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An alternative Daugavet Property

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• Daugavet equation

X espacio de Banach, $T \in L(X)$

$$\|Id + T\| = 1 + \|T\| \qquad \text{(DE)}$$

☐ **Daugavet, 1963 :** $T \in K\big(C[0,1]\big)$ satisfies (DE)

☐ **Lozanoskii, 1966 :** $T \in K\big(L_1[0,1]\big)$ satisfies (DE)

☐ **Abramovich, Holub and more, 80's :**
$X = C(K)$, K perfect compact space or
$X = L_1(\mu)$, μ atomless positive measure,
then every weakly compact $T \in L(X)$ satisfies (DE).

• Daugavet Property

A Banach space X has the *Daugavet Property* if every rank-one operator $T \in L(X)$ satisfies (DE)

(Kadets-Shvidkoy-Sirotkin-Werner, 2000)

☐ K perfect compact space, μ atomless positive measure
$\Rightarrow C(K)$ and $L_1(\mu)$ have the Daugavet property

☐ A C^*-algebra has the Daugavet property if and only if it is nonatomic.
The predual of a nonatomic von Neumann algebra has the Daugavet property

(Oikhberg, 200?)

☐ More examples: $A(\mathbb{D})$ and H^∞

(Werner, 1997)

Theorem

If X has the Daugavet property then every weakly compact operator on X satisfies (DE)

(Kadets-Shvidkoy-Sirotkin-Werner, 2000)

Consequences

X with the Daugavet property, then

- ☐ X does not have the RNP and it is not an Asplund space
- ☐ $X \supset l_1$
- ☐ $X^* \supset L_1[0,1]$ isometrically
- ☐ X does not embed into any space having an unconditional basis

Some results

◇ $C(K, X)$ has the Daugavet property iff
X has or K is perfect

(Kadets, 1996 — Martín-Payá, 2000)

◇ $L_1(\mu, X)$ has the Daugavet property iff
X has or μ is atomless

(many authors, 80's and 90's)

◇ $L_\infty(\mu, X)$ has the Daugavet property iff
X has or μ is atomless

(Martín-Villena, preprint)

• The numerical index 1

X has *numerical index 1* if the numerical radius and the norm coincide on $L(X)$, that is,

$$\|T\| = \sup\{|x^*(Tx)| \,:\, \|x^*\| = \|x\| = x^*(x) = 1\}$$

for every $T \in L(X)$.
Equivalently, X has numerical index 1 iff

$$\max_{|\lambda|=1} \|Id + \lambda T\| = 1 + \|T\| \qquad \text{(aDE)}$$

for every $T \in L(X)$

(Lumer, 1968)

EXAMPLES

☐ K compact, μ positive measure

$\Rightarrow C(K)$ and $L_1(\mu)$ has numerical index 1

(Duncan-McGregor-Pryce-White, 1970)

☐ A C^*-algebra is commutative iff it has numerical index 1

(Crab-Duncan-McGregor, 1974)

☐ $C(K,X)$ (resp. $L_1(\mu,X)$ and $L_\infty(\mu,X)$) has numerical index 1 iff X has

(Martín-Payá, 2000 — Martín-Villena, preprint)

• An "alternative" Daugavet property

☐ Daugavet property:

$$\|Id + T\| = 1 + \|T\| \qquad \forall\, T \in K(X)$$

☐ Numerical index 1:

$$\max_{|\lambda|=1} \|Id + \lambda T\| = 1 + \|T\| \qquad \forall\, T \in L(X)$$

☐ *An alternative Daugavet property*:

$$\max_{|\lambda|=1} \|Id + \lambda T\| = 1 + \|T\| \qquad \forall\, T \in K(X)$$

Examples

- ☐ $l_1 \oplus_\infty C([0,1], l_2)$ has the alternative Daugavet property but it does not have numerical index 1 and it does not have the Daugavet property

- ☐ $C(K, X)$ has the alternative Daugavet property iff X has or K is perfect

- ☐ $L_1(\mu, X)$ has the alternative Daugavet property iff X has or μ is atomless

- ☐ $L_\infty(\mu, X)$ has the alternative Daugavet property iff X has or μ is atomless

- ☐ Tensor products:

  $l_1^4 \widetilde{\otimes}_\varepsilon l_1^4$ and $l_\infty^4 \widetilde{\otimes}_\pi l_\infty^4$ does not have the alternative Daugavet property

(Lima, 1981 — Martín-Payá, 2000)

Theorem

X real Banach space with the alternative Daugavet property.

Then:

(i) If $\sharp \text{dent}\,(B_X) = \infty \quad \Rightarrow \quad X \supset c_0$ o $X \supset l_1$

(ii) If $\sharp w^*\text{–dent}\,(B_{X^*}) = \infty \quad \Rightarrow \quad X^* \supset l_1$

Open problems

- ◇ Characterize the C^*-algebras which have the alternative Daugavet property
- ◇ If X has the alternative Daugavet property,
  $\Rightarrow X \supset c_0$ or $X \supset l_1$ **?**