Workshop : WOMEN IN GEOMETRY

SHULI CHEN

TITLE: Positive scalar curvature metrics and aspherical summands

ABSTRACT: A closed manifold is called aspherical if it has contractible universal cover. It has been conjectured since the 80s that all closed aspherical manifolds do not admit metrics with positive scalar curvature (PSC). In dimensions $3, 4, 5$ this conjecture is solved by works of Schoen-Yau, Gromov-Lawson, Chodosh-Li, and Gromov. We prove for $n = 3, 4, 5$ that the connected sum of a closed aspherical n-manifold with an arbitrary non-compact manifold does not admit a complete metric with PSC. For $n = 3, 4$, we further look at the manifold $M \setminus S$ obtained from a closed aspherical n-manifold M by deleting a subset S consisting of disjoint incompressible embedded closed aspherical submanifolds (possibly with different dimensions), and show it does not admit a complete metric with PSC. This is joint work with Jianchun Chu and Jintian Zhu.

AZAHARA DE LA TORRE PEDRAZA

TITLE: Prescribing non-constant Q and T curvatures on the four-dimensional half sphere

ABSTRACT: Given a 2-dimensional closed Riemannian surface, a classical problem in Geometry consists on prescribing its Gaussian curvature to be a given function via conformal changes of the background metric. Such metrics arise, for instance, from diffeormorphisms that preserve the angles between the tangent vectors. The resulting equation has been studied for a long time, but the case of the sphere, known as the Nirenberg problem, is still partially open. If the surface has boundary, it is natural to prescribe also the boundary geodesic curvature. In higher dimensions, the geometry becomes richer and we can prescribe different contractions of the curvature tensor. The most natural one is prescribing the scalar curvature on the interior and the mean curvature on the boundary. To explore further conformal and topological properties of curvatures, a new operator, leading to the definition of the Q-curvature, was introduced by Branson in 1985. It was generalised to dimension 4 by Branson and Ørsted in 1991. When the manifold has a boundary, Chang and Qing introduced a boundary operator which leads to the T curvature. In this talk, we will show the existence of a conformal metric with prescribed non-constant Q and boundary T curvatures on the upper hemisphere, which represents the analogue to the Nirenberg problem. Using a (non-usual) variational formulation, although the functional is not coercive, we will see the existence of minimizers by imposing symmetry conditions (inspired by Moser's work on the Nirenberg problem). We will focus mainly on the case of non-negative curvatures. The talk is based on a work done in collaboration with Sergio Cruz-Blázquez.

ANNA MARIA FINO

TITLE: An overview on strong geometries with torsion

ABSTRACT: A strong geometry with torsion is a Riemannian manifold carrying a metric connection with closed skew-symmetric torsion. In the seminar I will first review general properties of metric connections with closed skew-symmetric torsion. Then I will focus on the case of Hermitian manifolds and 7-manifolds endowed with a G_2-structure.

ALBA DOLORES GARCÍA RUÍZ

TITLE: High-Energy Laplace Eigenfunctions on Integrable Billiards

ABSTRACT: In this talk, we consider a bounded domain in the Euclidean plane and examine the Laplace eigenvalue problem with specific boundary conditions. A famous conjecture by Berry suggests that in chaotic dynamical systems, eigenfunctions resemble random monochromatic waves; however, this behavior is generally not expected in integrable dynamical systems. Here, we explore the behavior of high-energy eigenfunctions and their connection to Berry?s random wave model. In particular, we study a related property, which we call Inverse Localization, describing how eigenfunctions can approximate monochromatic waves in small regions of the domain.

LILIA MEHIDI

TITLE: Plane waves from multiple perspectives: Auslander's conjecture, conformal geometry, and Lichnerowicz conjecture

ABSTRACT: To any topological space, one can associate its fundamental group, a topological invariant. A natural question then arises: which groups can appear as fundamental groups of compact manifolds? The assumption that the manifold admits a geometric structure further restricts the possibilities. Auslander conjectured that the fundamental groups of compact affine manifolds, that is, compact manifolds modeled on the affine space, are solvable. In this talk, we will review existing results in the classical affine case and examine how this conjecture extends to manifolds modeled on a different geometric structure, one that can be seen as a deformation of special affine structures: the plane wave structure. We will also explore their conformal geometry in connection with the Lorentzian conformal Lichnerowicz conjecture. It turns out that these plane wave manifolds play an important role in conformal geometry in the locally homogeneous setting.

ARTEMIS AIKATERINI VOGIATZI

TITLE: Quartically pinched submanifolds for the mean curvature flow in the sphere

ABSTRACT: We introduce a new sharp quartic curvature pinching for submanifolds in $\mathbb{S}^{n+m}$, $m \geq 2$, which is preserved by the mean curvature flow. Using a blow up argument, we prove a codimension and a cylindrical estimate, where in regions of high curvature, the submanifold becomes approximately codimension one, quantitatively, and is weakly convex and moves by translation or is a self shrinker. With a decay estimate, the rescaling converges smoothly to a totally geodesic limit in infinite time, without using Stampacchia iteration or integral analysis.