Locally strongly convex surfaces with complete flat affine metric

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We study locally strongly convex surfaces with complete flat Blaschke metric. We show how we can characterize all known examples by a tensorial condition involving the covariant derivative of the shape operator and the gradient of the Pick invariant, see [1].

We prove that if M is an affine complete locally strongly convex surface with flat equiaffine metric satisfying

$$trace_h(\nabla S) + \mu grad_h(J) = 0,$$

for some constant μ. Then M is affine congruent to either the elliptic paraboloid, the surface $x_1x_2x_3 = 1$ or the surface

$$x(u,v) = \frac{-1}{\sqrt{3}}((cosh(3u))^{\frac{1}{3}}cosh(\sqrt{3}v), (cosh(3u))^{\frac{1}{3}}sinh(\sqrt{3}v), \int_0^u (cosh(3t))^{\frac{-2}{3}}dt).$$

References

[1] A. Martínez, F. Milán and L. Vrancken, A Class of Surfaces with Flat Blaschke Metric and Their Characterization, *Ann. Glob. Anal. Geom.* **28** (2005), 35-57.