

First order semi-local invariants of stable mappings from 3-manifolds to \mathbb{R}^3

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In [6] Vassiliev introduced a method to obtain topological invariants on function spaces. This method has proven to be very useful and has given interesting results in several cases:

- i) Knots in \mathbb{R}^3 (Vassiliev in [6]).
- ii) Immersed plane curves (Arnol'd in [1]).
- iii) Stable mappings from surfaces to \mathbb{R}^3 (Goryunov in [2]).
- iv) Stable mappings from the plane to the plane (Ohmoto and Aicardi in [4]).
- v) Stable mappings from 3-manifolds to the plane (Yamamoto in [7]).

In this paper we apply this method to stable mappings from 3-manifolds to \mathbb{R}^3 .

Starting from the classification of germs obtained by Marar and Tari in [3] we determine a complete list of germs and multigerms up to codimension 2. The analysis of the different unfoldings allows us to determine the structure of the discriminant subset (non-stable mappings) in a neighbourhood of each of the codimension 2 strata as well as to provide suitable coorientations to the codimension 1 strata. In this way we obtain 5 cocycles that form a complete set of generators for the cohomology ring $H^0(E(\mathbb{R}^3, \mathbb{R}^3), \mathbb{Z})$ (first order Vassiliev type invariants), where E stands for stable mappings.

Besides the obvious invariants (number of triple points, number of swallowtails and number of intersections between cuspidal edges and fold planes) we provide a geometrical interpretation for the other two invariants related to Euler characteristics and links of Legendrian lifts in $ST^*\mathbb{R}^3$.

Keywords: Vassiliev invariants, stable mappings,...

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