On the Affine Maximal Surfaces Equation

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Resumen

As Blaschke showed in 1923, the Euler-Lagrange equation of the equiaffine area functional is the following fourth-order and nonlinear equation

$$L[\phi] := \phi_{yy}\rho_{xx} - 2\phi_{xy}\rho_{xy} + \phi_{xx}\rho_{yy} = 0, \qquad \rho = \left(det\left(\nabla^2\phi\right)\right)^{-3/4}, \tag{1}$$

$\nabla^2\phi > 0$ being the positive definite Hessian matrix of ϕ. This equation is equivalent to the vanishing of the affine mean curvature, which along with the fact that, for locally strongly convex surfaces, the second variation formula is always negative led to the notion of *affine maximal surfaces.* Equation (1) has been widely studied from a global point of view. For instance, Trudinger and Wang proved that the entire convex solution of (1) are quadratic polynomials, answering in this way the so called *Affine Bernstein Problem* conjectured by Chern in 1978. Another celebrated global result is the one characterizing the elliptic paraboloid as the only affine complete affine maximal surface. Motivated by the lack of global examples, as becomes clear in view of the results above, the study of singularities of Equation (1) has lately received many contributions which has revealed an interesting global theory for this class of surfaces.

The aim of this paper is to investigate the behavior of solutions of (1) around isolated singularities. To be more precise, we shall consider the following problem:

$$L[\phi] = 0, \qquad \text{in a punctured domain } \mathcal{U}^\star = \mathcal{U} \setminus \{(0,0)\}, \tag{2}$$

where $\mathcal{U} \subseteq \mathbb{R}^2$ ia a planar domain containing the origin. We will also study global solutions of (1) with some *admissible* singularities. We call such solutions *affine maximal maps.*

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