Complementary Romanovski-Routh polynomials

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For $b = \lambda + i\eta$, $\lambda > 0$, the complementary Romanovski-Routh polynomials $\mathcal{P}_n(b;.)$ can be given by the hypergeometric expression

$$\mathcal{P}_n(b;x) = \frac{(x-i)^n}{2^n} \frac{(2\lambda)_n}{(\lambda)_n} \, {}_2F_1\Big(-n, b;\, b+\bar{b};\, \frac{-2i}{x-i}\Big), \quad n \geq 1.$$

They satisfy the three term recurrence

$$\mathcal{P}_{n+1}(b;x) = (x - c_{n+1}^{(b)})\mathcal{P}_n(b;x) - d_{n+1}^{(b)}(x^2+1)\mathcal{P}_{n-1}(b;x), \quad n \geq 1, \tag{1}$$

with $\mathcal{P}_0(b;x) = 1$ and $\mathcal{P}_1(b;x) = x - c_1^{(b)}$, where

$$c_n^{(b)} = \frac{\eta}{\lambda+n-1} \quad \text{and} \quad d_{n+1}^{(b)} = d_{n+1}^{(\lambda)} = \frac{1}{4}\frac{n(2\lambda+n-1)}{(\lambda+n-1)(\lambda+n)}, \quad n \geq 1. \tag{2}$$

Moreover, if $\lambda > 1/2$ then they also satisfy the varying orthogonality

$$\int_{-\infty}^{\infty} x^m \mathcal{P}_n(b;x) \frac{1}{(1+x^2)^n} \frac{(e^{-\operatorname{arccot} x})^{2\eta}}{(1+x^2)^{\lambda}} dx = \gamma_n^{(\lambda)} \delta_{m,n}, \quad m = 0, 1, \ldots, n.$$

In this talk we will look at some recent developments with respect these polynomials.