GEOMETRY AND PDE IN FRONT OF THE ALHAMBRA
BOOK OF ABSTRACTS

A class of very degenerate operators: existence through convexity

Isabeau Birindelli
Università di Roma I

In a recent paper with Galise and Ishii we have proved existence of solutions for the Truncated Laplacians, a class of very degenerate fully nonlinear operators, when the domain is hula hoop i.e. uniformly convex and the first order term is small. We shall see which of these conditions can be removed and which can't.

A gradient estimate for nonlocal minimal graphs

Xavier Cabré
ICREA and UPC, Barcelona

The talk will be concerned with s-minimal surfaces, that is, hypersurfaces of $\mathbb{R}^n$ with zero nonlocal mean curvature. These are the equations associated to critical points of the fractional s-perimeter. We will present a recent result in collaboration with M. Cozzi in which we establish, in any dimension, a gradient estimate for nonlocal minimal graphs. It leads to their smoothness, a result that was only known for $n = 2$ and 3 (but without a quantitative bound); in higher dimensions only their continuity had been established. We will also present a work with E. Cinti and J. Serra in which we prove that half spaces are the only stable s-minimal cones in $\mathbb{R}^3$ for s sufficiently close to 1.

Gluing methods for Vortex dynamics in Euler flows

Manuel del Pino
University of Bath

We consider the two-dimensional Euler flow for an incompressible fluid confined to a smooth domain. We construct smooth solutions with concentrated vorticities around k points which evolve according to the Hamiltonian system for the Kirkhoff-Routh energy, using an outer-inner solution gluing approach. The asymptotically singular profile around each point resembles a scaled finite mass solution of Liouville's equation. We also discuss the *vortex filament conjecture* for the three-dimensional case. This is joint work with Juan Dávila, Monica Musso and Juncheng Wei.

Characterization of f-extremal disks

José María Espinar
Universidad de Cádiz

In this talk we show uniqueness for overdetermined elliptic problems defined on topological disks with regular boundary, i.e., positive solutions u to $\Delta u +$

$f(u) = 0$ in $\Omega \subset (M^2, g)$ so that $u = 0$ and $\frac{\partial u}{\partial \vec{\eta}} = cte$ along $\partial\Omega$, $\vec{\eta}$ the unit outward normal along $\partial\Omega$ under the assumption of the existence of a candidate family. In particular, this gives a positive answer to the Schiffer conjecture for the first Dirichlet eigenvalue and classifies simply-connected harmonic domains, also called *Serrin Problem*, in $\mathbb{S}^2$. This is a joint work with L. Mazet.

Critical domains for the first nonzero Neumann eigenvalue in Riemannian manifolds

Mouhamed Moustapha Fall
African Institute of Mathematical Sciences

We will discuss the first order optimality condition for the first nonzero Neumann eigenvalue in Riemannian manifolds, as set dependent functional. We then consider the shapes of domains maximizing the first nonzero Neumann eigenvalue on cylindrical manifolds, under small and large volume constraints. A rigidity result related to an overdetermined problem arising from the Hadamard formula for this eigenvalue will be also presented.

Joint work with Tobias Weth.

A sharp Bernstein-type theorem for entire minimal graphs

Alberto Farina
Université Picardie Jules Verne.

We consider entire solutions u to the minimal surface equation in $\mathbb{R}^N$, with $N \geq 8$, and we prove the following sharp result : *if $N - 7$ partial derivatives $\frac{\partial u}{\partial x_j}$ are bounded on one side (not necessarily the same), then u is necessarily an affine function.*

On higher dimensional singularities for the fractional Yamabe problem: a non-local Mazzeo-Pacard program

María del Mar González
Universidad Autónoma de Madrid

We consider the problem of constructing solutions to the fractional Yamabe problem that are singular at a given smooth sub-manifold, for which we establish the classical gluing method of Mazzeo and Pacard for the scalar curvature in the fractional setting. This proof is based on the analysis of the model linearized operator, which amounts to the study of a fractional order ODE, and thus our main contribution here is the development of new methods coming from conformal geometry and scattering theory for the study of non-local ODEs. Note, however, that no traditional phase-plane analysis is available here. Instead, first, we provide a rigorous construction of radial fast-decaying solutions by a blow-up argument and a bifurcation method.

Second, we use conformal geometry to reinterpret this equation, and third, for the linear theory, we use complex analysis and some non-Euclidean harmonic analysis to examine a fractional Schrödinger equation with a Hardy type critical potential. We construct its Green's function, deduce Fredholm properties, and analyze its asymptotics at the singular points in the spirit of Frobenius method. Surprisingly enough, a fractional linear ODE may still have a two-dimensional kernel as in the second order case.

This is joint work with W. Ao, H. Chan, A. DelaTorre, M. Fontelos and J. Wei.

One-dimensional symmetry for the Euler equations and related semilinear elliptic equations

François Hamel
Aix-Marseille Université

In this talk, I will discuss one-dimensional symmetry properties for the solutions of some PDEs in dimension 2 and in higher dimensions. I will show that a steady flow of an ideal incompressible fluid with no stagnation point and tangential boundary conditions in a two-dimensional strip is a shear flow. The same conclusion holds for a bounded steady flow in a half-plane and in the whole plane. I will also discuss the case of circular flows in annular domains. The proofs are based on the study of the geometric properties of the streamlines of the flow and on one-dimensional and radial symmetry results for the solutions of some semilinear elliptic equations. The talk is based on some joint works with N. Nadirashvili.

References

[1] F. Hamel, N. Nadirashvili, Shear flows of an ideal fluid and elliptic equations in unbounded domains, *Comm. Pure Appl. Math.* **70** (2017), 590-608.
[2] F. Hamel, N. Nadirashvili, A Liouville theorem for the Euler equations in the plane, *preprint hal.archives-ouvertes.fr/hal-01491806*.
[3] F. Hamel, N. Nadirashvili, Parallel and circular flows for the two-dimensional Euler equations, *Sémin. Laurent Schwartz EDP Appl.* 2017-2018, exp. V, 1-13.
[4] F. Hamel, N. Nadirashvili, Circular flows for the Euler equations in two-dimensional annular domains, *in preparation*.

TBA

Laurent Hauswirth
Université Paris-Est

Prescribing Morse scalar curvatures in high dimension

Andrea Malchiodi
Scuola Normale Superiore, Pisa

We consider the classical problem of prescribing the scalar curvature of a manifold via conformal deformation of the metric, dating back to works by Kazdan and Warner. This problem is mainly understood in low dimensions, where blow-ups of solutions are proven to be "isolated simple". We find natural conditions to guarantee this also in arbitrary dimensions, when the prescribed curvatures are Morse functions. As a consequence, we improve some pinching conditions in the literature and derive existence results of new type. This is joint work with M. Mayer.

Prescribing the tangent space of a minimal plane

Laurent Mazet
Université de Tours

In a recent paper, Chodosh and Ketover proved that in an asymptoticaly Euclidean 3-manifold there exist properly embedded minimal planes. More precisely, for any point in this manifold, there exists a minimal plane containing that point. In this talk, I will explain how one can also prescribe the unit normal to the minimal plane at that point. Besides, one can show that giving three points there is always a minimal plane containing these three points.
This is a joint work with H. Rosenberg.

Phase Transitions and Minimal Hypersurfaces

Marco Méndez Guaraco
University of Chicago

Long standing questions in the theory of minimal hypersurfaces have been solved in the past few years. This progress can be explained and enriched through a strong analogy with the theory of phase transitions. I will present the current state of these ideas, discuss my contributions to the subject and share directions for future developments.

Immersed spheres and overdetermined problems

Pablo Mira
Universidad Politécnica de Cartagena

In this talk we will give some classification results for immersed spheres and overdetermined problems in simply connected domains, for fully nonlinear PDEs in dimension two, by means of the Poincare-Hopf index theorem. We will also explain how our results deepen the connection between surface theory and overdetermined problems.

TBA

Frank Pacard
École Polytechnique, Paris

Overdetermined problems and constant mean curvature surfaces in isoperimetric cones

Filomena Pacella
Università di Roma I

We present some recent results about the characterization of domains in a cone for which a partial overdetermined problem admits a solution. As for the classical Serrin's overdetermined problem this question is connected to the study of constant mean curvature surfaces (CMC surfaces) which, in our setting, are surfaces with their boundary intersecting a cone. Hence we also characterize this type of CMC surfaces, in particular when polar graphs are considered. Finally we show connections between the above questions and an isoperimetric inequality which allows to identify the "minimal" solutions of the partial overdetermined problem in cones having an isoperimetric property.
These results have been obtained in collaboration with Giulio Tralli.

Approximation theorems for parabolic equations and movement of local hot spots

Daniel Peralta-Salas
Instituto de Ciencias Matemáticas (CSIC)

In the first part of the talk I will present a global approximation theorem for a general parabolic operator L, which asserts that if v satisfies the equation $Lv = 0$ in a spacetime region $\Omega \subset \mathbb{R}^{n+1}$ satisfying certain necessary topological condition, then it can be approximated in a Hölder norm by a global solution u to the equation. If Ω is compact and L is the usual heat operator, one can instead approximate the local solution v by the unique solution that falls off at infinity to the Cauchy problem with a suitably chosen smooth, compactly supported initial datum. In the second part, this result is applied to prove the existence of solutions to the equation $Lu = 0$ in $\mathbb{R}^{n+1}$ with a local hot spot that moves along a prescribed curve for all time, up to a uniformly small error.

References

[1] A. Enciso, M.A. García-Ferrero, D. Peralta-Salas, Approximation theorems for parabolic equations and movement of local hot spots. Duke Math. J. to appear.

Maximal solution of the Liouville equation in doubly connected domains

Angela Pistoia
Università di Roma I

We prove the existence of solutions to the classical Liouville equation which blows-up along a smooth, simple and closed curve inside a doubly connected domain. The result has been obtained in collaboration with Giusi Vaira and Mike Kowalczyk.

Classification results for critical problems involving the p-laplacian

Berardino Sciunzi
Università della Calabria

The adaptation of the moving plane procedure to p-Laplace equations is an hard issue. I will start the talk describing some known techniques and then I will discuss classification results for the critical p-Laplace equation in the whole space. In particular I shall also present some new results in collaboration with F. Oliva and G. Vaira regarding the doubly critical equation in the whole space involving the Hardy potential.

An optimal boundary Harnack estimate for uniformly elliptic PDE in divergence form

Boyan Sirakov
PUC-Rio

This talk is devoted to global estimates and properties of nonnegative supersolutions of uniformly elliptic PDE in divergence form. We study bounds in terms of the distance to the boundary, as well as integrability and L^p-estimates up to the boundary, of supersolutions and their gradients. We find the optimal form of the boundary weak Harnack inequality recently proved in [1], for equations in divergence form. If time permits, we will present some applications to uniform a priori bounds of positive solutions.

References

[1] Boundary Harnack Estimates and Quantitative Strong Maximum Principles for Uniformly Elliptic PDE, *International Mathematics Research Notices*, rnx107, https://doi.org/10.1093/imrn/rnx107

Normalized harmonic map flow

Michael Struwe
ETH Zurich

Finding non-constant harmonic 3-spheres for a closed target manifold N is a prototype of a super-critical variational problem. In fact, the direct method fails, as the infimum of the Dirichlet energy in any homotopy class of maps from the 3-sphere to any closed N is zero; moreover, the harmonic map heat flow may blow up in finite time, and even the identity map from the 3-sphere to itself is not stable under this flow.

To overcome these difficulties, we propose the normalized harmonic map flow as a new tool, and we show that for this flow the identity map from the 3-sphere to itself now, indeed, is stable; moreover, the flow converges to a harmonic 3-sphere also when we perturb the target geometry. While our results are strongest in the perturbative setting, we also outline a possible global theory.

Minimal inmersions of closed surfaces in hyperbolic 3-manifold

Gabriella Tarantello
Università di Roma II

Motivated by the the work of K. Ulenbeck, we discuss minimal immersions of closed surfaces of genus larger than 1 on hyperbolic 3-manifold. In this respect we establish multiple existence for the Gauss –Codazzi equation and describe the asymptotic behaviour of the solutions in terms of the marked conformal structure on the surface and the (prescribed) second fundamental form of the minimal immersion.

Correspondances for overdetermined elliptic problems

Martin Traizet
Université de Tours

We consider the following over-determined problem in the plane:

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ \dfrac{\partial u}{\partial \nu} = \text{constant} & \text{on } \partial\Omega \end{cases}$$

Under suitable hypothesis, we establish a correspondance between the solutions to this problem and the solutions to an over-determined problem of prescribed mean curvature type.

Recent results on Allen-Cahn equation

Juncheng Wei
University of British Columbia

In this talk I will first talk about a uniform $C^{2,\theta}$ estimate for level sets of stable solutions to the singularly perturbed Allen-Cahn equation in dimensions $n \leq 10$ (which is optimal). The proof combines two ingredients: one is the infinite dimensional reduction method which enables us to reduce the $C^{2,\theta}$ estimate for these level sets to a corresponding one on solutions of Toda system; the other one uses a small regularity theorem on stable solutions of Toda system to establish various decay estimates on these solutions, which gives a lower bound on distances between different sheets of solutions to Toda system or level sets of solutions to Allen-Cahn equation. ([2].) As a consequence we prove that finite Morse index solutions in $\mathbb{R}^2$ must be finitely ended ([3]). We also give a complete classification of finite Morse index solutions in $\mathbb{R}^2$ for a special bistable nonlinearity ([1]).

References

[1] Y. Liu and Juncheng Wei, A complete classification of finite Morse index solutions to elliptic sine-Gordon equation in the plane, arXiv:1806.06921
[2] Kelei Wang and Juncheng Wei, Second order estimates on transition layers, arXiv:1810.09599
[3] Kelei Wang and Juncheng Wei. Finite morse index implies finite ends. *to appear in Communication on Pure and Applied Mathematics*, arXiv preprint arXiv:1705.06831, 2017.