Space, time, Spacetime

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Outline

The vanishing of time

Relativity is the theory of the vanishing of time.
Only space-time exists and it is not possible to define canonically a time.
One defines time-related notions (time-functions, proper times...) but thy do not have the usual properties of time.
The theory is invariant w.r.t. time reparametrization.

A Newtonian analogy

- basic physics Before Newton :
  horizontals (2d) + vertical (1d)
  (Aristote : the natural motion of element earth is the vertical)
  Fundamental *anisotropy* of " space".
- Newton introduces *3D isotropic space*:
  no distinction between dimensions
  all rotations permitted (including those mixing vertical and horizontal)
- No way to define the vertical (excepted on earth)
  On earth, Newton attributes the vertical to something new, not a property of space : the (local) *gravitation* force

Where is the vertical in empty space ?

A Newtonian analogy

Where is the vertical in empty space ?

- direction of earth ?
- of the nearest planet ? star ?
- the vertical direction of my body ?
- I can fix it thanks, e.g., to a gravitational force (something *in addition* to space) (where is vertical at the center of earth ?)

2d + 1d $\rightarrow$ 3d *isotropic* Euclidean space (no other choice)
2d rotations : $\rightarrow$ 3d-rotations ; groups : SO(2)$\rightarrow$ SO(3)

Special relativity

3d space + 1d time $\rightarrow$ 4d Minkowski spacetime

isotropy of space-time: all directions are equivalent (including the temporal ones)

groups : SO(3) $\rightarrow$ SO(3,1) = Lorentz group

status of time in Minkowski spacetime

$\simeq$ **status of vertical in Newtonian space**

No way to define *time*. But

- One may chose arbitrary *time functions* in space-time: timelike gradient

- *proper time* = metric length defined along each time like line.

Proper time

Each *time-like line* in space-time is potentially the world line of an observer: (the line of his successive positions in space-time)

An observer (or a particle) *is* a world line.

the metric structure of space-time associates to each observer his *proper time*; but

- defined along his own world line only (where the observer stands): not elsewhere in space-time .
- only for him : even at the same position, two observers O and O' do not share the same proper time.
- it is not possible to compare directly proper times of different observers

(excepted in the trivial case when they share the same world line)

no meaning to speak about time dilatation (or contraction)

No time in space-time

No way to define directly a time in Minkowski spacetime.
One may chose an *arbitrary time function*. Nothing physical could depend on this choice.
Extend the proper time of an observer (defined on his world line) to whole space-time? (to construct a time function)?
No canonical way either to do so.

(excepted in some particular cases like inertial observer in Minkowski spacetime.)

This is dangerous (see below) and leads to wrong interpretations;
and not possible in general (in general relativity).

Time properties ?

The basic property of Newtonian time is *datation*. All other properties follow. I propose to call

- *time* a process which permits datation with all the consequences (below), like Newtonian time.
- and *time function* a datation without necessarily al these properties.

Time functions exist in special relativity, and locally in general relativity.

In great number with no one privilegiated.

None has the properties of time.

Time properties ?

datation: to each event, Nw time assigns a *date*.
chronology a total ordering of events
causality: what events can cause an event ? (= chronology in Nw physics)
simultaneity : what events are simultaneous ?
synchronization: can one compare clock times and clock rates ?
duration = between two events ; of a process
space = set of synchronous events ?
In Newtonian Physics, all these properties result from datation.

Special $\rightarrow$ general relativity

The situation is worse

- Time functions may not exist globally

In any case, remain *arbitrary* : one needs something external to chose one.

Different time functions give different *incompatible datations*;

- proper time always well defined along world lines

gives no datation (except along the corresponding world line)

$\rightarrow$ no datation of events $\rightarrow$ no chronology

no absolute simultaneity ? (Einstein's starting point)

One may define "simultaneity *from the point of view of an observer*" (Einstein procedure).

Events simultaneous for O are not for O': different and contradictory notions of simultaneity for different observers ; no one privileged (except me, of course)

e.g., GPS: speaking about "the position of the satellite now" has no meaning

simultaneity

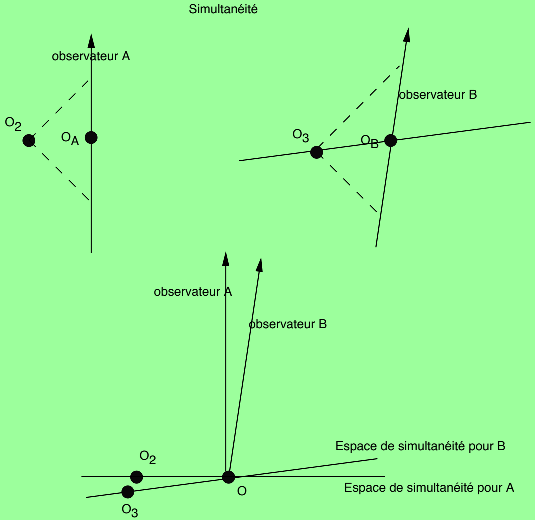


Figure: simultaneity

Time in general relativity

Time in general relativity

Time functions arbitrarily / Proper time local and subjective
But absolute *causal structure*(= *conformal part* of the metric)
(global causality may be violated);
not related to any chronology or time function ;
not a total but a *partial* order (two events may be non causally related) → no present.

durations

[proper]duration = metric length along a time-like curve.
The duration between two events depends on the history between them: *Langevin twins*

(exactly like the distance between Paris and Marseille depends on the road you chose).

There is a curve with longest duration : the geodesic (straight in flat space-time).

Proper durations differ (in general) from time [function]-intervals.

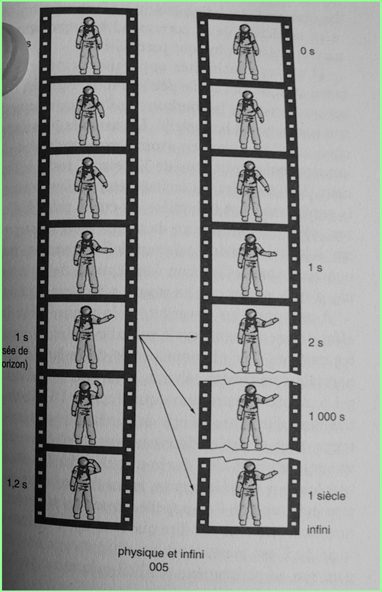
→ no time in general in GR.

Time functions or proper times do not have the properties of time.

Time related notions in GR

not time functions, no time ;
but causality, proper times (can be extended only approximatively and subjectively);
and redshifts
You cannot analyze an "extended" system by assuming space, time and velocities; "extended" depends on the required precision
- For ordinary life, the Solar System (even the Galaxy) is OK.
- For the plane experiments testing Langevin twins "paradox", for the GPS, for Pioneer effect (see Giulini's talk) Solar System is too big.
- When using the last atomic clocks (optical fountains), you cannot assume time flowing at a scale of *one meter*.
One must carefulwhen trying to define *velocities* between objects at different locations
Time travel perfectly possible in the future (no paradox); a general statement : " time travel "is impossible only if time exists."

black holetime



general relativity: special cases

Minkowski spacetime appears as a vacuum solution of general relativity: maximally symmetric, flat; no matter
de Sitter space-time : maximally symmetric, with constant curvature; requires a cosmological constant; describes approximatively our universe ; no matter, cosmological constant
Friedmann-Lemaître models : admit maximally symmetric spatial sections (cosmological principle)
Other simple models :
Schwarzschild solution (vacuum around a spherically symmetric source)
perturbations of these models

Time and GR

We would like time (?)

- to interpret and express evolution
- to quantize (see below); but then there is no metric (there is, i.e., a wave function in the space of metrics), so not even proper time and causality can exist.

Observables commute with the Hamiltonian $\rightarrow$ constant quantities: no evolution ? What this means ?

"TIME PROBLEM"

$\rightarrow$ Relational time, thermal time

Relational

Any measurement is in fact a correlation;
- a measurement *of time* is a correlation between some event and the indication of a clock.
- A measurement *at time t* : the same (implicitly)
One always monitors correlations between systems; most often one system is a *clock*. **There is no time; there are clocks**.
Every observable is such a correlation; and can be expressed as a *time independent quantity* (Rovelli)→ express usual physics (including quantum) with only *time-independent observables*.
This is only a reinterpretation of physics, with the same results and predictions, expressed differently.
But it generalizes to the case without time : general relativity (cf GPS, Pioneer effect...), quantum gravity
How to recover time ? (one aspect of the pb of time) : material clocks, scale factor as a clock in cosmology, thermal time...

Thermal time

Tomita-Takesaki, Alain Connes...: modular group

Any non-commutative algebra admits a one-parameter group of external isomorphisms (canonical up to internal endomorphisms).

They interpret it as a *dynamical evolution*, with one parameter called *"thermal time"*.

Every physical system is related to a non-commutative algebra : *Poisson* (classical) or *C⋆-algebra* (quantum).

Also *causality* is expressed by *non-commutativity*.

General ansatz? time = non commutativity

Applications (Connes, Rovelli, Smerlak, Martinetti...)

Thermal time/2

- *classical system at equilibrium* : thermal time = usual time × temperature.
- *Unruh observer* : → thermal time = proper time × temperature
- *Hawking observer* → Hawking temperature of black hole, generalize to any situation with *horizons*
- Use networks of algebras of operator-valued functions, with commutation relations given by *causality*.

(note: *equivalence geometry – algebra* by Gel'fand theorem, interpreted as a categorical equivalence.)

A physical system generates its own evolution with a thermal time.

When a temperature is present, this gives a physical time.

When a physical time (i.e., proper time) is present, this gives a temperature.

→ Links with thermodynamics and gravitation, temperature and entropy of horizons (generalizing black holes), entropic forces, holographic conjecture...

Thermal time/3

also related to *categorical (topos) interpretation of quantum physics* (Isham, Butterfield, Doering...)
commutative sub-algebras →
Heyting (not Boole) algebra of quantum propositions : intuitionist logic

(co-Heyting → para-consistent logic ?)

Also topos interpretation of causal sets (= discrete models of space-time) (Markopoulou);

n.b. causal sets as an answer to the difficulties with infinite mentioned in G Ellis talk)?)

intuitionist logic for relativistic propositions ? Work to be done

(*Categories and Physics*, colloque FFP10, Frontiers of Fundamental Physics, Perth (Australie) 2010,

American Institute of Physics conference proceedings vol 1246, p 114-126

(`http://proceedings.aip.org/proceedings`,

`http://scitation.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1246&Issue=1`)

Cosmology

Cosmological principle $\rightarrow$ space-time admits homogeneous isotropic sections : *Friedmann-Lemaître models*.
This simple symmetric situation allows us to select a particular time function called cosmic time:
(may not be generalized to arbitrary space-time).
The sections foliate space-time

$\rightarrow$ a family of particular *time functions*.

(= level surfaces are the symmetric spatial sections)

The world lines of *comoving matter* are orthogonal to these sections.
Normalize (time function = proper time along comoving observer's world line)

$\rightarrow$ *cosmic time*

Simpler definition

cosmic time (O) = max (all proper durations from any M to O)

Cosmic time

Cosmic time does not share the properties of time !

- defines a chronology
- identifies with "physical time" only for a class of observers (including us) (equivalence principle) but not for the general observer (even inertial)
- durations between events *are not* the cosmic time differences
- events at same value of cosmic time *are not* simultaneous (surfaces of simultaneity have constant conformal time)

(*Space and Observers in Cosmology*, Marc Lachièze-Rey 2001, A&A, 376, 17-27 (gr-qc/0107010))

only approximately valid for *perturbed cosmology*, i.e., with real matter in it (osculating models)

Cosmological effects in the local static frame, M. Mizony and Marc Lachièze-Rey 2004, A&A 434 1 (2005) 45-52, (gr-qc/0412084)

In cosmology

even if cosmic time gives a convenient chronology, better use *scale factor* (accessible through *redshift*, after subtracting proper motion)

Cosmic time
- is not accessible,
- is model dependent (any expression in cosmic time is a model dependent reconstitution)
- and *gives a false illusion of time*: adapted to common sense but not to physical cosmology.

One also may use radar (=GPS) coordinates which have a covariant character

The covariance of GPS coordinates and frames, Lachièze-Rey M. 2005, Class. Quantum Grav. 23 (2006) 3531-3544 http://fr.arxiv.org/abs/gr-qc/0602052

Intermede: "real quantities" in GR

(*Réalisme relativiste*, Lachièze-Rey M. 2008,

in "Réalisme et thories physiques", Cahiers de philosophie de l'Université de Caen n. 45)

They must be covariant, lead to measurable quantities (real, thus covariant, and scalar).

Such quantities can only be obtained by contractions of covariant tensors.

$\rightarrow$ "real quantities" are "natural operators"

natural means canonically transformed by diffeos : tensors and connections.

operators mean that they can act on each other.

Examples: the metric tensor, the curvature, the connection, the 4-velocity, the electromagnetic [Maxwell] 2-form , the energy-momentum tensor...

no space, no time

(In fact the *real* quantity is the diffeomorphism-class of such objects.)

Quantize gravity (naive approach)

dynamical variable = the metric g (in given manifold with fixed topology).
canonically quantization:
- functions $\rightarrow$ operators
acting on Hilbert space of states (states $\simeq$ wave-functions)
(here functions of the possible values of g)
requires a canonical form; requires time.
$\rightarrow$ Introduce an *arbitrary* time function t (= foliation) : ADM formalism
Write usual Einstein-Hilbert Lagrangian (diffeo- and Lorentz invariant).
No time derivatives of g_{00} or g_{0i} enter in action.
$\rightarrow$ 4 local *constraints* (the conjugate momenta are identically zero (or imposed by matter): the sum of the gravitational and matter energy-momentum densities =0)
The gravitational dof's are the components of the spatial metric g_{ij}: a 3-geometry at fixed T
Configuration space = *superspace* ($\rightarrow$ mini-superspace in cosmology)

Quantize gravity

The constraints impose zero Hamiltonian. This due due to time-reparametrization invariance (the time component of diffeo-invariance).
(the technical expression of absence of time), at the origin of the so called " time problem".
For practical reasons , express gravity not by metric but by tetrads +connection (Palatini)
Introduce Ashtekar connections ; find Hilbert space as spin networks; turn to fluxes [of tetrads] and *holonomies* [of connections] description ... ...

Quantize cosmology

The cosmological principle imposes symmetries and reduces the nb of degrees of freedom.
For FrL cosmology, only scale factor a +matter (generally described as a scalar field): *minisuperspace*
- canonical quantization → Wheeler-de Wittequation
- following LQG (loop quantum gravity), one can adopt the connection representation, → holonomy representation
→ LQC (loop quantum cosmology)
Solutions appear to be "nearly classic";
→ described as *effective FrL models*.
(= FrL models with corrected Einstein equations)
They have generically cosmic bounce (independently of matter content); inflation (useless), super-inflation...
study fluctuations → observable tracks ?