theorem 5].

Now, if K is perfect, Example 3 gives us that $\operatorname{daug}\big(C(K,X)\big) = \operatorname{daug}\big(C(K)\big) = 1$. Otherwise, K has an isolated point so, on one hand Example 4 gives $\operatorname{daug}\big(C(K)\big) = 0$ and, on the other hand, we can write $C(K,X) = X \oplus_\infty Z$ for some Banach space Z, so $\operatorname{daug}\big(C(K,X)\big) \leqslant \operatorname{daug}(X)$ by Proposition 7.

(ii). If μ is atomless, by Example 3 we have

$$\operatorname{daug}\big(L_1(\mu,X)\big) = \operatorname{daug}\big(L_1(\mu)\big) = 1.$$

Otherwise, we may write $L_1(\mu,X)$ in the form $L_1(\nu,X) \oplus_1 [\oplus_{i\in I} X]_{l_1}$ for a non-empty set I and an atomless measure ν. Now, $\operatorname{daug}\big(L_1(\nu,X)\big) = 1$ so, by Proposition 7, we have

$$\operatorname{daug}\big(L_1(\mu,X)\big) = \operatorname{daug}\big([\oplus_{i\in I} X]_{l_1}\big) = \operatorname{daug}(X).$$

Now, Example 4 shows that $\operatorname{daug}\big(L_1(\mu)\big) = 0$ and the result follows. The proof of (iii) is completely analogous. $\square$

If X is a finite-dimensional space, Theorem 10 is still true if we consider $\operatorname{daug}(X) = 0$. Actually, in this case, $\operatorname{daug}\big(C(K,X)\big)$ —resp. $\operatorname{daug}\big(L_1(\mu,X)\big)$, $\operatorname{daug}\big(L_\infty(\mu,X)\big)$— is equal to 0 or 1 depending on whether or not K has isolated points —resp. μ has atoms—.

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