# “CAT WITHIN THE IMAGE”

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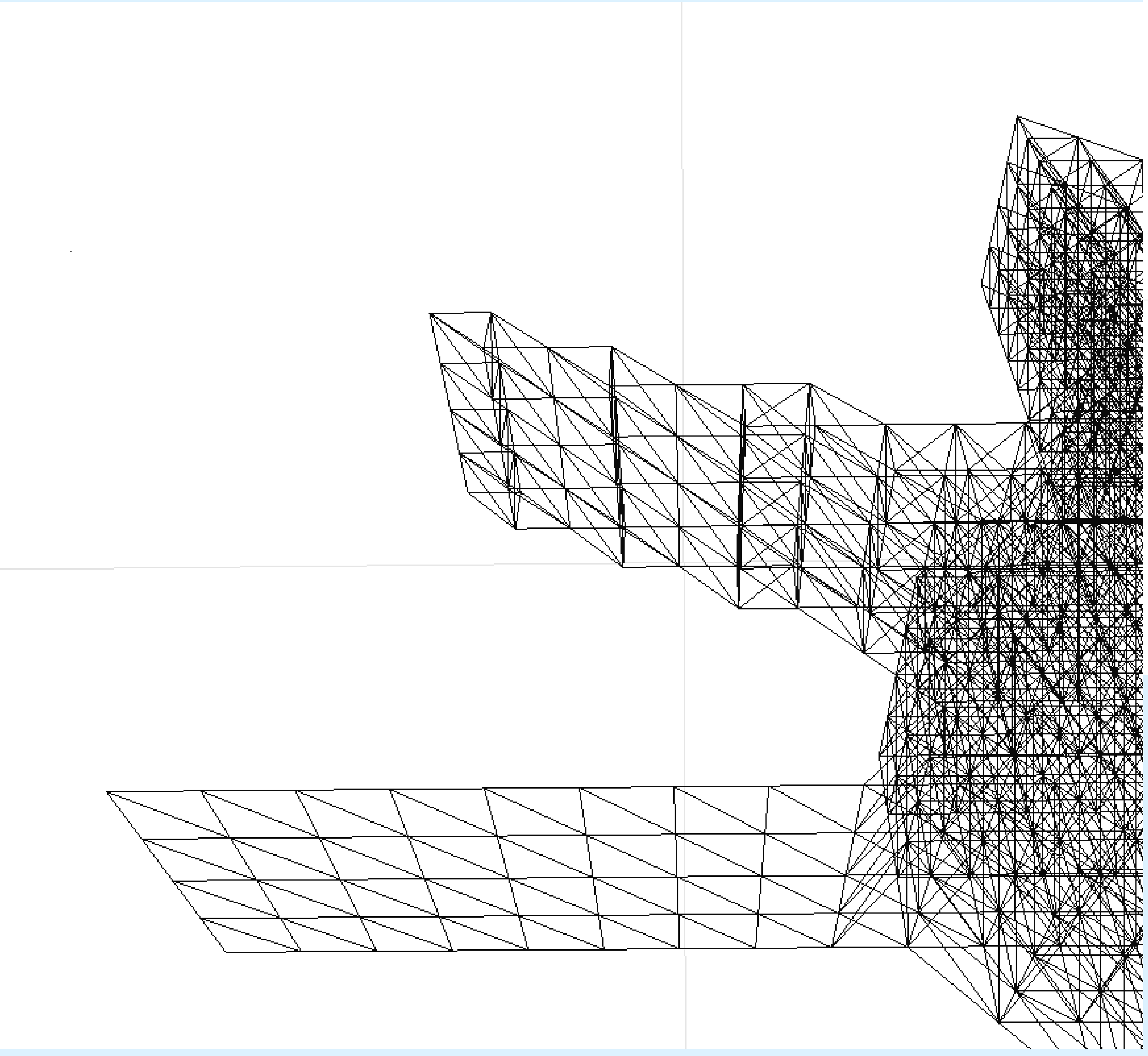
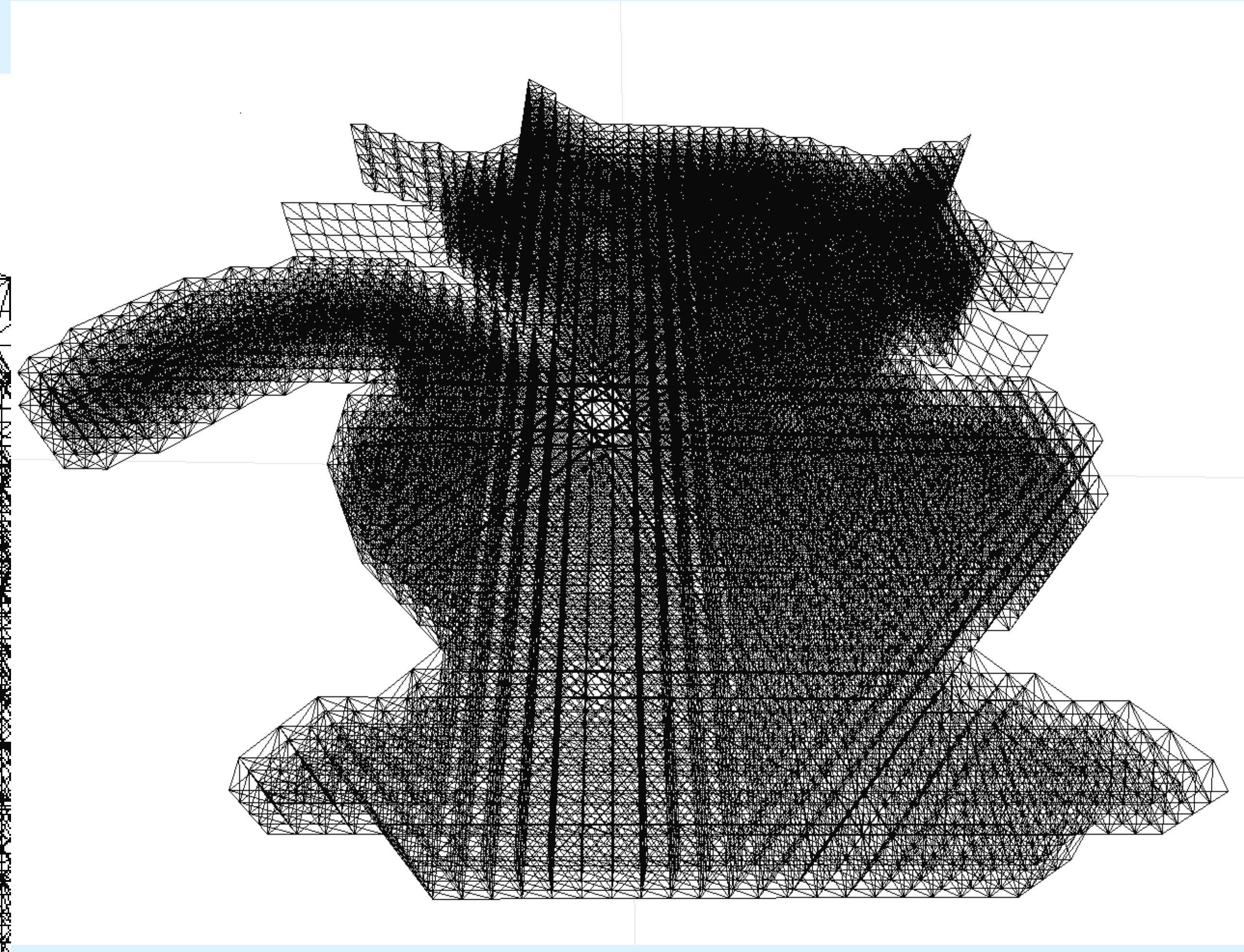
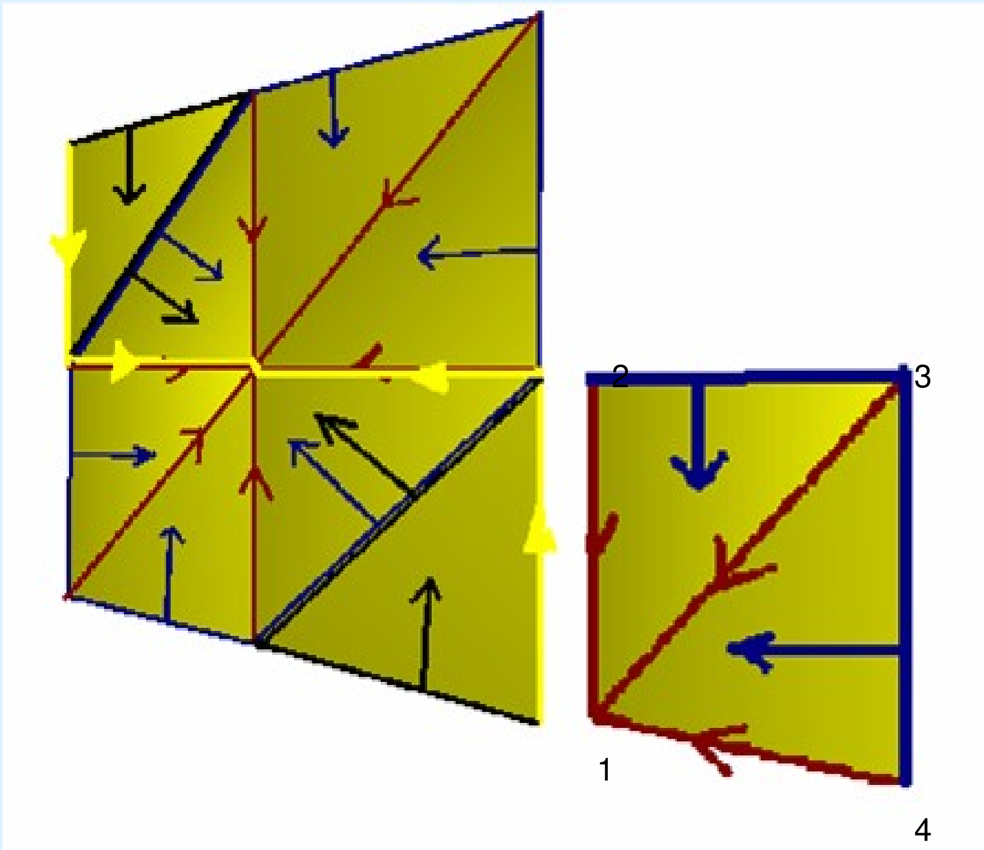
**We visually describe the AT-model method (González-Díaz & Real,2003) that, roughly speaking, allows us to represent a digital binary nD image by a homotopy operator connecting in a strong way the object and its homology. This technique allows to use advanced computational tools from CAT (Computational Algebraic Topology) into the discrete and combinatorial context of the nD digital images. This philosophy of representing a combinatorial object using an algebraic format has its origins in classical topological methods (Eilenberg-Mac Lane, Effective Homology, Discrete Morse Theory,…).**



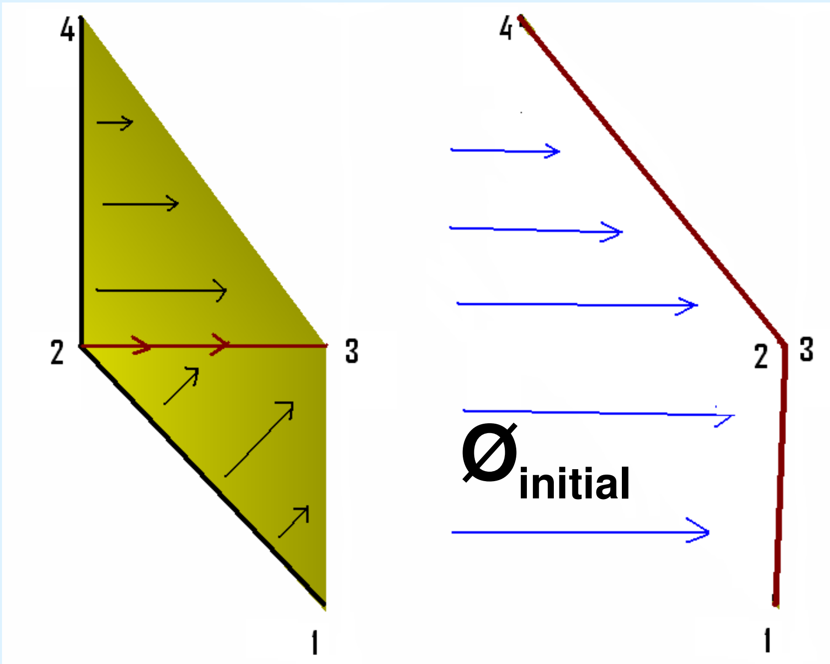





d(<a,b,c>= <b,c>-<a,c>+<a,b>
d(<a,b>)= <b>-<a>
such that
**d d=0** (2-nilpotency)
**H(d) = Ker d/Im d**
$H_0$(d) ~ connected components
$H_1$(d) ~“holes” or “tunnels”
$H_2$(d) ~ cavities…

| |
|---|
| **Voxels 4** |
| **1 (1,1,0)** |
| **2 (0,1,0)** |
| **3 (0,0,0)** |
| **4 (1,0,0)** |
| **Points 0** |
| **Edges 0** |
| **Triangles 2** |
| **<1,2,3>** |
| **<1,3,4>** |
| **Tetrahedron 0** |

| | | |
|---|---|---|
| 1 | (1) | Øalg: 0 |
| 0 | (2) | Øalg: { <(1 2)>} |
| 0 | (3) | Øalg: { <(1 3)>} |
| 0 | (4) | Øalg: { <(1 4)>} |
| 0 | <(1 2)> | Øalg: 0 |
| 0 | <(1 3)> | Øalg: 0 |
| 0 | <(1 4)> | Øalg: 0 |
| 0 | <(2 3)> | Øalg: { <(1 2 3)>} |
| 0 | <(3 4)> | Øalg: { <(1 3 4)>} |
| 0 | <(1 2 3)> | Øalg: 0 |
| 0 | <(1 3 4)> | Øalg: 0 |




**ØØ=0**

**d d=0; (initial conditions) ØØ=0;**
**ØdØ=Ø; (homology gradient) dØd= d;**
**Flow=1+ dØ+Ød**
**H(d)=H(Ø)**