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The Future of Spanish Pensions

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Abstract

We use an overlapping generations model economy with endogenous retirement to study the 2011 and 2013 reforms of the Spanish public pension system. We find that this latest reforms, which extend the number of years os contributions used to compute the pensions, delay the retirement ages, introduce two sustainability factors, and effectively transform the Spanish pay-as-you-go system into a defined-contribution system, succeed in making Spanish pensions sustainable until 2037, but they fail to do so afterwards. The success until 2037 is achieved reducing the real value of the average pension and leaving the many loopholes of the contributivity and the transparency of the system unchanged. This reduction in pensions is progressive and, by 2037, the average pension will be approximately 20 percent smaller in real terms than what it would have been under the pension rules prevailing in 2010. The 2013 pension reform fails after 2037 because, from that year onwards, approximately 50 percent of the Spanish retirees will be paid the minimum pension, which is exempt from the sustainability factors. We conjecture that further reforms lurk in the future of Spanish pensions.

Keywords: Computable general equilibrium, social security reform, retirement

JEL classification: C68, H55, J26

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1 Introduction

The Spanish Pensions Prevailing in 2010. At least since the year 2000 academic researchers have concluded unanimously that the Spanish pay-as-you-go defined-benefit public pension system, which was in force before the 2011 Reform, was unsustainable.[1] Moreover, they reached this conclusion *before* the 2008 recession. This means that the sustainability problem of Spanish pensions was *structural* and was not related to the business cycle. In Díaz-Giménez and Díaz-Saavedra (2006) we showed that the cause of this problem was the demographic transition that will increase the expenditures of the Spanish pension system steadily during the next four decades. The literature mentioned in Footnote 1 reaches the same conclusion without exceptions.

In this article we calibrate an enhanced version of the model economy described in Díaz-Giménez and Díaz-Saavedra (2009) to the Spanish post-recession macroeconomic scenario in 2010, we simulate the latest demographic scenario published in 2012 by the *Instituto Nacional de Estadística (INE)* and we reach the same conclusion. Specifically, we show that, under the rules prevailing in 2010, the pension system revenues would have remained virtually unchanged between 2010 and 2050, and the pension system expenditures would have almost doubled.

Pension expenditures increase for three reasons: because the size of the retiree cohorts increases, because longevity increases, and because the retiree cohorts become more educated. According to the INE's 2012 demographic scenario, the share of Spanish residents aged 65 or more was 20.9 percent in 2010, and it will be 43.6 percent in 2050; life-expectancy at age 65 was 17.4 years in 2010, and it will be 23.4 years in 2050; and, according to our estimations of the Spanish educational transition, the share of Spanish workers who had completed college was 20.7 percent in 2010, and it will be 26.0 percent in 2050. The natures of these changes is demographic and completely unrelated to the 2008 recession, and their severity spells doom for any pay-as-you-go, defined-benefit pension system such as the one prevailing in Spain in 2010.

The 2011 Reforms. In spite of this overwhelming scientific evidence, until 2011, every political party represented in the Spanish parliament denied steadfastly that Spanish pensions had a structural sustainability problem. They did so hiding behind the secrecy of a parliamentary agreement known as the *Pacto de Toledo,* that excludes pensions from political discussion. During this period, Spanish trade unions and business associations, and most of the insurance sector joined ranks in the defense of the *status quo* of the Spanish pension system and insisted in this denial.

The political change of mind took place in January 2010 when the Spanish Government sent

[1] See, for example, the aggregate accounting models of Jimeno (2000), Balmaseda, Melguizo, and Taguas (2006), and Gil, López-García, Onrubia, Patxot, and Souto (2007); the individual life-cycle models of Alonso and Herce (2003), Jimeno (2003), Da Rocha and Lores (2005), and González, Conde-Ruiz, and M. Boldrin (2009); and the general equilibrium models of Rojas (2005), Sánchez-Martín (2010), and Díaz-Giménez and Díaz-Saavedra (2006 and 2009).

its *2011-2014 Stability Plan* to the European Commission. Amongst other measures, this plan proposed a parametric reform of the Spanish pension system. The reform that was finally approved in 2011 enacted three main parametric changes: a gradual increase in the number of years of contributions that are used to calculate the retirement pension, a delay of the first retirement age from 61 to 63 for workers who retire voluntarily, and a gradual delay of the normal retirement age from 65 to 67. The adoption of these changes started in 2013.[2]

Three recent articles have studied the 2011 Reform of the Spanish pension system. Conde-Ruiz and González (2013) and De la Fuente and Doménech (2013) simulate two individual life-cycle models and they conclude that the 2011 Reform will reduce the expenditure in pensions somewhat, but that it is insufficient to solve the middle and long-term sustainability problems faced by the Spanish pension system. And a report published by economists form the Spanish Finance Ministry (MEH, 2011) simulates an aggregate accounting model economy and reaches a similar conclusion.

The 2013 Delay. In 2013, when the 2011 reforms were starting to be implemented, the Spanish government enacted a further gradual delay of the first retirement age. This delay increases the first retirement age from 63 to 65 years, one month per year starting in 2013, for workers who retire voluntarily.

In the second model economy that we study in this article we simulate the 2011 Reform and the 2013 delay simultaneously and we find that this reform extends in five years —from 2018 to 2023— the duration of the pension reserve fund, that it reduces the total debt that would have been accumulated by the pension system until 2050, from 212 to 124 percent of that year GDP, and that it reduces the consumption tax rate needed to finance the pensions from 45 percent to 39 percent. Consequently, we conclude that this reform is insufficient to solve the sustainability problem of the Spanish pension system.

The 2013 Sustainability Factors. The 2011 Reform made provisions to add a sustainability factor to Spanish pensions. This factor would take into account the expected duration of retirement and would reduce the real value of pensions as needed to ensure the financial sustainability of the system, effectively changing the Spanish pension system from a defined-benefit system to a defined-contribution system.

In 2013 the Spanish government named a *Committee of Experts* to make a proposal with the details of this sustainability factor. The committee proposed to use two factors instead of one. The first factor is an *Intergenerational Equity Factor* (IEF). This factor is the ratio that obtains when we divide the life-expectancy at retirement of the workers of a given age who retire in a reference year by the life-expectancy at retirement of the workers of the same age who retire later.[3]

[2] For the details of the reforms, see http://www.seg-social.es/prdi00/groups/public/documents/normativa/150460.pdf.

[3] For instance, if the reference year is 2014, the life expectancy of the 65 year-olds who will retire that year will be 20.27 and the life expectancy of the 65 year-olds who retire, say, in 2020 will be 21.14, then the intergenerational equity factor for that cohort is $IEF_{65,2020} = 20.27/21.14 = 0.9588$. Therefore, the pensions of the second group of

This factor applies to new pensions only and minimum pensions are exempt from its application and are revalued discretionally by the government each year. The second factor is an *Annual Revaluation Factor* (ARF). This factor reduces the value of every pension in payment except for minimum pensions, in an amount that makes the pension system sustainable. Specifically, pensions in payment are reduced to equate a moving average of past and future pension system outlays to a moving average of past and future pension system revenues.

In the third model economy that we simulate in this article we use the ARF to revaluate pensions from 2013 onwards and we use the IEF to compute the value of new pensions from 2019 onwards. We find that this reform extends the duration of the pension reserve fund from 2018 to 2038, that it reduces the total debt that would have been accumulated by the pension system until 2050 from 212 to 20 percent of GDP, and that it reduces the consumption tax rate needed to finance the pensions from 45 to 24 percent.

We conclude that this reform improves the sustainability of the Spanish pension system, but that it does so with a startling reduction in the real value of pensions. In 2050, the real value of the average pension in this model economy is 34 percent smaller than in the benchmark model economy, the pension substitution rate falls from 39 to 28 percent of the average wages of households in the 60 to 64 age group, and the share of the retirees who are paid the minimum pension increases by a startling 30 percentage points, from 29 percent to 59 percent. Moreover, because the sustainability factors do not apply to minimum pensions, the long term deficits of the system reappear after 2037 and the long term sustainability of pensions is not completely resolved.

The Pension Revaluation Index. In December 2013 the Spanish government approved a bounded version of the Annual Revaluation Factor and it called it the *Pension Revaluation Index* (PRI). The PRI imposes two bounds on the ARF. In real terms, the lower bound is 0.25 minus the inflation rate and the upper bound is 0.5.

In the last model economy that we simulate in this article we use the PRI to revaluate pensions from 2013 onwards and we use the IEF to compute the value of new pensions from 2019 onwards. To do so, we assume that the inflation rate in Spain will be 2.32 percent which was the average inflation between 1998 and 2013. This assumption implies that the lower bound on the PRI in our model economy is $-2.07(= 0.25 - 2.32)$ in real terms. When we compare the ARF reform with the PRI reform, we find that the PRI reform reduces the duration of the pension reserve fund from 2038 to 2037, that it increases the total debt that would have been accumulated by the pension system from 19 to 28 percent of GDP in 2050, and that it increases the consumption tax rate needed to finance the pensions from 24 to 27 percent. On the positive side, in 2050 the PRI reform increases the value of the average pension from 66 percent to 71 percent of the real value of the average pension in the benchmark model economy, and the pension substitution rate from 28 to 31

retirees will be approximately 4 percent smaller than those of the first group.

percent. Of course, this makes the Spanish pension system less sustainable than under the ARF reform. In Table 1 we summarize our findings.

The Model Economy. As we have already mentioned, in this article we simulate an enhanced version of the general equilibrium, multiperiod, overlapping generations model economy populated by heterogeneous households described in Díaz-Giménez and Díaz-Saavedra (2009). The model economy that we study here differs from the one that we used in that article in two fundamental ways. First, we have substituted the proportional tax on labor income of the previous version with a progressive household income tax. This allows us to exploit the heterogeneity in our model economy and to avoid the ongoing discussion of whether the calibration should replicate the marginal or the average tax rates of the progressive personal income taxes of real economies. Second, we have updated our calibration year to 2010. This allows us to account for the first two years of the current Spanish recession.

Other differences between this version of our model economy and the previous one are the following: The new version of our model economy replicates de 2010 distribution by age and education of the population in Spain; it updates the deterministic component of the life-cycle profile of earnings to reflect the decrease in the education wage-premium; it simulates the INE's 2012 Spanish demographic scenario; it replicates the World Economic Outlook's October 2013 growth scenario for Spain; it delivers a value for the Frisch labor supply elasticity, which is more in line with recent estimates of this variable; and it improves the measurement of the key aggregates and ratios. Moreover, our simulation exercises are the first to quantify the consequences of the latest reforms for the future of Spanish pensions and to uncover their shortcomings.

Conclusions. We conclude that the 2011 and 2013 reforms of the Spanish pension system improve its sustainability, that they do so at the expense of startling reductions in the real value of pensions, and that they fail to solve the long term sustainability problems of the system completely. This leads us to conjecture that further reforms lurk in the future of the Spanish pensions.

2 The Model Economy

We study an overlapping generations model economy with a continuum of heterogeneous households, a representative firm, and a government, which we describe below.

2.1 Population and Endowment Dynamics

We assume that the households in our model economy differ in their age, $j \in J$; in their education, $h \in H$; in their employment status, $e \in \mathcal{E}$; in their assets, $a \in A$; in their pension rights, $b_t \in B_t$,

Table 1: The Reference and Reformed Economies in 2010, 2030, and 2050

Model	Results in 2010											
	Rev	Exp	Def	PRF	τ_c	AvP	RAvP	PSR	MinP	Y	K	L
All	11.1	11.3	0.2	6.7	21.1	100.0	100.0	50.5	21.5	100.0	100.0	100.0
	Results in 2030											
P2010	10.3	13.7	3.4	–28.7	29.3	123.7	100.0	36.1	35.0	129.4	133.0	102.7
R2013	10.6	12.2	1.6	–7.4	24.9	121.7	98.4	40.5	34.8	131.6	133.7	105.2
RARF	10.5	10.9	0.4	3.1	21.1	110.1	89.0	36.2	41.3	135.0	138.3	107.7
RPRI	10.5	10.8	0.3	3.0	21.1	110.3	89.2	36.4	41.6	134.9	137.6	107.8
	Results in 2050											
P2010	9.6	20.1	10.5	–212.5	45.5	187.0	100.0	38.8	29.2	166.1	177.7	87.5
R2013	9.9	17.7	7.8	–123.6	39.1	178.1	95.2	40.9	30.3	169.8	179.8	89.8
RARF	9.3	10.7	1.4	–19.6	24.4	123.6	66.1	28.5	59.2	184.1	192.0	98.0
RPRI	9.4	11.7	2.4	–27.7	26.6	133.4	71.3	30.8	52.6	182.0	190.2	96.7

Rev: Revenues (%GDP); Exp: Expenditures (%GDP); Def: Pension system deficit (%GDP); PRF: Pension reserve fund or pension system debt (%GDP); τ_c: Consumption tax rate needed to finance the pension system (%). AvP: Average pension (2010=100); RAvP: Average pension relative to the average pension in Model Economy P2010 (2010=100); PSR: Pension Substitution Rate (AvP/W(60-64)); MinP: Share of the retirees who receive the minimum pension (%); Y: Output index (2010=100); K: Capital index (2010=100); L: Labor input index (2010=100);

P2010: This is the benchmark model economy. Its pension system replicates the pay-as-you-go, defined-benefit pension system that prevailed in Spain before the 2011 Reform and the 2013 delay in the first retirement age.

R2013: The pension system of this model economy replicates the pay-as-you-go, defined-benefit pension system that resulted from the 2011 Reform and the 2013 delay in the first retirement age.

RARF: The pension system of this model economy replicates the pay-as-you-go, defined-contribution pension system that results from the application of the Intergenerational Equity Factor and the Annual Revaluation Factor.

RPRI: The pension system of this model economy replicates the pay-as-you-go, defined-contribution pension system that results from the application of the Intergenerational Equity Factor and the Pension Revaluation Index.

and in their pensions $p_t \in P_t$.[4] Sets J, H, $\mathcal{E}$, A, B_t, and P_t are all finite sets which we describe below. We use $\mu_{j,h,e,a,b,p,t}$ to denote the measure of households of type (j, h, e, a, b, p) at period t. For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

Age. Every household enters the economy when it is 20 years old and it is forced to exit the economy at age 100. Consequently, $J = \{20, 21, \ldots, 100\}$. We also assume that each period every household faces a conditional probability of surviving from age j to age $j+1$, which we denote by ψ_{jt}. This probability depends on the age of the household and it varies with time, but it does not depend on the household's education.

Education. We abstract from the education decision, and we assume that the education of every household is determined forever when they enter the economy. We consider three educational levels. Therefore, $H = \{1, 2, 3\}$. Educational level $h = 1$ denotes that the household has dropped out of high school; educational level $h = 2$ denotes that the household has completed high school but has not completed college[5]; and educational level $h = 3$ denotes that the household has completed college.

Population Dynamics. In the real world the age distribution of the population changes because of changes in fertility, survival rates, and migratory flows. The population dynamics in our model economy are exogenous and we describe them in Section 4 below.

Employment status. Households in our economy are either workers, retirees, or disabled households. We denote workers by ω, retirees by ρ, and disabled households by d. Consequently, $\mathcal{E} = \{\omega, \rho, d\}$. Every household enters the economy as a worker. The workers face a positive probability of becoming disabled at the end of each period of their working lives. And they decide whether to retire at the beginning of each period once they have reached the first retirement age, which we denote by R_0. In our model economy, both the disability shock and the retirement decision are irreversible and there is no mandatory retirement age.

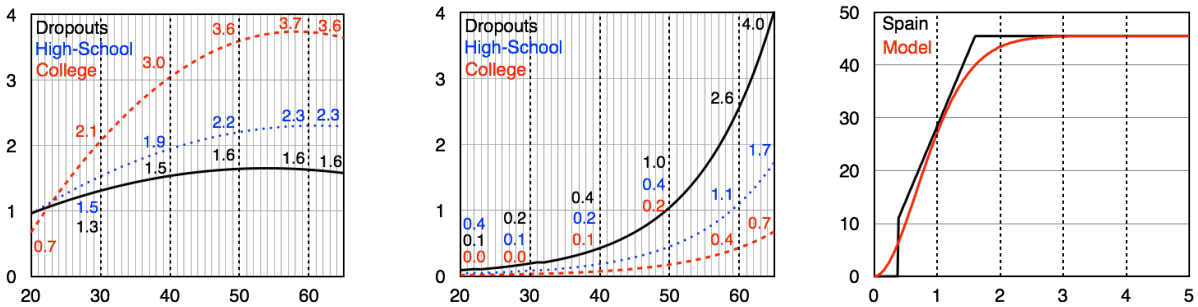
Workers. Workers receive an endowment of efficiency labor units every period. This endowment has two components: a deterministic component, which we denote by ϵ_{jh}, and a stochastic idiosyncratic component, which we denote by s.

We use the deterministic component to characterize the life-cycle profiles of earnings. These profiles are different for each educational group, and we model them using the following family of

[4] To calibrate our model economy, we use data per person older than 20. Therefore our model economy households are really individual people.

[5] In this group we include every household that has completed the first level of secondary education.

Figure 1: The Endowment of Efficiency Labor Units, the Disability Risk, and the Payroll Tax



A: The Endowment of ELU B: The Disability Risk (%) C: The Payroll Tax*

*In the vertical axis of this panel we plot payroll tax collections expressed as the percentage share of GDP per person over 20 and in the horizontal axis we plot labor income expressed as a multiple of GDP per person over 20.

quadratic functions:[6]

$$\epsilon_{jh} = a_{1h} + a_{2h}j - a_{3h}j^2 \tag{1}$$

We choose this functional form because it allows us to represent the life-cycle profiles of the productivity of workers in a very parsimonious way. We represent the calibrated versions of these functions in Panel A of Figure 1.

We use the stochastic component of the endowment shock, s, to generate earnings and wealth inequality within the age cohorts. We assume that s is independent and identically distributed across the households, that it does not depend on the education level, and that it follows a first order, finite state, Markov chain with conditional transition probabilities given by

$$\Gamma[s' \mid s] = \Pr\{s_{t+1} = s' \mid s_t = s\}, \text{ where } s, s' \in \omega = \{s_1, s_1, \ldots, s_n\}. \tag{2}$$

We assume that the process on s takes three values and, consequently, that $s \in \omega = \{s_1, s_2, s_3\}$. We make this assumption because it turns our that three states are sufficient to account for the Lorenz curves of the Spanish distributions of income and labor earnings in enough detail, and because we want to keep this process as parsimonious as possible.

Retirees. As we have already mentioned, workers who are R_0 years old or older decide whether to remain in the labor force, or whether to retire and start collecting their retirement pension. They make this decision after they observe their endowment of efficiency labor units for the period. In our model economy retirement pensions are incompatible with labor earnings and, consequently, retirees receive no endowment of efficiency labor units.

[6]In the expressions that follow the letters a denote parameters.

Disabled households. We assume that workers of education level h and age j face a probability φ_{jh} of becoming disabled from age $j+1$ onwards. The workers find out whether they have become disabled at the end of the period, once they have made their labor and consumption decisions. When a worker becomes disabled, she exits the labor market and she receives no further endowments of efficiency labor units, but she is entitled to receive a disability pension until she dies.

To determine the values of the probabilities of becoming disabled, we proceed in two stages. First we model the aggregate probability of becoming disabled. We denote it by q_j, and we assume that it is determined by the following function:

$$q_j = a_4 e^{(a_5 \times j)} \tag{3}$$

We choose this functional form because the number of disabled people in Spain increases more than proportionally with age, according to the *Boletín de Estadísticas Laborales* (2007).

Once we know the value of q_j we solve the following system of equations:

$$\begin{cases} q_j \mu_{j,2007} & = & \sum_h \varphi_{jh} \mu_{jh,2007} \\ \varphi_{j2} & = & a_6 \varphi_{j1} \\ \varphi_{j3} & = & a_7 \varphi_{j1} \end{cases} \tag{4}$$

This procedure allows us to make the disability process dependent on the educational level as is the case in Spain. We represent our calibrated values for φ_{jh} in Panel B of Figure 1.[7]

2.2 Preferences

We assume that households derive utility from consumption, $c_{jht} \geq 0$, and from non-market uses of their time, $(1 - l_{jht})$, and that their preferences can be described by the following standard Cobb-Douglas expected utility function:

$$\max E \left\{ \sum_{j=20}^{100} \beta^{j-20} \, \psi_{jt} \, [c_{jht}^{\alpha} (1 - l_{jht})^{(1-\alpha)}]^{(1-\sigma)} / 1 - \sigma \right\} \tag{5}$$

where $0 < \beta$ is the time-discount factor; 1 is the normalized endowment of productive time; and $0 \leq l_{jht} \leq 1$ is labor.

2.3 Technology

We assume that aggregate output, Y_t, depends on aggregate capital, K_t, and on the aggregate labor input, L_t, through a constant returns to scale, Cobb-Douglas, aggregate production function of the form

$$Y_t = K_t^{\theta} (A_t L_t)^{1-\theta} \tag{6}$$

[7] The data on disability can be found at www.empleo.gob.es/es/estadisticas.

where A_t denotes an exogenous labor-augmenting productivity factor whose law of motion is $A_{t+1} = (1+\gamma_t)\,A_t$, and where $A_0 > 0$. Aggregate capital is obtained aggregating the capital stock owned by every household, and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate, δ, and we use r and w to denote the prices of capital and of the efficiency units of labor before all taxes.

2.4 Government Policy

The government in our model economy taxes capital income, household income and consumption, and it confiscates unintentional bequests. It uses its revenues to consume, and to make transfers other than pensions. In addition, the government runs a pay-as-you-go pension system.

In this model economy the consolidated government and pension system budget constraint is

$$G_t + P_t + Z_t = T_{kt} + T_{st} + T_{yt} + T_{ct} + E_t + (F_t - F_{t+1}) \tag{7}$$

where G_t denotes government consumption, P_t denotes pensions, Z_t denotes government transfers other than pensions, T_{kt}, T_{st}, T_{yt}, and T_{ct}, denote the revenues collected by the capital income tax, the payroll tax, the household income tax, and the consumption tax, E_t denotes unintentional bequests, and $F_t > 0$ denotes the value of the pension reserve fund at the beginning of period t. Finally, $(F_t - F_{t+1})$ denotes the revenues that the government obtains from the pension reserve fund or deposits into it.

We assume that the pension reserve fund must be non-negative and that transfers other than pensions are thrown to the sea so that they create no distortions in the household decisions.

2.4.1 Taxes

Capital income taxes are described by the function

$$\tau_k(y_t^k) = a_8 y_t^k \tag{8}$$

where y_t^k denotes before-tax capital income.

Household income taxes are described by the function

$$\tau_y(y_t^b) = a_9 \left\{ y_t^b - \left[a_{10} + (y_t^b)^{-a_{11}} \right])^{-1/a_{11}} \right\} \tag{9}$$

where the tax base is

$$y_t^b = y_t^k + y_t^l + p_t - \tau_k(y_t^k) - \tau_s(y_t^l) \tag{10}$$

where y_t^l denotes before-tax labor income, p_t denotes pensions, and τ_s denotes the payroll tax function that we describe below. Expression (9) is the function chosen by Gouveia and Strauss (1994) to model effective personal income taxes in the United States, and it is also the functional form chosen by Calonge and Conesa (2003) to model effective personal income taxes in Spain.

Consumption taxes are described by the function

$$\tau_c(c_t) = a_{12t} c_t. \tag{11}$$

Finally, we assume that at the end of each period, once they have made their labor and consumption decisions, a share $(1 - \psi_{jt})$ of all households of age j die and that their assets are confiscated by the government.

2.4.2 The Pension System

Payroll taxes. In Spain the payroll tax is capped and it has a tax-exempt minimum. In our model economy the payroll tax function is the following:

$$\tau_s(y_t^l) = \begin{cases} a_{13}\bar{y}_t - \left[a_{13}\bar{y}_t \left(1 + \frac{a_{14} y_t^l}{a_{13}\bar{y}_t}\right)^{-y_t^l / a_{13}\bar{y}_t}\right] & \text{if } j < R_1 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where parameter a_{13} is the cap of the payroll tax, $\bar{y}_t$ is per capita output *at market prices* in period t, and y_t^l is labor income that same period before taxes. This function allows us to replicate the Spanish payroll tax cap, but it does not allow us to replicate the tax exempt minimum. In Panel C of Figure 1 we represent the payroll tax function for our calibrated values of a_{13} and a_{14}.

Retirement pensions. A household of age $j \geq R_0$, who chooses to retire, receives a retirement pension which is calculated according to the following formula:

$$p_t = \phi(1.03)^v (1 - \lambda_j) \frac{1}{N_b} \sum_{t=j-N_b}^{j-1} \min\{a_{15}\bar{y}_t, y_t^l\} \tag{13}$$

In this expression, parameter N_b denotes the number of consecutive years immediately before retirement that are used to compute the retirement pensions; parameter $0 < \phi \leq 1$ denotes the pension system replacement rate; variable v denotes the number of years that the worker remains in the labor force after reaching the normal retirement age;[8] function $0 \leq \lambda_j < 1$ is the penalty paid for early retirement; and $a_{15}\bar{y}_t$ is the maximum covered earnings. Expression (13) replicates the main features of Spanish retirement pensions:

[8] This late retirement premium was introduced in the 2002 reform of the Spanish public pension system.

Pensions in our model economy are computed upon retirement and their real value remains unchanged. We also model minimum and maximum retirement pensions. Formally, we require that $p_{0t} \leq p_t \leq p_{mt}$, where p_{0t} denotes the minimum pension and p_{mt} denotes the maximum pension. We update the minimum pension so that it remains a constant proportion of output per capita.[9]

The Spanish *Régimen General de la Seguridad Social* establishes a set of penalties for early retirement that are a linear function of the retirement age. To replicate this rule, our choice for the early retirement penalty function is the following

$$\lambda_j = \begin{cases} a_{16} - a_{17}(j - R_0) & \text{if } j < R_1 \\ 0 & \text{if } j \geq R_1 \end{cases} \tag{14}$$

Finally, the Spanish pension replacement rate is a function of the number of years of contributions. In our model economy we abstract from this feature because it requires an additional state variable. It turns out that this last assumption is not very important because, in our our model economy, 99.6 of all workers aged 20-64 in our benchmark model economy choose to work in our calibration year. This suggests that the number of workers who would have been penalized for having short working histories in our model economy is very small.

Disability pensions. We model disability pensions explicitly for two reasons: because they represent a large share of all Spanish pensions (10.7 percent of all pensions in 2010), and because disability pensions are used as an alternative route to early retirement in many cases.[10] To replicate the current Spanish rules, we assume that there is a minimum disability pension which coincides with the minimum retirement pension. And that the disability pensions are 75 percent of the households' retirement claims. Formally, we compute the disability pensions as follows:

$$p_t = \max\{p_{0t}, 0.75b_t\}. \tag{15}$$

The pension reserve fund. We assume that pension system surpluses, $(T_{st} - P_t)$, are deposited into a non-negative pension reserve fund which evolves according to

$$F_{t+1} = (1 + r^*)F_t + T_{st} - P_t \tag{16}$$

where parameter r^* is the exogenous rate of return of the fund's assets. We assume that the government changes the consumption tax rate as needed in order to finance the pensions when the pension reserve fund runs out.

[9] In Spain normal and maximum pensions are adjusted using the inflation rate and minimum pensions are increased discretionally. This has implied that over the last decade or so the Spanish minimum pension has roughly kept up with per capita GDP, and that the maximum pension and normal pensions have decreased as a share of per capita GDP. This little known fact is known as *the silent reform* of Spanish pensions.

[10] See Boldrin and Jiménez-Martín (2003) for an elaboration of this argument.

2.5 Market Arrangements

Insurance Markets. We assume that there are no insurance markets for the stochastic component of the endowment shock. This is a key feature of our model economy. When insurance markets are allowed to operate, every household of the same age and education level is identical, and the earnings and wealth inequality disappears almost completely.

Assets. We assume that the households in our model economy cannot borrow. Since leisure is an argument of their utility function, this borrowing constraint can be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world. These restrictions give the households a precautionary motive to save. They do so accumulating real assets, which we denote by a_t, and which take the form of productive capital. For computational reasons we restrict the asset holdings to belong to the discrete set $\mathcal{A}=\{a_0, a_1, \ldots, a_n\}$. We choose $n=100$, and assume that $a_0=0$, that $a_{100}=75$, and that the spacing between points in set $\mathcal{A}$ is increasing.[11]

Pension Rights. We assume that the workers' pension rights belong to the discrete set $B_t = \{b_{0t}, b_{1t}, \ldots, b_{mt}\}$.[12] Let parameter N_b denote the number of years of contributions that are taken into account to calculate the pension. Then, when a worker's age is $R_0 - N_b < j < R_0$, the b_{it} record the average labor income earned by that worker since age $R_0 - N_b$. And when a worker is older than R_0, the b_{it} record the average labor income earned by that worker during the previous N_b years. We assume that $b_{0t} = 0$, and that $b_{mt} = a_{15}\bar{y}_t$, where $a_{15}\bar{y}_t$, is the maximum earnings covered by the Spanish pension system. We also assume that $m = 9$ and that the spacing between points in set B_t is increasing.

Pensions. We assume that both the disability and retirement pensions belong to set $P_t=\{p_{0t}, p_{1t}, \ldots, p_{mt}\}$. The rules of the pension system determine the mapping from pension rights into pensions, and workers take into account this mapping when they decide how much to work and when to retire. Since this mapping is single valued, and the cardinality of the set of pension rights, B_t, is 10, we let $m = 9$ also for P_t. Finally, we assume that the distances between any two consecutive points in P_t is increasing. This is because minimum pensions play a large role in the Spanish system and this suggests that we should have a tight grid in the low end of P_t.

[11] In overlapping generation models with finite lives and no altruism there is no need to impose an upper bound for set $\mathcal{A}$ since households who reach the maximum age will optimally consume all their assets. İmrohoroğlu, İmrohoroğlu, and Joines (1995) make a similar point.

[12] Set B_t changes with time because its upper bound is the maximum covered earnings which are proportional to per capita output.

2.6 The Households' Decision Problem

We assume that the households in our model economy solve the following decision problem:

$$\max E\left\{\sum_{j=20}^{100}\beta^{j-20}\,\psi_{jt}\,[c_{jht}^{\alpha}(1-l_{jht})^{(1-\alpha)}]^{(1-\sigma)}/1-\sigma\right\} \tag{17}$$

subject to

$$c_{jht}+a_{jht+1}+\tau_{jht}=y_{jht}+a_{jht} \tag{18}$$

and where

$$\tau_{jht}=\tau_k(y_{jht}^k)+\tau_{st}(y_{jht}^l)+\tau_y(y_{jht}^b)+\tau_{ct}(c_{jht}) \tag{19}$$

$$y_{jht}=y_{jht}^k+y_{jht}^l+p_t \tag{20}$$

$$y_{jht}^k=a_{jht}r_t \tag{21}$$

$$y_{jht}^l=\epsilon_{jh}s_tl_{jht}w_t \tag{22}$$

$$y_{jht}^b=y_{jht}^k+y_{jht}^l+p_t-\tau_a(y_t^k)-\tau_s(y_t^l) \tag{23}$$

where $a_{jht}\in\mathcal{A}$, $p_t\in P_t$, $s_t\in\omega$ for all t, and a_{jh0} is given. Notice that every household can earn capital income, only workers can earn labor income, and only retirees and disabled households receive pensions.

A relevant feature of the households decision problem that we have not mentioned here is that households decide optimally when to retire. As we have already mentioned, in the model economy, the pension rights of workers who are between 20 and R_0 years old evolve according to the following expression:

$$b_{t+1}=\begin{cases}0 & \text{if} \quad j<R_0-N_b\\ (b_t+y_t^l)/[j-(R_0-N_b-1)] & \text{if} \quad R_0-N_b\leq j<R_0,\end{cases} \tag{24}$$

When households reach age R_0 they decide whether or not to retire. This decision depends on their pension rights, on their other state variables, j, h, a_t and s_t, and on the benefits and costs of continuing to work. The benefits are the labor earnings and the avoidance of the early retirement penalties, if any, and the costs are the forgone leisure and the forgone pension. And they also take into account the change in their pension rights, $b_{t+1}-b_t$, which could be a benefit or a cost depending on the values of b_t and of the current and expected future endowments of efficiency labor units.

2.7 Definition of Equilibrium

Let $j\in J$, $h\in H$, $e\in\mathcal{E}$, $a\in\mathcal{A}$, $b_t\in B_t$, and $p_t\in P_t$, and let $\mu_{j,h,e,a,b,p,t}$ be a probability measure defined on $\Re=J\times H\times\mathcal{E}\times\mathcal{A}\times B_t\times P_t$.[13] Then, given initial conditions μ_0, A_0, E_0, F_0, and K_0, a competitive

[13] Recall that, for convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

equilibrium for this economy is a government policy, $\{G_t, P_t, Z_t, T_{kt}, T_{st}, T_{yt}, T_{ct}, E_{t+1}, F_{t+1}\}_{t=0}^{\infty}$, a household policy, $\{c_t(j,h,e,a,b,p),\ l_t(j,h,e,a,b,p),\ a_{t+1}(j,h,e,a,b,p)\}_{t=0}^{\infty}$, a sequence of measures, $\{\mu_t\}_{t=0}^{\infty}$, a sequence of factor prices, $\{r_t, w_t\}_{t=0}^{\infty}$, a sequence of macroeconomic aggregates, $\{C_t, I_t, Y_t, K_{t+1}, L_t\}_{t=0}^{\infty}$, a function, Q, and a number, r^*, such that:

(i) The government policy and r^* satisfy the consolidated government and pension system budget constraint described in Expression (7) and the the law of motion of the pension system fund described in Expression (16).

(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r_t = f_1(K_t, A_t L_t) - \delta$ and $w_t = f_2(K_t, A_t L_t)$.

(iii) Given the initial conditions, the government policy, and factor prices, the household policy solves the households' decision problem defined in Expressions (17), through (23).

(iv) The stock of capital, consumption, the aggregate labor input, pension payments, tax revenues, and accidental bequests are obtained aggregating over the model economy households as follows:

$$K_t = \int a_{jht} d\mu_t \tag{25}$$

$$C_t = \int c_{jht} d\mu_t \tag{26}$$

$$L_t = \int \epsilon_{jh} s_t l_{jht} d\mu_t \tag{27}$$

$$P_t = \int p_t d\mu_t \tag{28}$$

$$T_{ct} = \int \tau_{ct}(c_{jht}) d\mu_t \tag{29}$$

$$T_{kt} = \int \tau_k(y^k_{jht}) d\mu_t \tag{30}$$

$$T_{st} = \int \tau_s(y^l_{jht}) d\mu_t \tag{31}$$

$$T_{yt} = \int \tau_y(y^b_{jht}) d\mu_t \tag{32}$$

$$E_t = \int (1-\psi_{jt}) a_{jht+1} d\mu_t \tag{33}$$

where $y^k_{jht} = a_{jht} r_t$, $y^l_{jht} = \epsilon_{jh} s_t l_{jht} w_t$, and $y^b_{jht} = y^k_{jht} + y^l_{jht} + p_t - \tau_k(y^k_t) - \tau_s(y^l_t)$, and all the integrals are defined over the state space $\Re$.

(v) Net investment I_t is

$$I_t = K_{t+1} - (1-\delta) K_t \tag{34}$$

Notice that in this model economy $I_t \neq \int (a_{jht+1} - (1-\delta)a_{jht})d\mu_t$. This is because some households die and their end of period capital is confiscated and because immigrants who are older than 20 bring into the economy the same amount of capital as the current residents who are in the same state.

(vi) The goods market clears:

$$C_t + \int (a_{jht+1} - a_{jht})d\mu_t + G_t + [Z_t + (F_{t+1} - F_t)] = F(K_t, A_t L_t). \tag{35}$$

The last term of the left-hand side of this expression is not standard. Transfers other than pensions, Z_t, show up in this expression because we assume that the government throws them into the sea. And the change in the value of the pension reserve fund, $(F_{t+1} - F_t)$ shows up because pension system surpluses are invested in the pension fund and pension system deficits are financed with the fund until it is depleted.[14]

(vii) The law of motion for μ_t is:

$$\mu_{t+1} = \int_{\Re} Q_t d\mu_t. \tag{36}$$

Describing function Q formally is complicated because it specifies the transitions of the measure of households along its six dimensions: age, education level, employment status, assets holdings, pension rights, and pensions. An informal description of this function is the following:

We assume that new-entrants, who are 20 years old, enter the economy as able-bodied workers, that they draw the stochastic component of their endowment of efficiency labor units from its invariant distribution, and that they own zero assets and zero pension rights. Their educational shares are exogenous and they determine the evolution of μ_{ht}. We also assume that new-entrants who are older than 20 replicate the age, education, employment status, wealth, pension rights, and pensions share distribution of the existing population.

The evolution of μ_{jht} is exogenous, it replicates the Spanish demographic projections, and we compute it following a procedure that we describe in Section 4 below. The evolution of μ_{et} is governed by the conditional transition probability matrix of its stochastic component, by the probability of becoming disabled, and by the optimal decision to retire. The evolution of μ_{at} is determined by the optimal savings decision and by the changes in the population. The evolution of μ_{bt} is determined by the rules of the Spanish public pension system which we have described in Section 2.1. Finally, we assume that once a household retires or becomes disabled its retirement or disability pensions never change.

[14] The last term of the left-hand side of Expression (35) would show up as net exports in the standard national income and product accounts.

3 Calibration

To calibrate our model economy we do the following: First, we choose a calibration target country —Spain in this article— and a calibration target year —2010 in this article. Then we choose the initial conditions and the parameter values that allow our model economy to replicate as closely as possible selected macroeconomic aggregates and ratios, distributional statistics, and the institutional details of our chosen country in our target year. We describe these steps in the subsections below.

3.1 Initial conditions

To determine the initial conditions, first we choose an initial distribution of households, μ_0. In Appendix 1 we provide a detailed description about how we obtain that distribution. The initial distribution of households implies an initial value for the capital stock. This value is $K_{2010} = 12.1602$. The initial distribution of households and the initial survival probabilities determine the initial value of unintentional bequests, E_{2010}. We must also specify the initial values for the productivity process, A_{2010}, and for the pension reserve fund F_{2010}. Since A_{2010} determines the units which we use to measure output and does nothing else, we choose $A_{2010} = 1.0$. Finally, our choice for the initial value of the pension reserve fund is $F_{2010} = 0.0612\ Y^*_{2010}$, where Y^*_t denotes output at market prices, which we define as $Y^*_t = Y_t + T_{ct}$. Our choice for F_{2010} replicates the value of the Spanish pension reserve fund at the end of 2010.

3.2 Parameters

Once the initial conditions are specified, to characterize our model economy fully, we must choose the values of a total of 50 parameters. Of these 50 parameters, 3 describe the household preferences, 21 the process on the endowment of efficiency labor units, 4 the disability risk, 3 the production technology, 12 the pension system rules, and 7 the remaining components of the government policy. To choose the values of these 50 parameters we need 50 equations or calibration targets which we describe below.

3.3 Equations

To determine the values of the 50 parameters that identify our model economy, we do the following. First, we determine the values of a group of 31 parameters directly using equations that involve either one parameter only, or one parameter and our guesses for (K, L). To determine the values of the remaining 19 parameters we construct a system of 19 non-linear equations. Most of these equations require that various statistics in our model economy replicate the values of the corresponding Spanish statistics in 2010. We describe the determination of both sets of parameters in

the subsections below.

3.3.1 Parameters determined solving single equations

The life-cycle profile of earnings. We measure the deterministic component of the process on the endowment of efficiency labor units independently of the rest of the model. We estimate the values of the parameters of the three quadratic functions that we describe in Expression (1), using the age and educational distributions of hourly wages reported by the *Instituto Nacional de Estadística* (INE) in the *Encuesta de Estructura Salarial* (2010) for Spain.[15] This procedure allows us to identify the values of 9 parameters directly.

The disability risk. We want the probability of becoming disabled to approximate the data reported by the *Boletín de Estadísticas Laborales* (2007) for the Spanish economy. We use this dataset to estimate the values of parameters a_4 and a_5 of Expression (3) using an ordinary least squares regression of q_j on j. According to the *Instituto de Mayores y Servicios Sociales*, in 2008 in Spain 62.6 percent of the total number of disabled people aged 25 to 44 years old had not completed high school, 26.9 percent had completed high school, and the remaining 10.5 percent had completed college. We use these shares to determine the values of parameters a_6 and a_7 of Expression (4). Specifically, we choose $a_6 = 0.269/0.626 = 0.4297$ and $a_7 = 0.105/0.626 = 0.1677$. This procedure allows us determine the values of 4 parameters directly.

The pension system. In 2010 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 44,772 euros of annual gross labor income. Hence, the maximum contribution was 12,670 euros which correspond to 45,53 percent of the Spanish GDP per person who was 20 or older. To replicate this feature of the Spanish pension system we choose the value of parameter a_{13} of our payroll tax function to be $a_{13} = 0.4553$.

Our choice for the number of years used to compute the retirement pensions in our benchmark model economy is $N_b = 15$. This is because in 2010 the Spanish *Régimen General de la Seguridad Social* took into account the last 15 years of contributions prior to retirement to compute the pension.

We assume that the minimum pension, the maximum pension, and the maximum covered earnings are directly proportional to per capita income. Our targets for the proportionality coefficients are $b_{0t} = 0.1731$, $b_{mt} = 1.2567$, and $a_{15} = 1.6089$. These numbers correspond to their values in 2010 for workers included in the *Régimen General*.[16]

[15] Since we only have data until age 64, we estimate the quadratic functions for workers in the 20–64 age cohort and we project the resulting functions from age 65 onwards.

[16] Specifically, in 2010 the minimum retirement pension in Spain was 4,817 euros, the maximum pension was 34,970 euros, the maximum covered earnings were 44,772 euros, and GDP per person who was 20 or older was 27,827 euros. All these data are yearly.

Table 2: Parameters determined solving single equations

	Parameter	Value
Parameters determined directly		
Earnings Life-Cycle		
	$a_{1,1}$	0.9189
	$a_{1,2}$	0.8826
	$a_{1,3}$	0.5064
	$a_{2,1}$	0.0419
	$a_{2,2}$	0.0674
	$a_{2,3}$	0.1648
	$a_{3,1}$	0.0006
	$a_{3,2}$	0.0008
	$a_{3,3}$	0.0021
Disability Risk		
	a_4	0.000449
	a_5	0.0924
	a_6	0.4297
	a_7	0.1677
Preferences		
Curvature	σ	4.0000
Technology		
Capital share	θ	0.3669
Productivity growth rate	γ	0.0000
Public Pension System		
Maximum early retirement penalty	a_{16}	0.4000
Early retirement penalty per year	a_{17}	0.0800
Number of years of contributions	N_b	15
First retirement age	R_0	60
Normal retirement age	R_1	65
Rate of return for the pension fund	r^*	0.0200
Government Policy		
Household Income Tax function		
	a_9	0.4500
	a_{11}	1.0710
Parameters determined by guesses for (K, L)		
Public Pension System		
Payroll tax cap	a_{13}	0.4553
Maximum covered earnings	a_{15}	1.6089
Minimum retirement pension	b_{0t}	0.6326
Maximum retirement pension	b_{mt}	4.5927
Government Policy		
Government consumption	G	0.7546
Capital income tax rate	a_8	0.1898
Consumption tax rate	a_{12}	0.2105

In the benchmark model economy we choose the first and the normal retirement ages to be $R_0 = 60$ and $R_1 = 65$. In Spain the first retirement age was 60 until 2002. This rule was changed in 2002 when the first retirement age was changed to 61, with some exceptions. We choose $R_0 = 60$ because in 2010 a large number of workers were still retiring at that age.[17]

To identify the early retirement penalty function, we choose $a_{16} = 0.4$, and $a_{17} = 0.08$. This is because we have chosen $R_0 = 60$, and because in Spain the penalties for early retirement are 8 percent for every year before age 65. Finally, for the rate of return on the pension reserve fund's assets we choose $r^* = 0.02$.[18] These choices allow us to determine the values of 10 parameters.

Government policy. To specify the government policy, we must choose the values of government consumption, G_t, of the tax rate on capital income, a_8, of parameters a_9 and a_{11} of the household income tax function, and of the tax rate on consumption, a_{12t}. We describe our procedure to choose the value of these 5 parameters in Appendix 2.

Preferences. Of the four parameters in the utility function, we choose the value of only σ directly. Specifically, we choose $\sigma = 4.0$. This choice and the value of the share of consumption in the utility function, imply that the relative risk aversion in consumption is 1.8937, which falls within the 1.5-3 range which is standard in the literature.

Technology. According to the OECD data, the capital income share in Spanish GDP was 0.3669 in 2008. Consequently, we choose $\theta = 0.3669$. We also choose the growth rate of total factor productivity directly. We discuss this choice in Section 4 below.

Adding up. So far we have determined the values of 31 parameters either directly or as functions of our guesses for (K, L) only. We report their values in the first two blocks of Table 2.

3.3.2 Parameters determined solving a system of equations

We still have to determine the values of 19 parameters. To find the values of those 19 parameters we need 19 equations. Of those equations, 14 require that model economy statistics replicate the value of the corresponding statistics for the Spanish economy in 2010, 4 are normalization conditions, and the last one is the government budget constraint that allows us to determine the value of Z/Y^* residually.

[17] In 2010 in Spain 22.4 percent of the people who opted for early retirement were 60 years old or younger. And 5.78 percent of the total number of retirees were 60 or younger. See *Ministerio de Trabajo e Inmigración (MTIN), Anuario de Estadísticas* 2010 (http://www.empleo.gob.es/estadisticas/ANUARIO2010/PEN/index.htm).

[18] In Díaz-Giménez and Díaz-Saavedra (2009) we also run simulations for $r^* = 1$, 3, and 4 percent. We found that the changes implied by the various values of r^* were small and that they did not modify the qualitative conclusions of that article.

Table 3: Macroeconomic Aggregates and Ratios in 2010 (%)

	C/Y^{*a}	T_y/Y^*	T_s/Y^*	P/Y^*	K/Y^{*b}	h^c
Spain	51.5	7.4	10.1	10.3	3.28	37.5

[a]Variable Y^* denotes GDP at market prices.
[b]The target for K/Y^* is in model units and not in percentage terms.
[c]Variable h denotes the average share of disposable time allocated to the market.

Aggregate Targets.

According to the Spanish *Encuesta de Empleo del Tiempo (2010)*, the average number of hours worked per worker was 36.79 per week. If we consider the endowment of disposable time to be 14 hours per day, the total amount of disposable time is 98 hours per week. Dividing 36.79 by 98 we obtain 37.5 percent which is the share of disposable time allocated to working in the market that we target. Consequently, the Frisch elasticity of labour supply implied in our model economy is 0.77, which is in the middle of the range of recent econometric estimates of this parameter.[19]

According to the BBVA database, in 2010 the value of the Spanish capital stock was 3,454,401 million 2000 euros.[20] According to the *Instituto Nacional de Estadística* in 2010 the Spanish Gross Domestic Product at market prices was 1,051,342 million 2000 euros. Dividing these two numbers, we obtain $K/Y = 3.28$, which is our target value for the model economy capital to output ratio. In Appendix 2 we describe how we obtain the values of the first four macroeconomic ratios that we report in Table 3 .

Distributional Targets. We target the 3 Gini indexes and 5 points of the Lorenz curves of the Spanish distributions of earnings, income and wealth for 2004. We have taken these statistics from Budría and Díaz-Giménez (2006), and we report them in bold face in Table 8. Castañeda et al. (2003) argue in favor of this calibration procedure to replicate the inequality reported in the data. These targets give us a total of 8 additional equations.

Normalization conditions. In our model economy there are 4 normalization conditions. The transition probability matrix on the stochastic component of the endowment of efficiency labor units process is a Markov matrix and therefore its rows must add up to one. This gives us three normalization conditions. We also normalize the first realization of this process to be $s(1)=1$.

The Government Budget. Finally, the government budget is an additional equation that allows us to obtain residually the government transfers to output ratio, Z_t/Y_t^*.

[19]See, for example, Fuster et al. (2007).
[20]This number can be found at $http://www.fbbva.es/TLFU/microsites/stock09/fbbva_stock08_index.html$.

3.3.3 Computation

To determine the values of these 19 parameters, first we solve the system of 14 non-linear equations in 14 unknowns that we obtain when we equate the relevant statistics of the model economy to their corresponding Spanish targets. Once we have chosen the best solution to this system, we obtain the values of the remaining 5 parameters from the government budget and from our normalization conditions. We describe our computational algorithm in Appendix 3. We report the values of the 19 unknowns in Table 4 and in the first two blocks of Table 5.

Table 4: Parameters determined solving a system of equations

	Parameter	Value
Preferences		
Leisure share	α	0.2979
Time discount factor	β	1.0460
Technology		
Capital depreciation rate	δ	0.0724
Public Pension System		
Payroll tax rate	a_{14}	0.2385
Pension replacement rate	ϕ	0.8279
Government Policy		
Household Income tax function	a_{10}	0.0672
Government transfers	Z	0.0807

3.4 Calibration results

We now show that our model economy is consistent with Spanish 2010 data. We start by describing the stochastic component of the endowment process. Next we discuss the retirement behavior because it is the main concern of this article. Then we describe the main macroeconomic aggregate and ratios and, finally, we discuss the distributions of earnings, income, and wealth. We find that our model economy replicates reasonably well most of the Spanish statistics that we target.

3.4.1 The stochastic component of the endowment process

The procedure that we have used to calibrate our model economy identifies the stochastic component of the endowment of efficiency labor units process, s. In Table 5 we report its main features. Recall that we have restricted to three the number of realizations of s. We find that, in order to replicate the Spanish Lorenz curves of the income and earnings distributions, the value of the highest realization of s is 11.3 times that of its lowest value. We find also that the process on s is very persistent. Specifically, the expected durations of the shocks are 17.2, 31.3, and an astonishing 3333.3 years. In the last column of Table 5 we report the invariant distributions of the shocks. We

find that approximately 89 percent of the workers are either in state $s = s_1$ or in state $s = s_2$, and that about 11 percent are in state $s = s_3$.[21]

Table 5: The Stochastic Component of the Endowment Process

		Transition Probabilities			
	Values	$s' = s_1$	$s' = s_2$	$s' = s_3$	$\pi^*(s)^a$
$s = s_1$	1.0000	0.9417	0.0582	0.0000	31.41
$s = s_2$	2.0856	0.0319	0.9680	0.0000	57.25
$s = s_3$	11.2892	0.0000	0.0002	0.9997	11.32

[a] $\pi^*(s)\%$ denotes the invariant distribution of s.

3.4.2 Retirement behavior

In Table 6 we report the average retirement ages and the participation rates of people aged from 60 to 64. The average retirement age in our model economy is 63.5 years, 1.2 years higher than in Spain. We also find that the average retirement ages are increasing in the number of years of education. Unfortunately, we could not find these data for Spain, but we think that this increasing relationship is very plausible, since the Spanish participation rates of the 60–64 age cohort are strongly increasing in education (see the third column of Table 6).

Table 6: Retirement Ages And Participation Rates of Older Workers

	Avg Ret Ages		Part rates at 60-64 (%)	
	Spain[a]	Model	Spain[b]	Model
All	62.3	63.5	56.6	53.5
Dropouts	*n.a.*	63.1	45.5	40.2
High School	*n.a.*	63.8	61.0	64.7
College	*n.a.*	64.4	75.2	80.5

[a] The Spanish data is for both males and females in 2010 (Source: Eurostat).
[b] The Spanish data is from both the *Encuesta de la Población Activa*, and the *Encuesta de Empleo del Tiempo 2010*, excluding the unemployed and non-participants who do not collect either retirement or disability pensions.

The total participation rate of the households in the 60 to 64 age cohort is 53.9 percent in our model economy, and 56.6 percent in Spain. As we have already mentioned, the participation rates both in Spain and in our model economy are increasing in education. This is because, even though all educational types value leisure equally, the foregone labor income—which is the opportunity

[21] The process on s is very different from the one we found in Díaz-Giménez and Díaz-Saavedra (2009). Specifically, we find that the range of the values of the realizations is larger and that the shocks are more persistent. These differences are mostly due to the progressivity of the personal income tax, the double taxation of capital income, the increase in the share of college educated workers, the change in distributional targets that occur because we delay the calibration year, and the assumption that transfers other than pensions are thrown into the sea.

cost of leisure—is lower for less educated workers and, therefore they tend to retire earlier. Our model economy replicates this behavior even though it has fewer labor market categories than Spain. In Spain people of working age can be employed, unemployed, retired, disabled, and other non-participants. In our model economy, we abstract from the unemployed and from the other non-participants.

In Panel A of Figure 2 we illustrate the age-profiles of the retirement hazards. The Spanish profile, which displays a small peak at the first retirement age and a much larger one at the normal retirement age, is a common stylized fact in countries that run defined-benefit pension systems (see Gruber and Wise, 1999). At first sight, our model economy replicates this pattern. A closer scrutiny reveals that the hazards are three and five percentage points higher in our model economy at those ages.[22]

In Panel B of Figure 2 we show that high-school dropouts have a higher probability to exit the labor force at age 60 than more educated workers. Our results show that in our model economy 90 percent of those who retire at 60 are dropouts. This finding is consistent with those of Sánchez-Martín (2010), who reports that low-income workers have a higher probability of retiring at age 60 than high-income workers.

The details of the Spanish minimum retirement pension are one of the reasons behind this result. In 2010, about 27 percent of the Spanish retirees receive the minimum retirement pension—this share is 28 percent in our model economy. Workers who qualify for the minimum pension can start to collect it at 60 without paying an early retirement penalty. Moreover, for many of these workers, remaining in the labor force after age 60 does not increase their pensions. Since many of these typically low-wage earners gain very little from continued employment, many of them choose to retire as early as possible. In our model economy, 97 percent of the workers who retire at age 60 collect the minimum pension, while Jiménez-Martín and Sánchez-Martín (2006) report that in 1997 this number was 67 percent in Spain.

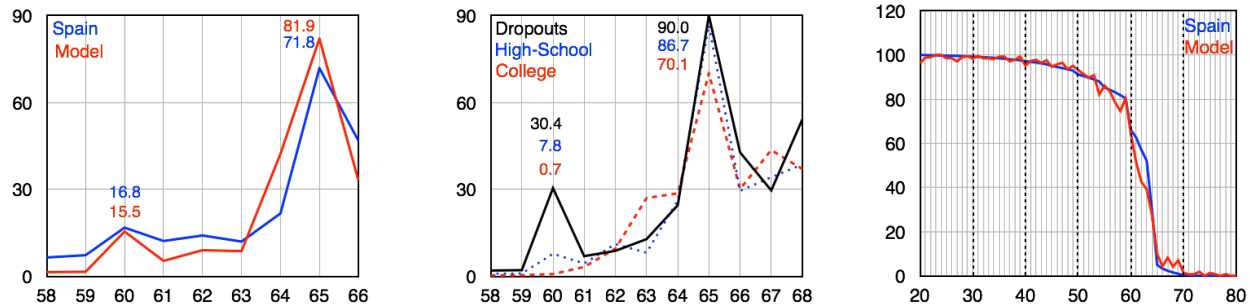
Retirement hazards are decreasing between ages 60 and 63, both in Spain and in our model economy. This is because workers who qualify to collect a pension that is higher than the minimum pension and who choose to work for one extra year after age 60 reduce the early retirement penalty by 8 percent. This means that these workers face an implicit subsidy if they continue to work between ages 60 and 64, and this subsidy may amount to as much as 25 percent of their net yearly salary, as shown by Boldrin et al. (1997)[23].

This behavior changes at age 65. This is because the incentives provided by the Spanish pension system to delay retirement beyond this age are very small. Since pension rights are a function of

[22]The Spanish data for the retirement hazards is taken from García Pérez and Sánchez-Martín (2010).

[23]This effect can be reversed in the case of workers who expect to earn an exceptionally low salary for whatever reason. These workers face an implicit tax on continued work, since their low salaries reduce their pension rights and, therefore, their pensions.

Figure 2: Retirement Hazards and Shares of Workers (%)*



A: Retirement Hazards (All) B: Retirement Hazards (Edu) C: Shares of Workers (All)

*The Spanish data for the retirement hazards is taken from García Pérez and Sánchez-Martín (2010). The shares of workers are the shares of workers in the sum of workers, disabled people, and retirees. We compute this share for Spain from the *Encuesta de Empleo del Tiempo (2010)*, reported by the INE.

the average wages in the last fifteen years prior to retirement, the reduction in wages that results from the life-cycle usually reduces the pensions. Therefore, most workers who continue to work after age 65 face an implicit tax on doing so and many choose to leave the labor force at 65 to avoid this tax. Finally, Boldrin et al. (1997), Argimón et al. (2009), and Sánchez-Martín (2010) find that the probability of retiring at age 65 is independent of salary level, and our model economy replicates this stylized fact. Panel B of Figure 2 shows that retirement hazards at 65 are similar for the three educational groups, and that they are larger than 75 percent for all of them.

In Panel C of Figure 2 we report the shares of workers in the sum of workers, disabled people and retirees. We find that the age distribution of this ratio is almost identical in Spain and in the benchmark model economy.

Overall, we find these results very encouraging. A trustworthy answer to the questions that we ask in this paper requires a model economy that captures the key institutional and economic forces that affect the retirement decision. Our model economy replicates in great detail both the Spanish tax system and the rules of the Spanish public pension system. Moreover, our calibration procedure allows us to obtain an earnings process that allows us to replicate the earnings, income and wealth inequality observed in Spain, as we discuss below. And we have just shown that our model economy replicates many of the features of retirement behavior found in Spanish data. This result is particularly remarkable, since we did not target explicitly any of these retirement behavior facts in our calibration procedure.

3.4.3 Aggregates and Ratios

In Table 7 we report the macroeconomic aggregates and ratios in Spain and in our benchmark model economy for 2010. We find that our benchmark model economy replicates most of the Spanish targets almost exactly. The largest relative difference is in the income tax collections to output ratio which is approximately 0.3 percentage points higher in the model economy.

Table 7: Macroeconomic Aggregates and Ratios in 2010 (%)

	C/Y^{*a}	K/Y^{*b}	h	T_y/Y^*	T_s/Y^*	P/Y^*
Spain	51.5	3.28	37.5	7.4	10.1	10.3
Model	51.4	3.28	37.6	7.7	10.1	10.2

[a] Variable Y^* denotes GDP at market prices.
[b] The target for K/Y^* is in model units and not in percentage terms.

3.4.4 Inequality

In Table 8 we report the Gini indices and selected points of the Lorenz curves for earnings, income, and wealth in our model economy and in Spain. The statistics reported in bold face are our eight calibration targets. The source for the Spanish data on earnings, income and wealth is the 2004 Financial Survey of Spanish Families, as reported in Budría and Díaz-Giménez (2006). The model economy statistics correspond to 2010.

Table 8: The Distributions of Earnings, Income, Pensions, and Wealth*

		Bottom Tail			Quintiles					Top Tail		
	Gini	1	1–5	5–10	1st	2nd	3rd	4th	5th	10–5	5–1	1
The Earnings Distributions (%)												
Spain	**0.49**	0.0	0.7	1.2	**5.3**	10.9	**16.2**	23.3	**44.3**	10.9	11.5	5.6
Model	0.48	0.1	0.8	1.3	5.2	9.4	13.5	16.0	55.7	17.5	18.1	6.6
The Income Distributions (%)												
Spain	**0.42**	0.0	0.7	1.1	5.1	10.1	15.2	22.5	47.1	11.1	12.8	6.7
Model	0.44	0.1	0.9	1.5	6.3	9.6	13.9	17.3	52.8	14.8	18.3	6.9
The Wealth Distributions (%)												
Spain	**0.57**	–0.1	0.0	0.0	0.9	**6.6**	12.5	**20.6**	59.5	12.5	16.4	13.6
Model	0.57	0.0	0.0	0.0	0.9	6.6	13.2	20.5	58.7	15.7	22.8	6.2

*The source for the Spanish data of earnings, income, and wealth is the 2004 *Encuesta Financiera de las Familias Españolas* as reported in Budría and Díaz-Giménez (2006). The model economy statistics correspond to 2010. The statistics in bold face have been targeted in our calibration procedure.

We find that our model economy replicates the Spanish Gini indices of earnings, income and wealth reasonably well —the largest difference is only 0.02. Moreover it also comes close to replicat-

ing the Gini index of pensions. According to Conde-Ruiz and Profeta (2007), in 2000 this number was 0.32 in Spain and in our model economy it is 0.36 in our calibration year. Once again, this result can be interpreted as an overidentification condition, since we did not use it as a calibration target.

When we compare the various quantiles of the distributions, we find that the model economy households in the first four quintiles of the earnings distribution earn less than the Spanish households and that the households in the top quintile earn sizably more —their share of earnings is almost 12 percentage points higher than the Spanish share. In contrast, our model economy replicates the Spanish wealth distribution very closely. And, predictably, the income distribution is in between the other two —for instance, the share of income earned by the households in the top quintile of the model is almost 6 percentage points larger than the Spanish share, which is almost half way between 12 and 1.

When we look at the top tails of the distributions we find that the share of wealth owned by the top 1 percent of the wealth distribution in the model economy is 7.4 percentage points higher in Spain. This disparity was to be expected, because it is a well-known result that overlapping generation model economies that abstract from bequests fail to account for the large shares of wealth owned by the very richest households in the data.[24]

4 The Simulation Scenarios

The benchmark model economy and the model economies with the reformed pensions share the initial conditions described above and the demographic, educational, growth, and fiscal policy scenarios that we now describe.

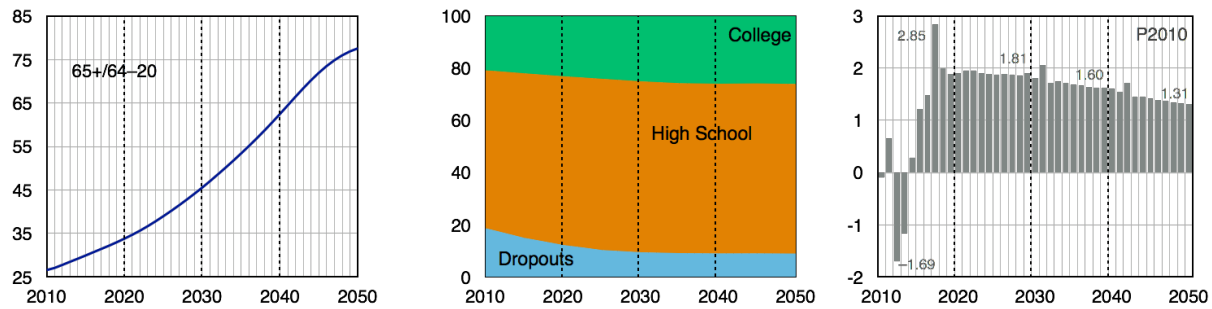
The Demographic Scenario. The demographic scenario of our four model economies replicates the demographic projections for Spain for the period 2010–2052 estimated by the *Instituto Nacional de Estadística* (INE). In Appendix 2 we describe in detail the procedure that we use to compute the age distribution, μ_{jt}, for $j = 20, 21, ..., 100$ and $t = 2010, 2011, ..., 2052$ from the INE's projections.[25] In Panel A of Figure 3 we plot the changes in the 65+ to 64–20 dependency ratio in the benchmark model economy.

The Educational Scenario. The initial educational distribution of our model economy replicates the educational distribution of the Spanish population in 2010, as reported by the INE in 2012. After 2010, we assume that the shares for the dropouts, high school, and college entrants are 8.65 percent, 63.53, and 27.82 percent forever. Those shares are the educational shares of the most

[24] See Castañeda et al. (2003) for an elaboration of this argument.

[25] The INE's demographic projections can be found at $http://www.ine.es/inebmenu/mnu_cifraspob.htm$.

Figure 3: The Simulation Scenarios



A: The Dependency Ratio B: The Distribution of Education C: The Growth Scenario

educated cohort ever in Spain, which corresponds to the 1980 to 1984 cohort.[26] In Panel B of Figure 3 we plot the changes in the distribution of education in the benchmark model economy.

The Growth Scenario. In 2010 and 2011 the growth rates of output at market prices in our model economy approximate the growth rates of Spanish GDP, which were –0.1 and 0.7 percent. For the 2012–2017 period, our model economy approximates the International Monetary Fund forecast for Spain published in its October 2013 World Economic Outlook. Specifically, the growth rates that we target for those years are –1.7, –1.2, 0.9, 1.6, 1.7, and 1.7 percent. After 2017, we assume that the growth rate of output at market prices is, approximately, 1.7 percent. In Panel C of Figure 3 we plot the growth rates of output in the benchmark model economy.

In our model economies there are three sources of output growth: demographic changes, educational changes, and changes in the labor-augmenting productivity factor, γ_t. To replicate the growth scenario described above, we iterate in the sequence of γ's until we obtain reasonable approximations to our targeted growth rates. The benchmark model economy and the three reformed economies share the educational and demographic transitions and the sequence of γ's. But output grows at different rates in the four model economies because of the endogenous changes in labor hours and savings brought about by their different pension systems.

The Inflation Rate Scenario. We assume that the inflation rate in our model economy will remain constant at 2.32 percent, which was the average value of the Spanish inflation rate in the 1998–2013 period.

The Fiscal Policy Scenario. Recall that the consolidated government and pension system budget constraint in our model economy is

$$G_t + P_t + Z_t = T_{kt} + T_{st} + T_{yt} + T_{ct} + (F_t - F_{t+1}) + E_t \tag{37}$$

In this expression G_t is exogenous and the remaining variables are endogenous. In every model

[26] Conde-Ruiz and González (2013) also follow this approach.

economy the capital income tax rates and the parameters that determine the payroll tax function and the household income tax function are identical and they remain unchanged at their 2010 values. The consumption tax rates differ across economies because we change them to finance the pensions once the pension reserve fund is exhausted. And every other variable in Expression (37) varies in time and differs across economies because they are all endogenous.

To determine the value of G, in the benchmark model economy we assume that G/Y^* is constant at its 2010 value, and we assume that all the reformed model economies share the $\{G_t\}$ sequence of the benchmark model economy. These assumptions imply that the G/Y^* ratios differ somewhat across the benchmark and the reformed model economies.

Minimum pensions, maximum pensions, and pension rights. To replicate what has been the traditional policy in Spain so far, in our benchmark model economy we assume that minimum and maximum pensions are updated each year so that their share of output per person remains constant. We also assume that the values of the minimum and maximum pensions that we have computed for the benchmark model economy remain unchanged in every reform.

We do not update pension rights. Therefore, their real value remains constant throughout our simulations.

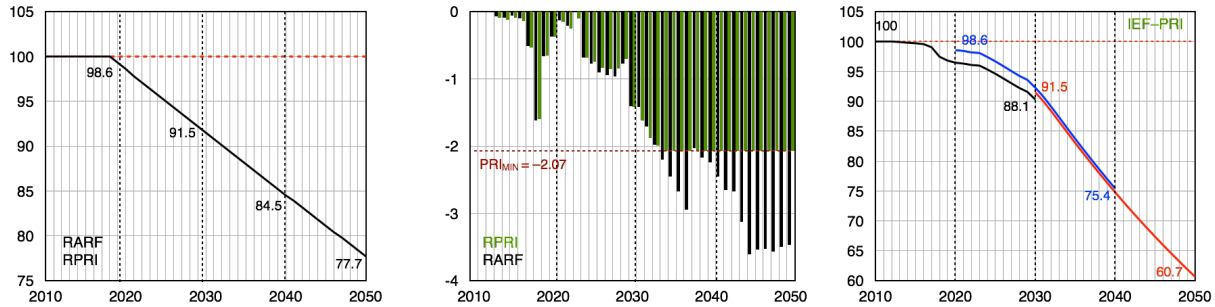
4.1 The Reforms

The 2011 Reform and the 2013 delay. The 2011 Reform of the Spanish pension system changed three of the main parameters of the pension system. It extended from 15 to 25 the number of years of earnings used to compute the retirement pension. It delayed from 61 to 63 years the first retirement age, and it delayed from 65 to 67 years the normal retirement age. The extension of the number of years used to compute the pensions was phased in gradually increasing that number in one year per year starting in 2013 and ending in 2022. The delay in the first retirement age was immediate. And the delays in the normal retirement were also gradual: one month per year between 2013 and 2018, and two months per year between 2019 and 2027. Consequently, the normal retirement age in Spain will be 66 in 2021 and 67 in 2027. Additionally, in 2013, the Spanish government approved a gradual delay of the first retirement age from 63 to 65 years.[27]

The Sustainability Factors. The 2011 Pension Reform made provisions to add a sustainability factor to Spanish pensions. This factor would take into account the expected duration of retirement and would revaluate pensions to ensure the financial sustainability of the system. This factor effectively changes the Spanish pay-as-you-go system from a defined-benefit to a defined-contribution system. In 2013, the Spanish Government created a *Committee of Experts* to make a specific proposal with the details of these factors. They proposed to break up the sustainability factor into two

[27]The changes in the first retirement ages apply only to voluntary retirees.

Figure 4: The Sustainability Factors (%)



A: The IEF[a] B: The ARF and the PRI C: The Value of Pensions[b]

[a]This panel plots the results of applying the IEF to the initial pension of a retiree whose pension claims are worth 100 in real terms.
[b]This panel plots the results of applying both the IEF and the PRI to the pensions of people whose pension claims are worth 100 in real terms and who retire at age 67 in 2010, 2020, and 2030.

components: an Intergenerational Equity Factor (IEF) and an Annual Revaluation Factor (ARF).

The Intergenerational Equity Factor. The idea behind the IEF is to adjust the pensions with the life-expectancy at retirement so that the life-time cost of retirement is approximately the same for every cohort. The specific formula finally adopted for this factor is the following

$$IEF_t = \varepsilon IEF_{t-1} \tag{38}$$

where ε is a time-varying mesure of the relative life-expectancy at age 67. Specifically, for the period 2019–2023 the value of ε will remain constant at

$$\varepsilon = \left[\frac{e_{67,2012}}{e_{67,2017}}\right]^{1/5} \tag{39}$$

In this expression variable $e_{67,t}$ denotes the life expectancy at age 67 in year t. For the period 2024–2028 the value of ε will be updated to

$$\varepsilon = \left[\frac{e_{67,2017}}{e_{67,2022}}\right]^{1/5} \tag{40}$$

and so on. The IEF will be applied for the first time in 2019, and it will apply to *new* pensions only. In Panel A of Figure 4 we represent the values of the IEF that we have computed using the 2012 mortality tables. It turns out that, by 2050 the IEF alone will have reduced the real yearly value of Spanish pensions in 22.3 percentage points.

The Annual Revaluation Factor. The second factor of the original proposal was the Annual Revaluation Factor (ARF). This factor reduces the value of every pension to adjust the pension system outlays to the pension system revenues. The formula originally proposed to define this factor is the

following:

$$g_{t+1} = \overline{\pi}_{t+1} + (\overline{g}_{c,t+1} - \overline{g}_{p,t+1} - \overline{g}_{s,t+1}) + \alpha(\frac{R_t^* - E_t^*}{E_t^*}) \tag{41}$$

where $\overline{x_t}$ indicates the moving arithmetic average of variable x_t computed between $t-5$ and $t+5$, x^* indicates the moving geometric average of variable x_t computed between $t-5$ and $t+5$, π is the inflation rate, $g_{c,t+1}$ is the growth rate of the pension system revenues, $g_{p,t+1}$ is the growth rate of the number of pensions, $g_{s,t+1}$ is the growth rate of the average pension due to the substitution of old pensions by new pensions, $0.25 \leq \alpha \leq 0.33$ is an adjustment coefficient, R_t denotes the pension system revenues, and E_t denotes pension system expenditures.

The Pension Revaluation Index. In December 2013 the Spanish government approved the final versions of the sustainability factors and changed their names. The IEF was unchanged except for its official name. The IEF is now formally called the *Sustainability Factor*. For the sake of clarity, in the rest of the article, we leave the name of this factor unchanged. The government also imposed a bound on the ARF and changed its name to *Pension Revaluation Index* (PRI). The PRI imposes two bounds on the ARF. The lower bound is 0.25 percent and the upper bound is 0.5 percent plus the inflation rate. In our model economy pensions are defined in real terms. Therefore, we modify the Spanish PRI bounds subtracting an estimate of the inflation rate. As we have already mentioned, our inflation scenario assumes that the inflation rate is constan and that its value is the Spanish average yearly inflation between 1998 and 2013, which was 2.32 percent. Consequently, the real value of the lower bound of the PRI is $-2.07(= 0.25 - 2.32)$ percent and its upper bound is 0.5 percent.

In Panel B of Figure 4 we plot the values of the ARF factor that obtain in Model Economy RARF and the values of the PRI factor that obtain in Model Economy RPRI. We describe these two model economies in the subsection below. Finally, in Panel C, we plot the result of applying both the IEF and the PRI to the pensions of people whose pension claims are worth 100 in real terms and who retire at age 67 in 2010, 2020, and 2030. We find that the IEF reduces the initial values of their pensions from 100 for those who retire in 2010 to 98.6 and 91.5 for those who retire in 2020 and 2030. Then, the application of the PRI reduces the value of their pensions every year. Thus, the real values of the pensions of 87 year olds are 81.1 in 2030, 75.4 in 2040, and 60.7 in 2050. If pensions had been revaluated with the rules prevailing in Spain in 2010, the real value of the pensions would have been 100 every year.

The Reform Announcements. We assume that the pension system reforms are announced at the beginning of 2011, and that they affect every household that had not retired by the end of that year.

5 Simulations

We use our model to simulate four economies that differ only in their pension systems. In the first model economy, which we label P2010, we replicate the pension system parameters that prevailed in Spain in 2010.[28] In the second model economy, which we label R2013, we replicate the details of the 2011 Pension Reform and we extend it to include the additional delay in the first retirement age enacted in 2013. In the third model economy, which we label RARF, we add to the previous reform the Intergenerational Equity Factor and the Annual Revaluation Factor as originally proposed in June 2013 by a *Committee of Experts* created to that effect. And in the last model economy, which we label RPRI, we substitute the Annual Revaluation Factor with the Pension Revaluation Index approved by the Spanish government in December of 2013. As we have already mentioned, these four model economies are identical except for their pension systems.

5.1 The Benchmark and the Reformed Model Economies

Model Economy P2010. Our benchmark model economy is Model Economy P2010. This model economy replicates the the values of parameters of the Spanish pay-as-you-go, defined benefit pension system that prevailed in Spain in 2010. These parameter values were the following: the first retirement age was 60, the normal retirement age was 65, and pension rights were computed taking into account the last 15 years of contributions previous to retirement. In Model Economy P2010 we replicate these parameter values exactly, and we keep them constant throughout the simulation with only one exception: we delay the first retirement age to 61 in 2015. This change is in line with what is happening in Spain as a result of regulatory changes enacted before 2010.

In this model economy we revaluate the minimum and the maximum pensions so that their share of output per person remains constant at is 2010 value. And we assume that the real value of pensionable rights does not change.

Model Economy R2013. In Model Economy R2013 we model the Spanish 2011 pension system reform and the 2013 delay that we have described above. As in Spain, we extend the number of years of earnings used to compute the pensions one year per year, from the 15 years previous to retirement in 2012 to 25 in 2022. In 2010, the first retirement age in Model Economy P2010 was 60 because many workers were still entitled to opt for early retirement at that age in Spain. In Model Economy R2013 we delay the first retirement age to 63 in 2012. We choose 2012 instead of 2011 because this change was approved in August of 2011; in 2018 we delay the first retirement age to 64, and in 2024 to 65. Finally, we delay the normal retirement age from 65 to 66 years in 2018, and from 66 to 67 years in 2024.

[28] In the simulations we report the model economy ratios as a share of output measured at factor cost. In the calibration we report output measured at market prices. Therefore, the P2010 ratios for 2010 differ in both sections.

Model Economy RARF. In Model Economy RARF we apply the Annual Revaluation Factor from 2013 onwards and the Intergenerational Equity Factor from 2019 onwards. Otherwise, the pension systems in Model Economies RARF and R2013 are identical. As is the case in Spain, the sustainability factors do not apply to minimum pensions.

Model Economy RPRI. In Model Economy RPRI we apply the Pension Revaluation Index from 2013 onwards and the Intergenerational Equity Factor from 2019 onwards. Otherwise, the pension systems in Model Economies RPRI and RARF are identical. And, once again, the sustainability factors do not apply to minimum pensions.

5.2 Findings

We simulate these four model economies using the demographic, educational, and economic scenarios that we have described in Section 4 and we illustrate the results of our simulations in Figure 5.

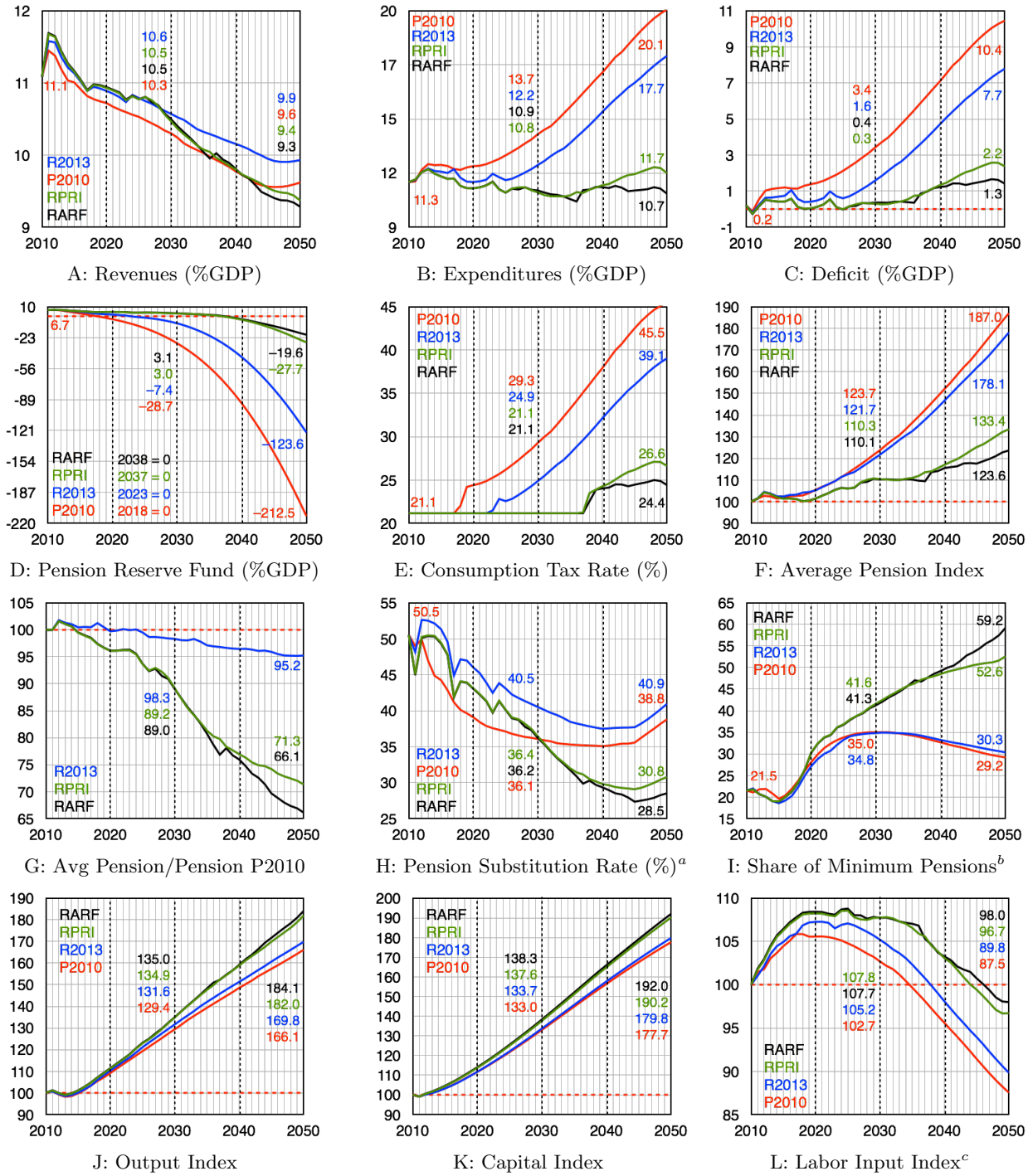
The Sustainability of the 2010 Pension System. Our simulations show that the pay-as-you-go, defined benefit pension system that prevailed in Spain in 2010 was completely unsustainable. We find that the pension reserve fund would have run out in 2018 (see Panel D of Figure 5), that the pension system deficit would have reached 10.5 percent of GDP by 2050 (see Panel C), and the accumulated pension system debt, 212 percent of GDP (see Panel D). In that same year, the consumption tax rate that would have been necessary to finance Spanish pensions would have reached 45.5 percent (see Panel E).

We also find that these sustainability problems were structural. We reach this conclusion because the pension deficits are due to the increase in pension expenditures, which are essentially unrelated to the 2008 recession. Specifically, we find that, while the pension system revenues remain virtually unchanged —in Model Economy P2010 they are 11.1 percent of GDP in 2010 and 9.6 percent in 2050 (see Panel A)— the pension system expenditures almost double —in Model Economy P2010 they are 11.3 percent of GDP in 2010 and 20.1 percent in 2050 (see Panel B).

Expenditures increase for three reasons: because of longevity increases, because cohort size increases, and because the cohorts become more educated. According to the INE's 2012 demographic scenario, life-expectancy at age 65 was 17.4 years in 2010 and it will be 23.4 years in 2050. The share of Spanish residents aged 65 or more was 20.9 percent in 2010 and it will be 43.6 percent in 2050. And the share of Spanish workers who had completed college was 20.7 percent in 2010 and it will be 26.0 percent in 2050.

These chantes imply that, in Model Economy P2010, the average pension is 87 percent higher in 2050 than in 2010 (see Panel F), while output is only 66 percent higher (see Panel J). This increase in the ratio of average pensions to output can be attributed both to ageing and to the educational transition. Ageing causes the capital labor ratio to increase and this results in an increase in wages

Figure 5: The Future of the 2010 Pension System and of the Reforms



[a]This statistic is the ratio between the average pension and the average wage of workers in the 60–64 age group.
[b]This statistic is the percentage share of the retirees who receive the minimum pension.
[c]This measure of the labor input does not include the exogenous, labour-augmenting productivity factor, A.

and, consequently, in an increase in pension claims. The educational transition delays the optimal retirement age, this reduces early retirement penalties and increases the real value of pensions further.

The Sustainability of the R2013, RARF, and RPRI Pension Systems. The three reforms improve the sustainability of the public pension system rather dramatically. The pension system deficits in 2050 fall from 10.5 percent of GDP in Model Economy P2010, to 7.8, 1.4 and 2.4 percent of GDP in Model Economies R2013, RARF, and RPRI (see Panel C of Figure 5). Resulting from these improvements, the accumulated pension system debts fall from 212 percent of GDP in Model Economy P2010 to 124, 20, and 28 percent in the reformed model economies (see Panel D), and the consumption tax rates needed to finance the pension system deficits fall from 45.5 percent in Model Economy 2010 to 39.1, 24.4, and 26.6 percent (see Panel E).

The sustainability factor reforms succeed in making the deficits sustainable until 2037. But they fail to do so after that year. This is because minimum pensions are exempt from the sustainability factors, because we assume that they are a constant share of per capita output in Model Economy 2010, and because the share of retirees who receive the minimum pension almost triples in the RARF and RPRI reforms. Specifically while this share is 21.5 percent in every model economy in 2010, in 2050 it increases to about 30 percent in Model Economies P2010 and R2011, and to 53 and 59 percent in Model Economies RPRI and RARF (see Panel I).

The Consequences for Public Pensions. But this improvement in the sustainability of public pensions is brought about with large reductions in the average pension. In Model Economy P2010, the average pension index reaches 187 in 2050 (see Panel F of Figure 5). This is the result of the pension revaluation rules, and of the demographic and educational transitions. In the reformed model economies the average value of pension also grows, but less. In 2050 the real value of the average pension in Model Economy R2013 is 95 percent of the real value of the average pension in Model Economy P2010 and, in Model Economies RPRI and RARF, the real values of the average pensions in 2050 are 71 and 66 percent of the average pension in Model Economy P2010 (see Panel G).

The pension substitution rates also decrease substantially. In 2010 the average pension amounts to 50.5 percent of the average wage of the 60–64 age cohort and, in 2050, this rate is 38.8 percent in Model Economy P2010, 40.9 percent in Model Economy R2013, 30.8 percent in Model Economy RPRI, and 28.5 percent in Model Economy RARF (see Panel H). The pension substitution rate profiles are decreasing because wages increase as the result of the exogenous increase in labor productivity and the endogenous increase of the capital labor ratio, and pensions increase less because in Model Economies P2010 and R2013 their real values remain constant and in Model Economies RARF and RPRI their real values are reduced by the ARF and the PRI.

The Consequences for Private Pensions. Savings and, consequently, capital accumulation increase substantially in the four model economies but at different rates (see Panel K of Figure 5). And most of these differences are justified by the increase in savings for retirement brought about by the reduction in pensions. In that panel we show that, in 2050, the capital stock indexes in Model Economies R2013, RARF, and RPRI are 1.2, 8.1, and 7.0 percentage points higher than in Model Economy P2010.

Macroeconomic Consequences. Panel J of Figure 5 shows that the three pension system reforms are expansionary, but in different degrees. The expansionary effects of the R2013 reform are minor: in 2050 its output index is only 2.2 percentage points higher than the output index of Model Economy P2010. And the expansionary effects of the RARF and RPRI reforms they are somewhat more substantial: in 2050 their output indexes are 10.8 and 9.6 percentage points higher than in Model Economy 2010. But the differences in the average growth rates of the four model economies are small (1.49, 1.49, 1.56, and 1.55 percent).

After 2046 these expansions are accounted for entirely by the exogenous increases in labor productivity and by the endogenous increases in capital accumulation (see Panel K), since by that year the values of the endogenous labor input indexes in the four model economies are below 100 (see Panel L).

6 Welfare

To quantify the welfare effects of the aforesaid reforms for the different cohorts, we use a consumption equivalent variation measure (CEV). Specifically, we compute how much a household's consumption has to change by during its remaining lifetime in Model Economy P2010, so that the household's expected lifetime utility in Model Economy P2010 equals its expected lifetime utility in the reformed economy. We start our computations in 2011, which is the year when all the reforms are announced. Therefore, we compute the CEV measure for those alive that year, and for the households born each year until 2030. We stop in 2030 because the households born that year will be 20 years old in 2050 and, consequently, they will enter our economy then.

Formally, we do the following: Let $z \in \Re = J \times H \times \mathcal{E} \times \mathcal{A} \times B_t \times P_t$. Then, we define $v^B[z, \Delta(z)]$ as the equilibrium value function of a household of type z in Model Economy P2010, whose equilibrium consumption allocation is changed by a fraction Δ every period and whose leisure remains unchanged. Then, for the households alive in 2011:

$$v^B[z, \Delta(z)] = \max E \left\{ \sum_{t=0}^{100-j} \beta^t \, \psi_{j+t,2011+t} \, u[c^B_{2011+t}(z)[1 + \Delta(z)], (1 - l^B_{2011+t}(z))] \right\} \tag{42}$$

where $c^B(z)$ and $l^B(z)$ are the solutions to the household decision problem.

For a household born in year t, who enters to the economy in year $t+20$:

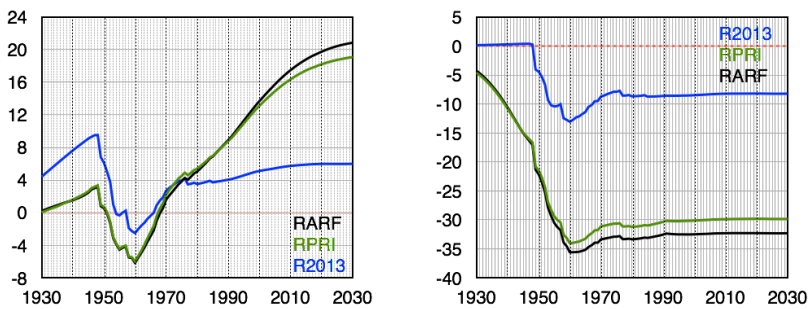
$$v^B[z, \Delta(z)] = \max E \left\{ \sum_{j=20}^{100} \beta^{j-20} \, \psi_{j,t+j} \, u[c^B_{t+j}(z)[1+\Delta(z)], (1 - l^B_{t+j}(z))] \right\} \tag{43}$$

Then $\Delta(z)$ is the number that solves

$$v^B[z, \Delta(z)] = v^R(z) \tag{44}$$

where $v^R(z)$ is the value of the optimal consumption and leisure allocations in the reformed model economy.[29]

Figure 6: The Welfare Costs of the Reforms



A: Financing Pensions with τ_c B: Financing Pensions with Debt

In Figure 6 and in Table 9 we report the results of our welfare comparisons for the cohorts born between 1930 and 2030. There are two ways to think about this exercise: an exercise in optimal taxation that trades-off higher pensions against the higher consumption taxes needed to finance them (see Panel A of Figure 6 and the first block of Table 9), or as an exercise in measuring the enormity of the breach of the feasibility constraint resulting from the P2010 pension system (see Panel B of Figure 6 and the second block of Table 9). The results of these two exercises are not surprising.

Financing pensions with consumption taxes. Financing pensions with a mix of payroll and consumption taxes in an economy inhabited by fully rational and fully forward-looking households is a bad idea. When households choose their life-time savings optimally, they almost always prefer the lower pensions and the lower consumption taxes. This is because in the low pension and low consumption tax regime they can use both the labor-leisure and the consumption-savings margins to optimize their life-time welfare. And in the high pension and high consumption tax regime they are more constrained. Another way to interpret this result is that the net life-time returns afforded

[29] The pension reforms that we study in this article change the pension rights of some workers. Therefore, a worker who is in state z in the benchmark model economy may find herself to be in state $\tilde{z}$ in a reformed model economy. In our welfare calculations we keep track of these changes.

Table 9: Compensating Variations between the Reforms and P2010 (%)

Cohort		Tax Financing (τ_c)			Debt Financing		
Birth	Age 65	R2013	RARF	RPRI	R2013	RARF	RPRI
1930	1995	4.4	0.2	0.0	0.1	–4.2	–4.6
1940	2005	7.6	1.5	1.5	0.3	–10.6	–10.8
1950	2015	6.1	0.5	0.7	–4.4	–22.1	–21.5
1960	2025	–2.5	–6.1	–6.0	–13.1	–35.6	–34.0
1970	2035	2.7	1.6	2.2	–8.7	–33.4	–31.1
1980	2045	3.5	5.0	5.4	–8.7	–33.4	–31.2
1990	2055	4.0	8.9	8.8	–8.6	–32.5	–30.3
2000	2065	5.1	13.7	13.1	–8.5	–32.5	–30.1
2010	2075	5.7	17.5	16.4	–8.2	–32.3	–29.9
2020	2085	6.0	19.7	18.2	–8.2	–32.3	–29.8
2030	2095	6.0	20.9	19.1	–8.2	–32.3	–29.8

by personal funded savings are better than those afforded by the Spanish pay-as-you go pension system.

In the R2013 pension reform every household except those born between 1954 and 1966 gains from the lower consumption taxes and lower pensions combinations afforded by that reform. The households born between 1942 and 1948, with welfare gains equivalent to an increase consumption between 8.2 and 9.5 percent, are the ones who profit the most from this reform.

The welfare gains in the R2013 pension reform are first increasing because most of the people born between 1930 and 1950 have already retired and the reform reduces their future tax liabilities but leaves their pensions unchanged. People born later benefit more because consumption taxes only start being lower than in the benchmark economy when the pension reserve fund runs out in 2018. In Model Economies RARF and RPRI the welfare gains are smaller because the sustainability factors reduce the value of pensions.

The cohorts born after 1950 are mostly workers when the reforms begin. The reforms delay the retirement ages and reduce the pension rights by increasing the number of years of contributions considered to compute the pensions. These losses are increasing because the reforms are phased in gradually. The cohorts born between 1955 and 1965 bear the largest losses because they bear the full force of these changes and they have little time to reoptimize. For households born after 1960 the welfare gains are increasing again because they start to benefit from the increasing reductions in the consumption tax rates.

In the RARF and RPRI reforms, the shapes of the welfare costs and gains are similar. Quantitatively, the households born between 1930 and 1974 are worse off with these two reforms than with the R2013 reform and the households born after 1974 are better off. Moreover the welfare

gains of the households born between 1930 and 1950 in the R2013 reform are small in the RARF and the RPRI reforms, and after that age the welfare gains increase rapidly until they reach 20.9 and 19.1 percent of consumption for the households born in 2030. This is more than three times the size of the welfare gains obtained by that cohort under the R2013 reform. These differences are the consequences of the different sizes of the trade-offs between lower consumption taxes and lower pensions.

Financing pensions with debt. The large sizes of the welfare losses brought about when we allow the pension deficits to be financed using debt help us understand the enormity of the free lunch implicit in the P2010 Spanish pension system. Under debt financing, the trade-off between lower consumption taxes and lower pensions disappears. The reforms delay the retirement ages and lower the pensions and they bring about almost no gains at all, other than those that may arise from the general equilibrium effects. The welfare losses are increasing with the age of birth until 1960 and they stabilize afterwards. They are highest in Model Economy RARF because in that model economy the pensions are the lowest. The small welfare gains brought about by the R2013 reform for the cohorts born before 1948 arise because the interest rates are higher than in the benchmark model economy, and because for those cohorts the pensions do not change.

7 Conclusion

We conclude that the 2011 Reform of the Spanish pension system, the 2013 delay in the first retirement age, the Intergenerational Equity Factor, and the Pension Revaluation Index, that effectively transforms the Spanish pay-as-you-go system into a defined-contribution system, solve the sustainability problems of Spanish pensions until 2037 but that they fail to do so after that date. They succeed until 2037 in the most perverse way: reducing the real value of the average pension, while leaving the many loopholes of the contributivity to the system unchanged. The reduction in pensions is progressive and we find that, by 2037, the value of the average pension will be approximately 20 percent smaller in real terms than what it would have been under the rules prevailing in 2010. These pension reforms fail after 2037 because, from that year onwards, approximately 50 percent of the Spanish retirees will be paid the minimum pension, which is exempt from the sustainability factors. We conjecture that further reforms await in the future of Spanish pensions.

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A Appendix 1: Population Dynamics

The initial distribution of households. Recall that $\mu_{j,h,e,a,b,p,t}$ denotes the measure of households of type (j, h, e, a, b, p) at period t and that, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript. To obtain μ_{2010}, we proceed as follows:

1. We take the measure $\mu_{j,2010}$ for all $j = \{20, 21, ..., 100\}$ directly from the latest demographic projection for the Spanish economy published by the *National Institute of Statistics* (INE). These demographic projections take into account the forecasts for the net migratory flows into Spain. However, to solve the households' decision problem we use the survival probabilities only.[30]

2. We obtain $\mu_{j,h,2010}$ directly from the *Encuesta de Población Activa*, which reports the educational distribution of the working age population for various age groups.

3. Next, we solve the decision problem of the model economy households. We obtain $\mu_{20,h,e,2010}$ from $\mu_{20,h,2010}$ and the invariant distribution of the stochastic component of the endowment of efficiency labor units process.[31]

   To compute $\mu_{j,h,e,2010}$ for $j = \{21, 22, ..., 100\}$, we use the conditional transition probability matrix of the stochastic component of the endowment of efficiency labor units process, the probability of becoming disabled, and the optimal decision to retire.

4. To obtain $\mu_{20,h,e,a,b,2010}$, we assume that new-entrants own zero assets and have zero pension claims. For $j = 21, 22, ..., 44$, we use the household's optimal saving decisions at age $j - 1$ and the pension system rules. From age $R_0 - N_b$ onwards, we average the labor income to determine the pension claims and the optimal labor supply decisions.

5. Finally, to obtain $\mu_{j,h,e,a,b,p,2010}$, we use the optimal retirement decisions and the pension system rules.

   Notice that steps 3, 4 and 5 must be computed simultaneously in the same loop.

Demographic Transition. We use the latest demographic projections of the *Instituto Nacional de Estadística* which correspond to 2012. The INE reports and projects the age distribution of Spanish residents from 2010 to 2052 for people aged from zero to 100 and more. Call those age cohorts N_{jt} and let $N_t = \sum_{20}^{100+} N_{jt}$. Then, the age distribution of the households in our model economy is $\mu_{jt} = N_{jt}/N_t$ for $j = 20, 21, ..., 99, 100+$ and for $t = 2010, 2011, ..., 2052$.[32]

[30] The survival probabilities can be found at $http : //www.ine.es/jaxi/menu.do?type = pcaxis\&path = \%2Ft20\%2Fp251\ \&file = inebase\&L = 0$.

[31] Note that we have assumed that there are no disabled households of age 20.

[32] This data can be found at $http : //www.ine.es/jaxi/menu.do?type = pcaxis\&path = \%2Ft20\%2Fp251\&file = inebase\ \&L = 0$.

Educational Transition. To update the distribution of education, we assume that from 2011 onwards, 8.65 percent of the 20 year-old entrants have not completed their secondary education, that 63.53 percent have completed their secondary education, and that 27.82 percent have completed college. This was the educational distribution of Spanish households born between 1980 and 1984, which was the most educated cohort in 2010.[33] We also assume that immigrants have the same educational distribution as the residents of the same age.

B Appendix 2: Calibration of the Model Economy Ratios

B.1 Calibration of the Macroeconomic Ratios

- The Spanish National Income and Product Data reported by the *Instituto Nacional de Estadística* (INE) fo 2010 are the following:

Table 10: Spanish GDP and its Components for 2010 at Current Market Prices

	Millon Euros	Shares of GDP (%)
Private Consumption	596,322	56.72
Public Consumption	221,715	21.08
Consumption of Non-Profits	10,589	1.00
Gross Capital Formation	244,987	23.30
Exports	283,936	27.00
Imports	306,207	29.12
Total (GDP)	1,051,342	100.00

- We adjust the amounts reported in Table 10 according to Cooley and Prescott (1995) and we obtain the following numbers:
  - Adjusted Private Consumption: Private Consumption – Private Consumption of Durables + Consumption of Non-Profits $= 596,322 - 54,127 + 10,589 = 552,784$ million euros.
  - Adjusted Public Consumption: Public Consumption = 221,715 million euros.
  - Adjusted Investment (Private and Public): Gross Capital Formation + Private Consumption of Durables $= 244,987 + 54,127 = 299,114$ million euros.
- The next adjustment is to allocate Net Exports to our measures of C, I, and G. To that purpose, we compute the shares of each of those three variables in the sum of the three and we allocate Net Exports according to those shares. The sum of the three variables is 1,073,613 million euros and the shares of C, I, and G are 51.49, 27.86, and 20.65 percent.

[33] This is the same approach used in Conde-Ruiz and González (2013).

- Net Exports are –22,271 million euros. When we allocate them to C, I, and G we obtain the final adjusted values for C, I, and G which are 541,317, 217,116, and 292,909. Naturally, this new adjusted values now add to Total GDP but the adjusted shares remain unchanged and they are 51.49, 27.86, y 20.65 percent of GDP.

- Next we redefine the model economy's output and consumption from factor cost to market prices as follows: $Y^* = Y + T_c$, where Y^* is the model economy's output at market prices and T_c is the consumption tax collections, and $C^* = C + T_c$, where C^* is the model economy's consumption at market prices.

- Finally we use $C^*/Y^* = 51.49$ and $G/Y^* = 20.65$ as targets.

B.2 Calibration of the Government Policy Ratios

Table 11: Spanish Public Sector Expenditures and Revenues in 2010*

Expenditures	Millions of euros	Percentage of GDP	Revenues	Millions of euros	Percentage of GDP
Consumption	221,715	21.08	Sales and gross receipts taxes[a]	94,234	8.96
Investment	40,091	3.81	Payroll taxes[b]	106,599	10.13
Pensions[c]	109,000	10.36	Individual income taxes	77,542	7.37
Interest payments	20,120	1.91	Corporate profit taxes	19,425	1.84
Other	88,719	8.44	Other revenues	83,626	9,96
			Deficit	98,218	9.33
Total	479,645	45.62	Total	479,645	45.62

Source: Spanish National Institute of Statistics, Spanish Social Security, and Eurostat.
*Shares of nominal GDP at market prices.
[a]It includes the tax collections from the Value Added Tax and other taxes on products.
[b]Total revenues from the Spanish Social Security.
[c]Total expenditure from the Spanish Social Security.

- In Table 11 we report the 2010 revenue and expenditure items of the consolidated Spanish public sector. Notice that the GDP share of Government consumption differs from the one that we have computed in Section A3.1 because here we use its unadjusted value.

- If we ignore the public pension system, the government budget in the model economy in 2010 is

$$G_{2010} + Z_{2010} = T_{c,2010} + T_{k,2010} + T_{y,2010} + E_{2010} \tag{45}$$

- Unitentional bequests, E_{2010}, are exogenous.
- We target the output shares of $T_{c,2010}, T_{k,2010}$,and $T_{y,2010}$ so that they replicate the GDP shares of Sales and Gross Receipt Taxes, Corportate Profit Taxes, and Individual Income taxes.
- We have already targeted the output ratio of government consumption and we have already accounted for government investment.
- We define the output share of transfers other than pensions, Z_{2010}, residually to satisfy the budget.
- We report the model economy government budget items in Table 12 below.

Table 12: Model Economy Public Sector Expenditures and Revenues in 2010 (%Y^*Shares)

Expenditures		Revenues	
Consumption and Investment (G)	20.65	Consumption taxes (T_c)	8.96
Pensions (P)	10.35	Payroll taxes (T_s)	10.12
Other Transfers (Z)	0.54	Household income taxes (T_y)	7.37
		Capital Income Taxes (T_k)	1.84
		Unitentional Bequests (E)	3.25
Total	31.54	Total	31.54

C Appendix 3: Computation

To solve our model economy, we must choose the values of 50 parameters. As we have already mentioned, we the obtain the values of 31 of these parameters directly because they are functions of single targets. Another 4 parameters normalization conditions and 1 is obtained residually from the government budget constraint. This gives us a total of 36 parameters and leaves us with 14 to be determined. To do so, we solve a system of 14 non-linear equations.

The 14 parameters determined by this system are the following:

- Preferences: β and γ.
- Technology: δ.
- Stochastic process for labor productivity: $s(2), s(3), s_{11}, s_{12}, s_{21}, s_{22}, s_{32}$, and s_{33}.
- Pension system: ϕ and a_{14}.

- Fiscal policy: a_{10}.

To solve this system of equations we use a standard non-linear equation solver. Specifically, we use a modification of Powell's hybrid method, implemented in subroutine DNSQ from the SLATEC package.

The DNSQ routine works as follows

1. Choose the weights that define the loss function that has to be minimized
2. Choose a vector of initial values for the 14 unknown parameters
3. Solve the model economy
4. Update the vector of parameters
5. Iterate until no further improvemets of the loss function can be found.

To solve the model economy, we proceed as follows:

1. We guess values for the interest rate, r, and for the effective labor input, N. Then, using the optimality conditions from the firm's maximization problem and the production function, we obtain the implied values for productive capital, K, output, Y, and the wage rate, w.
2. The value of output determines the values of the fiscal policy ratios, the values of the maximum and minimum pensions, the pension grid, These variables, the tax rates already determined uniquely by single targets, and the remaining 3 government variables which are unknowns determine the government policy.
3. Given the factor prices, the government policy, the age-dependent probabilities of surviving, and the initial values of the parameters that describe preferences and the stochastic process for labor productivity, we solve the household's decision problem backwards and obtain household's optimal decisions.
4. We aggregate these optimal decisions and obtain the implied values for the government revenue items (tax collections and accidental bequests), pension payments, and the new values for K, N, r, w and Y.
5. Finally, we update N and r, and we iterate until convergence.

Once that the model economy is solved, DNSQ compares the relevant statistics of the model economy with the corresponding targets, and changes the initial values of the parameters to reduce the values of the loss function. This procedure continues until DNSQ cannot find further

improvements of the loss function. At this point, the iteration stops and we have found a solution for the values of the 14 unknown parameters. Since the solutions to these very non-linear systems of equations are not guaranteed to exist and, when they do exist, they are not guaranteed to be unique, we try many different initial values for the 14 parameters and vectors of wights and we stop when we are convinced that we have found the best possible parameterization.

The system of equations is the following

$$
\begin{aligned}
0 &= 300 * ((C + T_c)/Y^* - 0.515) \\
0 &= 300 * (K/Y^* - 3.28) \\
0 &= 500 * (h - 0.375) \\
0 &= 500 * (P/Y^* - 0.103) \\
0 &= 30 * (T_s/Y^* - 0.101) \\
0 &= 30 * (T_y/Y^* - 0.0735) \\
0 &= 700 * (GY - 0.42) \\
0 &= 800 * (GE - 0.49) \\
0 &= 500 * (GW - 0.57) \\
0 &= 200 * (1QE - 0.053) \\
0 &= 100 * (5QY - 0.471) \\
0 &= 200 * (5QE - 0.443) \\
0 &= (2QW - 0.066) \\
0 &= (4QW - 0.206)
\end{aligned}
$$

and in Table 13 we report the initial values, the final values, and the weights that we have used to solve it and the errors that we have obtained.

Table 13: Initial Values, Final Values, Weights, and Errors.

Parameter	Initial Value	Final Value	Statistic	Weight (%)	Target	Result	Error (%)
δ	0.0653	0.0724	$(C+T_c)/Y^*$ (%)	300	0.515	0.514	-0.19
β	1.0459	1.0460	K/Y^*	300	3.28	3.28	0.00
γ	0.2900	0.2979	h (%)	500	37.50	37.62	0.32
ϕ	0.6308	0.8279	P/Y^* (%)	500	10.30	10.23	-0.68
a_{10}	0.2088	0.0672	T_s/Y^* (%)	30	10.13	10.10	-0.28
a_{14}	0.2373	0.2385	T_y/Y^* (%)	30	7.35	7.72	5.03
$s(2)$	2.4135	2.0856	GY	700	0.42	0.44	4.76
$s(3)$	5.6303	11.2892	GE	800	0.49	0.48	-2.04
s_{11}	0.9993	0.9417	GW	500	0.57	0.57	0.00
s_{12}	0.0006	0.0582	1QE (%)	200	5.30	5.20	-1.88
s_{21}	0.0007	0.0319	5QE (%)	200	44.30	55.70	25.73
s_{22}	0.9992	0.9680	5QY (%)	100	47.10	52.80	-12.1
s_{31}	0.0001	0.0000	2QW (%)	1	6.60	6.60	0.00
s_{32}	0.0007	0.0002	4QW (%)	1	20.60	20.50	-0.48