Clases de Master FisyMat, Desarrollos Actuales

The Standard Model of particle physics and Beyond

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Organizer and lecturer: Abdelhak Djouadi (adjouadi@ugr.es)

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2.1 The standard Higgs particle

We come to the Higgs boson itself, the scalar degree of freedom that was left after SSB. The kinetic part, $\frac{1}{2}(\partial_\mu H)^2$, comes from the covariant derivative $|D_\mu \Phi|^2$, while the mass and self–interaction parts, come from the potential $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$

$$V = \frac{\mu^2}{2}(0, v+H)\begin{pmatrix} 0 \\ v+H \end{pmatrix} + \frac{\lambda}{4}\left|(0, v+H)\begin{pmatrix} 0 \\ v+H \end{pmatrix}\right|^2 \tag{1}$$

Using the relation $v^2 = -\mu^2/\lambda$, one obtains

$$V = -\frac{1}{2}\lambda v^2\,(v+H)^2 + \frac{1}{4}\lambda(v+H)^4 \tag{2}$$

and finds that the Lagrangian containing the Higgs field H is given by

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V = \frac{1}{2}(\partial^\mu H)^2 - \lambda v^2\, H^2 - \lambda v\, H^3 - \frac{\lambda}{4} H^4 \tag{3}$$

From this Lagrangian, one can see that the Higgs boson mass simply reads

$$M_H^2 = 2\lambda v^2 = -2\mu^2 \tag{4}$$

and the Feynman rules[1] for the Higgs self–interaction vertices are given by

$$g_{H^3} = (3!) i\lambda v = 3i\,\frac{M_H^2}{v}\;,\;\; g_{H^4} = (4!) i\frac{\lambda}{4} = 3i\frac{M_H^2}{v^2} \tag{5}$$

[1] The Feynman rule for these vertices are obtained by multiplying the term involving the interaction by a factor $-i$. One includes also a factor $n!$ where n is the number of identical particles in the vertex.

As for the Higgs boson couplings to gauge bosons and fermions, they are related to their masses as can be seen from from Lagrangian describing these mass terms,

$$\mathcal{L}_{M_V} \sim M_V^2 \left(1+\frac{H}{v}\right)^2 \ , \ \ \mathcal{L}_{m_f} \sim -m_f \left(1+\frac{H}{v}\right) \tag{6}$$

from which one obtains also the Higgs boson couplings to gauge bosons and fermions

$$g_{Hff} = i\frac{m_f}{v} \ \ , \ g_{HVV} = -2i\frac{M_V^2}{v} \ \ , \ g_{HHVV} = -2i\frac{M_V^2}{v^2} \tag{7}$$

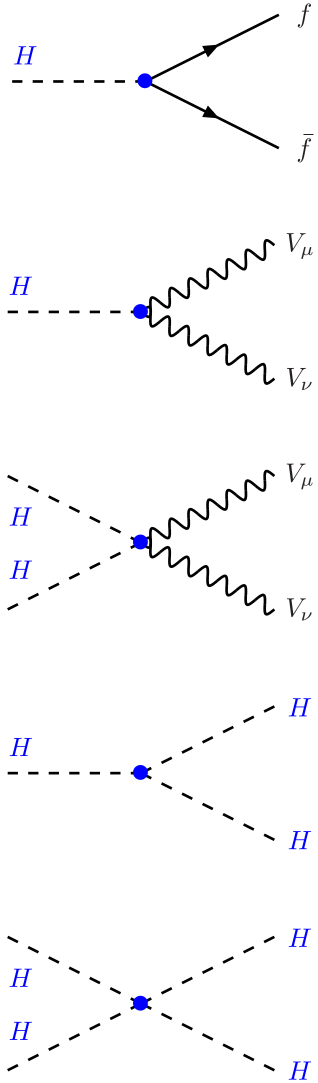
This form of the Higgs couplings ensures the unitarity of the theory as will be seen later. The vacuum expectation value v is fixed in terms of the W boson mass M_W or the Fermi constant G_μ precisely determined from muon decay, $G_\mu = 1.16637 \cdot 10^{-5}$ GeV^{-2},

$$M_W = \frac{1}{2} g_2 v = \left(\frac{\sqrt{2} g^2}{8 G_\mu}\right)^{1/2} \Rightarrow v = \frac{1}{(\sqrt{2} G_\mu)^{1/2}} \simeq 246 \text{ GeV} \tag{8}$$

Coming to the Feynman rules, the Higgs propagator in momentum space is given by

$$\Delta_{HH}(q^2) = \frac{i}{q^2 - M_H^2 + i\epsilon} \tag{9}$$

while the various Higgs couplings to fermions and gauge bosons and the self-couplings (using both v or G_μ) are given by



$$g_{Hff} \;=\; m_f/v \;=\; (\sqrt{2}G_\mu)^{1/2}\, m_f \qquad \times\,(i)$$

$$g_{HVV} \;=\; 2M_V^2/v \;=\; 2(\sqrt{2}G_\mu)^{1/2}\, M_V^2 \times\,(-ig_{\mu\nu})$$

$$g_{HHVV} \;=\; 2M_V^2/v^2 \;=\; 2\sqrt{2}G_\mu\, M_V^2 \times\,(-ig_{\mu\nu})$$

$$g_{HHH} \;=\; 3M_H^2/v \;=\; 3(\sqrt{2}G_\mu)^{1/2}\, M_H^2 \;\; \times\,(i)$$

$$g_{HHHH} = \; 3M_H^2/v^2 \;=\; 3\sqrt{2}G_\mu\, M_H^2 \qquad \times\,(i)$$

Figure 1: The Higgs boson couplings to fermions and gauge bosons and the Higgs self–couplings in the SM. The normalization factors of the Feynman rules are also displayed.

2.2 The gauge and the Goldstone bosons

In the unitary gauge, the SM physical spectrum consists of the fermions, the massless photon (and gluons) and massive $V = W^\pm, Z$ bosons (and no Goldstones) with propagators

$$\Delta_{VV}^{\mu\nu}(q) = \frac{-i}{q^2 - M_V^2 + i\epsilon}\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{M_V^2}\right] \tag{10}$$

The term $\propto g^{\mu\nu}$ is for the transverse component [the propagator of the photon is $-ig^{\mu\nu}/q^2$] and term $\propto q_\mu q_\nu$ for the longitudinal one which does not vanish at high q^2 (not $\propto 1/q^2$). This terms complicates calculations of amplitudes and makes renormalisation very tricky. It is more convenient to work in R_ξ gauges where gauge fixing terms are added to $\mathcal{L}_{\rm SM}$

$$\mathcal{L}_{\rm GF} = \frac{-1}{2\xi}\left[2(\partial^\mu W_\mu^+ - i\xi M_W w^+)(\partial^\mu W_\mu^- - i\xi M_W w^-) + (\partial^\mu Z_\mu - i\xi M_Z w^0)^2 + (\partial^\mu A_\mu)^2\right], \tag{11}$$

where $w^0 \equiv G^0, w^\pm \equiv G^\pm$ and different ξ correspond to different renormalizable gauges. In this case, the propagators of the massive gauge bosons are given by

$$\Delta_{VV}^{\mu\nu}(q) = \frac{-i}{q^2 - M_V^2 + i\epsilon}\left[g^{\mu\nu} + (\xi - 1)\frac{q^\mu q^\nu}{q^2 - \xi M_V^2}\right] \tag{12}$$

which in the unitary gauge, $\xi = \infty$, reduces to the expression given above. Usually, one uses the 't Hooft–Feynman gauge $\xi = 1$, where the $q^\mu q^\nu$ term is absent, to simplify the calculations; another choice is the Landau gauge, $\xi = 0$.

In renormalizable R_ξ gauges, the propagators of the Goldstone bosons are given by[2]

$$\Delta_{w^0w^0}(q^2) = \frac{i}{q^2 - \xi M_Z^2 + i\epsilon} \ , \quad \Delta_{w^\pm w^\pm}(q^2) = \frac{i}{q^2 - \xi M_W^2 + i\epsilon} \tag{13}$$

– In unitary gauge $\xi = \infty$, Goldstones do not propagate and decouple as they should.
– In the Landau gauge they are massless and do not interact with the Higgs particle.
– In 't Hooft–Feynman gauge, Goldstones are in the spectrum and have "masses" $\propto M_V$.

The couplings of Goldstones to fermions, as for the Higgs, are proportional to their masses

$$g_{G^0ff} = -2I_f^3 \frac{m_f}{v} \ , \quad g_{G^-ud} = \frac{-i}{\sqrt{2}v}[m_d(1-\gamma_5) - m_u(1+\gamma_5)] \tag{14}$$

The couplings of the Goldstones to gauge bosons are simply those of spin–zero particles.

The longitudinal components of the W and Z bosons give rise to interesting features at high energies. In the gauge boson rest frame, one can define the transverse and longitudinal polarization four–vectors as

$$\epsilon_{T_1}^\mu = (0,1,0,0) \, , \quad \epsilon_{T_2}^\mu = (0,0,1,0) \, , \quad \epsilon_L^\mu = (0,0,0,1) \tag{15}$$

For a four–momentum $p^\mu = (E, 0, 0, |\vec{p}|)$, after a boost along the z direction, the transverse

[2] Any dependence on ξ should however be absent from physical matrix elements squared, as the theory must be gauge invariant.

polarizations remain the same while the longitudinal polarization becomes

$$\epsilon_L^\mu = (|\vec{p}|/M_V, 0, 0, E/M_V) \stackrel{E \gg M_V}{\longrightarrow} p_\mu/M_V \tag{16}$$

Since this polarization is proportional to the gauge boson momentum, at very high energies, the longitudinal amplitudes will dominate in the scattering of gauge bosons.

In fact, there is a theorem, called the Electroweak Equivalence Theorem, which states that at very high energies, the longitudinal massive vector bosons can be replaced by the Goldstone bosons. And in many processes, such as vector boson scattering, the vector bosons themselves can by replaced by their longitudinal components.
The amplitude for the scattering of n gauge bosons in the initial state to n' gauge bosons in the final state is simply the amplitude for the scattering of the corresponding Goldstones

$$\begin{aligned} A(V^1 \cdots V^n \to V^1 \cdots V^{n'}) &\sim A(V_L^1 \cdots V_L^n \to V_L^1 \cdots V_L^{n'}) \\ &\sim A(w^1 \cdots w^n \to w^1 \cdots w^{n'}) \end{aligned} \tag{17}$$

Thus, in this limit, one can simply replace in the SM scalar potential, the W and Z bosons by their corresponding Goldstone bosons $w^\pm, w_0$, leading to a simple potential

$$V = \frac{M_H^2}{2v}(H^2 + w_0^2 + 2w^+w^-)H + \frac{M_H^2}{8v^2}(H^2 + w_0^2 + 2w^+w^-)^2 \tag{18}$$

which can be use to calculate the amplitudes for processes involving weak vector bosons. The calculations are then very simple, as one deals only with interactions among scalars.

2.3 The Standard Model and its parameters

The Standard Model (SM) refers to SU(3)×SU(2)×U(1) gauge invariance when combined with the electroweak symmetry breaking mechanism; we summarise here the main features. The rotations which led to the physical W, Z masses define the EW mixing angle $\sin\theta_W$

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}} = \frac{e}{g_2} = \sqrt{1-M_W^2/M_Z^2}\,, \tag{19}$$

Using the fermionic part of $\mathcal{L}_{\rm SM}$ with an explicitly covariant derivative, one obtains

$$\mathcal{L}_{\rm NC} = eJ_\mu^{\rm A}A^\mu + \frac{g_2}{\cos\theta_W}J_\mu^Z Z^\mu\ , \quad \mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}}(J_\mu^+ W^{+\mu}\,, + J_\mu^- W^{-\mu}) \tag{20}$$

for the neutral and charged current parts, respectively. The currents J_μ are then given by

$$J_\mu^A = Q_f\bar{f}\gamma_\mu f\ ,\ J_\mu^Z = \frac{1}{4}\bar{f}\gamma_\mu[(2I_f^3 - 4Q_f\sin^2\theta_W) - \gamma_5(2I_f^3)]f\ ,\ J_\mu^+ = \frac{1}{2}\bar{f}_u\gamma_\mu(1-\gamma_5)f_d\,, S \tag{21}$$

where $f_u(f_d)$ is the up–type (down–type) fermion of isospin $I_f^3 = +(-)\frac{1}{2}$ and charge Q_f. In terms of these and with $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$, the f couplings to gauge bosons are

$$Zf\bar{f}\ :\quad v_f = \frac{\hat{v}_f}{4s_Wc_W} = \frac{2I_f^3 - 4Q_fs_W^2}{4s_Wc_W}\quad ,\quad a_f = \frac{\hat{a}_f}{4s_Wc_W} = \frac{2I_f^3}{4s_Wc_W} \tag{22}$$

$$Wf\bar{f}'\ :\quad v_f = a_f = \frac{1}{2\sqrt{2}s_W} = \frac{\hat{a}_f}{4s_W} = \frac{\hat{v}_f}{4s_W} \tag{23}$$

In the charged case, there is a complication when extending to three quark families: current eigenstates q' are not identical to mass eigenstates q. If u–quarks are mass eigenstates, in the down–quark sector, the two sets are connected by a unitary transformation

$$(d', s', b') = V_{\rm CKM}(d, s, b) \tag{24}$$

where $V_{\rm CKM}$ is a 3×3 matrix. which is unitary, insuring that the neutral currents (NC) are diagonal in both bases: this is the GIM mechanism which ensures a natural absence of very constrained flavor changing neutral currents (FCNC) at tree–level in the SM.

For leptons, the mass and current eigenstates coincide since in the SM, the neutrinos are assumed to be massless, which is an excellent approximation in most purposes.

The relative strength of the CC and NC, $J_Z^\mu J_{\mu Z}/J^{\mu +} J_\mu^-$ can be measured by the parameter

$$\rho = M_W^2/(c_W^2 M_Z^2) \quad (= 1 \text{ at tree level in SM}) \tag{25}$$

This is a direct consequence of the choice of the Higgs representation that leads to EWSB. In a model with an arbitrary number of multiplets Φ_i with isospin I_i, and vevs v_i, one has

$$\rho = \frac{\sum_i \left[I_i(I_i+1) - (I_i^3)^2 \right] v_i^2}{2\sum_i (I_i^3)^2 v_i^2} \tag{26}$$

which equals also unity for an arbitrary number of doublet (as well as singlet) fields: SM has a custodial SU(2) global symmetry broken by hypercharge (in radiative corrections).

Finally, let us list the basic inputs of the Standard Model and their numerical values (which we will need in our discussion of Higgs phenomenology in the next lectures).

• The QED fine structure constant defined in the classical Thomson limit $q^2 \sim 0$ of Compton scattering, is one of the best measured quantities in Nature

$$\alpha(0) \equiv e^2/(4\pi) = 1/137.03599976\,(50) \tag{27}$$

However, it evolves with energies and the running between $q^2 \sim 0$ and scales $\mathcal{O}(M_Z)$ gives

$$\alpha(M_Z^2) = \frac{\alpha(0)}{1-\Delta\alpha}\,,\;\; \Delta\alpha(M_Z^2) = \Pi_{\gamma\gamma}(0) - \Pi_{\gamma\gamma}(M_Z^2) \simeq \sum_{\rm light f} \frac{\alpha}{3\pi}\left[\log\frac{M_Z^2}{m_f^2} - \frac{5}{3}\right] + \cdots \tag{28}$$

where only light particles contribute (decoupling theorem). For quarks one must include (complicated) higher orders in strong interactions. At the end, one gets $\alpha(M_Z^2) \simeq 1/129$.

• The Fermi coupling constant which is precisely measured in muon decays to be

$$G_\mu = (1.16637 \pm 0.00001)\cdot 10^{-5}\ {\rm GeV}^{-2} \tag{29}$$

In the SM, the decay $\mu^- \to \nu_\mu e^- \bar\nu_e$ is mediated by W bosons and one gets the relation

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_2}{2\sqrt{2}}\cdot\frac{1}{M_W^2}\cdot\frac{g_2}{2\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 s_W^2} = \frac{\pi\alpha}{2M_W^2(1-M_W^2/M_Z^2)} \tag{30}$$

• The strong coupling constant is precisely determined in various experiments to be

$$\alpha_s(M_Z^2) = 0.118 \pm 0.002 \tag{31}$$

for a QCD scale $\Lambda^5_{\rm QCD} = 216^{+25}_{-24}$ MeV and for 5 light flavors (the top is integrated out). In fact, the coupling also runs with energy and one has at scale μ at leading order

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ell_\mu}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log \ell_\mu}{\ell_\mu} + \cdots\right] \quad \text{with } \ell_\mu \equiv \log(\mu^2/\Lambda^2)\,,\; \beta_0 = 11 - \frac{2}{3}N_f \tag{32}$$

with N_f being the number of quarks that have a mass smaller than the energy scale μ. At scales close to a few times $\Lambda_{\rm QCD}$ the coupling is too large and perturbativity is lost.

- For the fermion masses, those of charged leptons are given by

$$m_\tau = 1.777 \text{ GeV}\,,\; m_\mu = 0.1056 \text{ GeV}\,,\; m_e = 0.511 \text{ MeV} \tag{33}$$

while their neutrinos partners are considered to massless (which is correct in most cases). The heavy quarks have masses (the pole masses defined in the on-shell scheme) are

$$m_t = 171.0 \pm 1 \text{ GeV}\,,\; m_b = 4.88 \pm 0.07 \text{ GeV}\,,\; m_c = 1.64 \pm 0.07 \text{ GeV} \tag{34}$$

while lighter quarks can be considered to be massless as $m_s \approx 0.1$ GeV, $m_{u,d} \approx$ few MeV. But again, these masses are running parameters and they decrease at higher energy (e.g, in a scheme called $\overline{\rm MS}$, $\bar{m}_b(M_Z) \simeq 3$ GeV, $\bar{m}_c(M_Z) \simeq 0.6$ GeV and m_t is almost unchanged).

- Finally, the masses of the weak gauge bosons are precisely determined to be

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}\,,\; M_W = 80.425 \pm 0.034 \text{ GeV} \tag{35}$$

while their total decay widths are approximately $\Gamma_Z = 2.5$ GeV and $\Gamma_W = 2.1$ GeV.

To completes this parameter list, we need to include M_H. Constraints?

2.4 Constraints from perturbative unitarity

Let us discuss the amplitude for WW scattering shown in Fig.3 in the limit $s \gg M_W^2$.

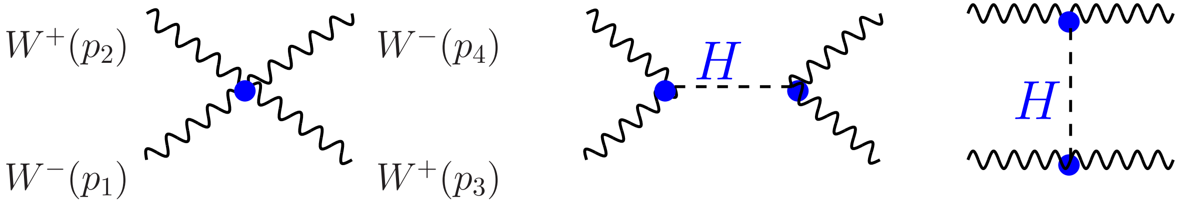


Figure 2: Some Feynman diagrams for the scattering of W bosons at high energy.

As seen in section 2.2, one can replace V bosons by their Goldstone partners and use the simple potential of eq. (18) for the interactions. One then obtains for the amplitude:

$$\begin{aligned} A(w^+w^- \to w^+w^-) &= -\left[2\frac{M_H^2}{v^2} + \left(\frac{M_H^2}{v}\right)^2 \frac{1}{s-M_H^2} + \left(\frac{M_H^2}{v}\right)^2 \frac{1}{t-M_H^2}\right] \\ &\xrightarrow{s \gg M_W^2} \frac{1}{v^2}\left[s+t-\frac{s^2}{s-M_H^2}-\frac{t^2}{t-M_H^2}\right] \end{aligned} \tag{36}$$

where s, t are the Mandelstam variables $s=(p_1+p_2)^2, t=(p_1-p_3)^2, u=(p_1-p_4)^2$. This will lead to a cross section $\sigma(W^+W^- \to W^+W^-)$ that violates the unitarity bound. To see it, decompose scattering amplitude A into partial waves a_ℓ of orbital angular momentum ℓ (P_ℓ are the Legendre polynomials and θ the scattering angle)

$$A = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)\, a_\ell \tag{37}$$

For a $2 \to 2$ process, one has $\mathrm{d}\sigma/\mathrm{d}\Omega = |A|^2/(64\pi^2 s)$ with $\mathrm{d}\Omega = 2\pi \mathrm{d}\cos\theta$, and thus

$$\sigma = \frac{8\pi}{s} \sum_{\ell=0}^{\infty} \sum_{\ell'=0}^{\infty} (2\ell+1)(2\ell'+1) a_\ell a_{\ell'} \int_{-1}^{1} \mathrm{d}\cos\theta P_\ell(\cos\theta) P_{\ell'}(\cos\theta) = \frac{16\pi}{s} \sum_{\ell=0}^{\infty} (2\ell+1)|a_\ell|^2 \quad (38)$$

where the orthogonality of Legendre polynomials, $\int \mathrm{d}\cos\theta P_\ell P_{\ell'} = \delta_{\ell\ell'}$, has been used.
The optical theorem: cross section is proportional to Im(A) in the forward direction,

$$\sigma = \frac{1}{s} \mathrm{Im}\,[\,A(\theta = 0)\,] = \frac{16\pi}{s} \sum_{\ell=0}^{\infty} (2\ell+1)|a_\ell|^2 \Rightarrow$$

$$|a_\ell|^2 = \mathrm{Im}(a_\ell) \Rightarrow [\mathrm{Re}(a_\ell)]^2 + [\mathrm{Im}(a_\ell)]^2 = \mathrm{Im}(a_\ell) \Rightarrow [\mathrm{Re}(a_\ell)]^2 + [\mathrm{Im}(a_\ell) - \frac{1}{2}]^2 = \frac{1}{4} \quad (39)$$

This is the equation of a circle of radius $\frac{1}{2}$ and center $(0, \frac{1}{2})$ in the plane $[\mathrm{Re}(a_\ell), \mathrm{Im}(a_\ell)]$.
The real part is between $-\frac{1}{2}$ and $\frac{1}{2}$. This is the unitarity condition: $|\mathrm{Re}(a_\ell)| < \frac{1}{2}$.
If one takes only the dominant $J = 0$ partial wave for the amplitude $A(w^+w^- \to w^+w^-)$

$$a_0 = \frac{1}{16\pi s} \int_s^0 \mathrm{d}t |A| = -\frac{M_H^2}{16\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log\left(1 + \frac{s}{M_H^2}\right)\right] \quad (40)$$

one obtains at c.m. energies much higher than the W and Higgs masses, $s \gg M_H^2 \gg M_W^2$,

$$a_0 \xrightarrow{s \gg M_H^2} -M_H^2/(8\pi v^2) \quad (41)$$

From the unitarity condition, $|\mathrm{Re}(a_0)| < \frac{1}{2}$, one has the upper bound $M_H \lesssim 870$ GeV.
This is the (perturbative as it is leading order only) unitarity bound on the Higgs mass.

In practice, the $W_L^+W_L^-$ scattering channel can be coupled with other channels like Z_LZ_L, HH, Z_LH and the charged channels $W_L^+H, W_L^+Z_L$ should be taken into account. The unitarity bound on the Higgs mass becomes then stronger and reads $M_H \lesssim 710\,\text{GeV}$.

$\Rightarrow$ In the SM, if $M_H \gtrsim \mathcal{O}(700)$, unitarity is violated unless new phenomena appear.[3]

In fact, one can use the unitarity argument discussed above in a different limit: if one assumes the Higgs mass to be much larger than $\sqrt{s}$ [which is much larger than M_W), the unitarity constraint writes, if one takes into account only the $W_L^+W_L^- \to W_L^+W_L^-$ channel,

$$a_0 \overset{s \ll M_H^2}{\longrightarrow} -s/(32\pi v^2) \tag{42}$$

and with the condition $|\text{Re}(a_0)| < \frac{1}{2}$, one obtains the constraint $\sqrt{s} \lesssim 1.7$ TeV

Again, a more stringent bound is obtained by considering all the coupled channels above

$$\sqrt{s} \lesssim 1.2 \text{ TeV} \tag{43}$$

This means that if the Higgs is too heavy [or, equivalently, not existing at all], New Physics beyond the SM should manifest itself at the TeV scale to restore unitary in VV scattering.

Hence, from the requirement that tree–level unitarity hold, one concludes that either:

- the Higgs boson is light, with a mass below the TeV scale, and the SM is fine;
- the Higgs is very heavy and new physics that plays its role should appear at 1 TeV;
- the breakdown is canceled by high–orders that signal the failure of perturbation theory.

$\Rightarrow$ Something should happen at the TeV scale!

[3]There is, however, a caveat as the analysis was only done at tree-level and the Higgs self–coupling becomes strong for large M_H, $\lambda = M_H^2/(2v^2)$. Hence, the radiative corrections can be very large and, eventually, render the theory non perturbative. The argument above should be thus called the tree–level unitarity or perturbative unitarity argument. Note however, that some non-perturbative simulations in lattice gauge theories support this result.

2.5 The triviality and stability bounds of the electroweak vacuum

As seen before, because of quantum corrections, the particle couplings and masses are not constant but depend on the considered energy scale: they are thus running parameters. The evolution is in general described by a Renormalization Group Equation (RGE).

This is the case for the quartic Higgs coupling $\lambda = M_H^2/(2v^2)$ which increases with $|q|$.

Leads to non-trivial constraints on the λ coupling and Higgs mass, that we summarize.

The triviality bound, on M_H when the couplings λ is large or the Higgs heavy.

The one-loop radiative corrections to Higgs self-coupling where only Higgs contributes:

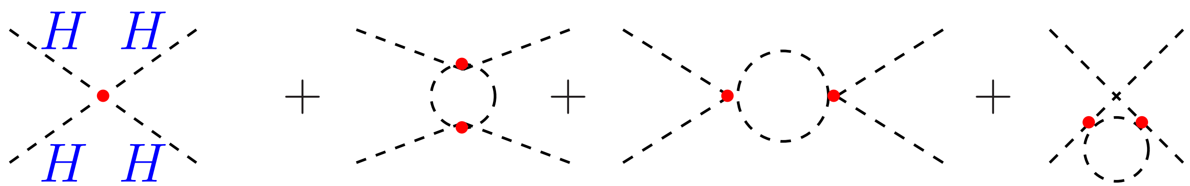


Figure 3: Typical Feynman diagrams for the tree–level and one–loop Higgs self–coupling.

The variation of λ with energy q described by an RGE that is given at one-loop by

$$\mathrm{d}\lambda(q^2)/\mathrm{d}q^2 = 3\lambda^2(q^2)/(4\pi^2) \; + \text{higher orders} \tag{44}$$

The solution of this equation, choosing the reference energy to the vev, $q_0 = v$, is then

$$\lambda(q^2) = \lambda(v^2)\left[1 - 3\lambda(v^2)/(4\pi^2)\log(q^2/v^2)\right]^{-1} \tag{45}$$

Hence, the quartic Higgs coupling varies logarithmically with the squared energy q^2.

If the energy is much smaller than the electroweak scale, $q^2 \ll v^2$, the quartic Higgs coupling is very small and eventually vanishes, $\lambda(q^2) \sim \lambda(v^2)/\log(\infty) \to 0_+$.

It is said that the theory is trivial, i.e. non interacting since the coupling is zero.

In the opposite limit, when the energy is much higher that weak scale, $q^2 \gg v^2$, the quartic coupling grows and eventually becomes infinite, $\lambda(q^2) \sim \lambda(v^2)/(1-1) \gg 1$. The point, called Landau pole, where the coupling becomes infinite is at the energy

$$\Lambda_C = v \exp\left(4\pi^2/3\lambda\right) = v \exp\left(4\pi^2 v^2/M_H^2\right) \tag{46}$$

The general triviality argument states that the scalar sector of the SM is a ϕ^4–theory, and for these to remain perturbative at all scales one needs a coupling $\lambda = 0$ (which in the SM, means that the Higgs is massless), thus rendering the theory trivial, i.e. non–interacting.

But one can turn the argument around: use the RGE for λ to establish the energy domain in which the SM is valid, i.e. the energy cut–off Λ_C below which λ remains finite.

In this case, from eq. (45), if $\Lambda_C \gg 1$, M_H should be small to avoid the Landau pole for which $3\lambda(v^2)/4\pi^2 \log(\Lambda^2/v^2) \approx 1$ and the coupling becomes infinite.

For instance, for $\Lambda_C \sim 10^{16}$ GeV, one needs a light Higgs, $M_H \lesssim 200$ GeV.

In turn, if the cut–off Λ_C is small, the Higgs boson mass can be rather large and for $\Lambda_C \sim 10^3$ GeV for instance, the Higgs mass is allowed to be of the order of 1 TeV.
In particular, if the cut–off is set at the Higgs boson mass itself, $\Lambda_C = M_H$, the requirement that the quartic coupling remains finite implies that $M_H \lesssim 700$ GeV.
Hence, we get a similar upper bound on M_H as from perturbative unitarity[4].

[4] But again, there is a caveat in the triviality argument: when $\lambda \gg 1$, one cannot use perturbation theory and the constraint is lost. However, from simulations of gauge theories on the lattice, where the non–perturbative effects are taken into account, one obtains the rigorous bound $M_H < 640$ GeV which is in agreement with the one above.

In the discussion before, only the Higgs contribution was included in the running of λ as we were interested in the regime $\lambda \gg 1$. Let us now discuss the regime where $\lambda \ll 1$. In this case, one needs to include fermion and gauge boson contributions in the running. Since H couples $\propto$ mass, only the top and massive boson contributions are important. Some one-loop Feynman diagrams for these additional contributions are given in Fig. 4.

Figure 4: Diagrams for the one–loop contributions of fermions and gauge bosons to λ.

The one–loop RGE for the quartic coupling, including these contributions, becomes

$$\frac{\mathrm{d}\lambda}{\mathrm{dlog}q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right] \quad (47)$$

where $\lambda_t = \sqrt{2}m_t/v$ is the top quark Yukawa. A first effect is that these contributions will slightly alter the triviality bound and the Λ_C value will depend on the precise value of m_t.

But it is for small λ values that these additional contributions can have a large impact and give some new information. Indeed, for $\lambda \ll \lambda_t, g_1, g_2$, the one-loop RGE becomes

$$\frac{\mathrm{d}\lambda}{\mathrm{dlog}Q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 - 12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right] \quad (48)$$

and its solution, taking again the electroweak scale v as the reference point, is

$$\lambda(q^2) = \lambda(v^2) + \frac{1}{16\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\frac{q^2}{v^2} \tag{49}$$

If λ is too small, the top contribution can be dominant and could make it negative $\lambda(q^2) < 0$, leading to a potential $V(q^2) < V(v)$. The vacuum is not stable anymore since it has no minimum. The stability argument tells us that to have a scalar potential that is bounded from below and, hence to keep $\lambda(Q^2) > 0$, the Higgs mass should be larger than

$$M_H^2 > \frac{v^2}{8\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\frac{q^2}{v^2} \tag{50}$$

This sets a strong constraint on M_H, which depends on the value of the cut–off scale Λ_C. For relatively low and very high values for this cut–off, one obtains

$$\begin{aligned} \Lambda_C \sim 10^3 \text{ GeV} \;&\Rightarrow\; M_H \gtrsim 70 \text{ GeV} \\ \Lambda_C \sim 10^{16} \text{ GeV} \;&\Rightarrow\; M_H \gtrsim 130 \text{ GeV} \end{aligned} \tag{51}$$

Note that the stability bound on M_H or Λ_C can be relaxed if the vacuum is metastable[5]

Thus, the positivity and the finiteness of the self–coupling λ impose, respectively, a lower bound $M_H \gtrsim 70$ GeV and an upper bound $M_H \lesssim 1$ TeV, for a cut–off $\mathcal{O}(1 \text{ TeV})$.

[5]The SM effective potential can have a minimum which is deeper than the standard electroweak minimum if the decay of the latter into the former, via thermal or quantum fluctuations, is suppressed. In this case, a lower bound on M_H follows from the requirement that no transition between the two vacua occurs and we always remain in the electroweak minimum. The obtained bound on M_H is in general much weaker than in the case of absolute stability of the vacuum.

Of course, the previous bounds are only approximative and to have more precise ones, some refinements must however be included. In particular one should:

- include the higher orders in the β functions of all SM couplings;
- take care of theory uncertainties in the cut–off close to perturbative breakdown;
- take care of the matching, ie the precise relation between particle masses and couplings;
- take care of the experimental uncertainties in the parameters (in particular m_t and α_s)

The results of a detailed analysis that includes all these effects is shown in Fig. 5. These are the triviality–stability bounds The width of the bands is due to the various experimental and theoretical errors. The allowed value of the Higgs mass are those that lie between the two bands.

If the New Physics scale $\Lambda_C \approx 1$ TeV, the Higgs mass is allowed to be in the range

$$50 \text{ GeV} \lesssim M_H \lesssim 800 \text{ GeV} \quad (52)$$

While requiring the SM to be valid up to the scale $\Lambda_{\rm GUT} \sim 10^{16}$ GeV, one has

$$130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV} \quad (53)$$

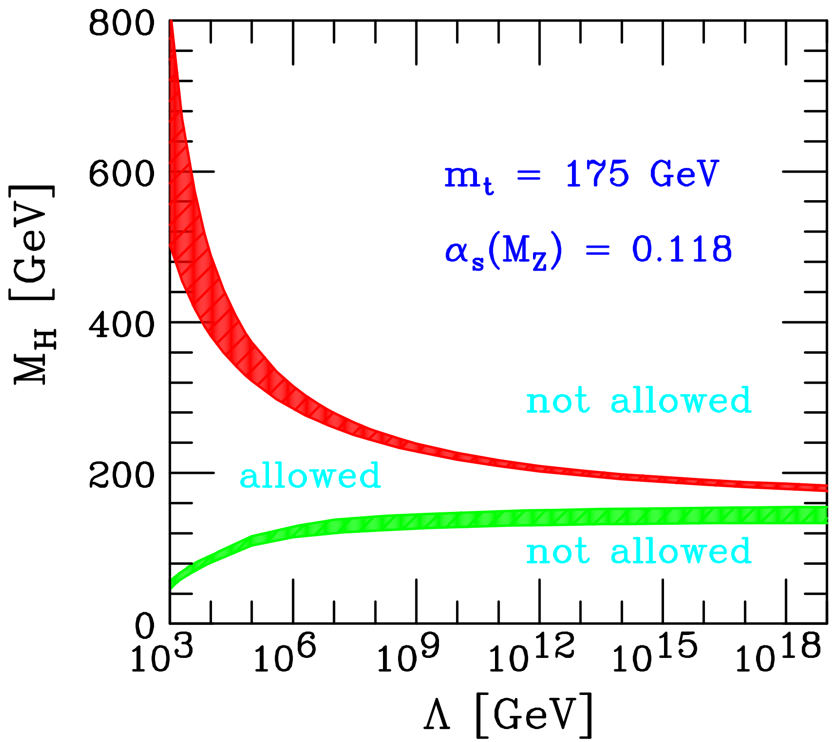


Figure 5: The triviality (upper) and vacuum stability (lower) bounds on M_H as a function of Λ for some SM parameters.