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Sharing the costs of cleaning a river: the Upstream Responsibility rule

Jorge Alcalde-Unzuy, María Gómez-Rúa, and Elena Molis
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Jorge Alcalde-Unzu†, María Gómez-Rúa‡, and Elena Molis§

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Abstract

The cleaning up of waste present in transboundary rivers, which requires the cooperation of different authorities, is a problematic issue, especially when responsibility for the discharge of the waste is not well-defined. Following Ni and Wang [12] we assume that a river is a segment divided into several regions from upstream to downstream. We show that when the transfer rate of the waste is unknown, the clean-up cost vector provides useful information for estimating some limits in regard to the responsibility of each region. We propose a cost allocation rule, the *Upstream Responsibility rule*, which takes into account these limits in distributing costs "fairly" and we provide an axiomatic characterization of this rule via certain properties based on basic ideas concerning the responsibility of regions.

JEL classification: C71; D61

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†Public University of Navarre, Department of Economics. Campus Arrosadia, 31006 Pamplona, Spain. E-mail: jorge.alcalde@unavarra.es.

‡University of Vigo, Department of Statistics and Operations Research. Campus Lagoas-Marcosende, 36310 Vigo. Spain. E-mail: mariarua@uvigo.es.

§University of Granada, Department of Economics. Campus la Cartuja, 18071 Granada, Spain. E-mail: emolis@ugr.es.

1 Introduction

Motivation The presence of waste in river channels is a common environmental problem. This is a major problem faced by authorities since waste can pollute water, which can be harmful for people, plants and animals, causing serious diseases and affecting ecosystems. In fact, the OECD has argued that it is necessary to assess the efficiency and effectiveness of water pollution abatement measures in the context of river basin management (OECD [13]). Around the world, about 200 rivers (see Ambec and Sprumont [2] and Barret [3]) flow across national borders, and a much greater number across borders between regions or municipalities. Cleaning transboundary rivers requires cooperation on the part of the different authorities involved and coordination of efforts if it is to be effective. However, the distribution of the costs of cleaning such rivers among the different regions may be a problematic issue, particularly when the extent to which each region is responsible for the waste discharged is not well-defined.

As far as we know, the first paper to analyze the problem of sharing the costs of cleaning a river among different regions from a theoretical point of view is that of Ni and Wang [12]. They model a river as a segment which is divided into subsegments from upstream to downstream such that each region is located in one of them. They assume that there is a central agency that determines the cost of cleaning each of these segments and they axiomatically propose two rules for allocating the total cleaning costs among all regions along the river. The first rule, called *Local Responsibility Sharing*, establishes that the cost of cleaning a segment of the river should be assigned to the region located in that segment. The second rule, called *Upstream Equal Sharing*, states that the cost of cleaning a segment must be distributed equally among the region in that segment and all the regions situated upstream from it.[1] We show that neither of these rules allocates the costs in a way that reflects the responsibility of each region in producing the waste present in river channels. The first does not take into consideration that the water of a river flows from one segment to another, taking part of the waste with it. The second implicitly assumes that the region in a segment and all the regions situated upstream from this have the same degree of responsibility for the waste present in the segment in question. However, this

[1] These rules are based on the theories of Absolute Territorial Sovereignty and Unlimited Territorial Integrity, respectively (see Godana [8] and Kilgour and Dinar [11]).

would only be "fair" if all regions have discharged exactly the same quantity of waste of the one present in that segment, which is not necessarily the case.

Overview of results In this paper, we seek to develop an alternative rule to those proposed by Ni and Wang [12] which takes into account the responsibility of the regions for the presence of the waste. We explicitly introduce into our model the fact that the waste is transferred, with the water, from upstream to downstream at a particular rate, an idea that is implicitly assumed in Ni and Wang [12]. If the social planner knew this rate, she could use the cost vector to accurately calculate the amount of waste discharged by each region into the river, and the costs could thus be distributed according to their actual responsibilities. However, in practice, the transfer rate may be unknown. In that case, we show that the social planner could estimate certain limits of that rate from the cost vector. Those limits provide useful information for distributing the costs fairly, since they enable certain limits of responsibility to be inferred for each region. The rules proposed by Ni and Wang [12] do not always assign costs in the intervals constructed with these limits, thus violating this basic principle of fairness.
We introduce a set of desirable properties taking into account this information concerning the responsibility of each region in discharging the waste. Those properties are: (i) *Limits of Responsibility*, which requires the cost paid by each region for cleaning its own segment always to be within its limits of responsibility; (ii) *No Downstream Responsibility*, which states that a region j situated downstream from another region i has no responsibility for the waste present in i and therefore does not have to pay anything towards the cost of cleaning it up; (iii) *Consistent Responsibility*, which ensures that the part of the cost of cleaning a segment paid by one region relative to the part paid by another region is consistent throughout all the segments situated downstream from both regions; and (iv) *Monotonicity with respect to Information on the Transfer Rate*, which states that when information on the transfer rate improves in such a way that it becomes natural to induce a higher (lower) estimated value for the real transfer rate, the amount of waste in any segment for which all its upstream regions are responsible must not be lower (higher) than before.

That set of properties characterize a new cost allocation rule, the *Upstream Responsibility rule*, which works as follows: first, it assigns to the region situated in a given segment the mid-point in the interval between its lower and its higher limits of responsibility. The remaining cost of cleaning the segment in question is divided among the upstream regions, maintaining the proportions of the allocation of the cost of cleaning the previous segment.

Related literature The study of allocation problems using game theoretical and/or axiomatic models to solve issues related to transboundary rivers has developed in two directions. On the one hand (the *harmful side*) some authors have developed models for studying how to share the costs of cleaning a river among the regions located along it. On the other hand (the *beneficial side*) some papers have analyzed models for determining how to share water resources among the different regions along a river.

Among the papers dealing with the harmful side, which is the body of literature into which our paper fits, there are two main approaches. Several papers, starting with Ni and Wang [12] and including ours, consider a river as a segment divided into different regions and assume that the cost of cleaning each region is exogenously given. Along these lines, Ni and Wang [12] propose and characterize the two rules - Local Responsibility Sharing and Upstream Equal Sharing - described above. They also defend these rules as the Shapley values of two appropriately defined TU games and as solutions belonging to the core of this problem. Van den Brink and van der Laan [17] show that these additional results are particularizations of certain well-known results of cooperative game theory and they provide an alternative axiomatic characterization of these rules. This model is extended by Dong et al. [6] by considering a river as a network. Based on a different principle (the "polluter-pays" principle), Gómez-Rúa [9] defines water taxes according to regions' responsibilities for pollution and characterizes several cost allocation rules based on properties of those taxes. Other papers such as Gengenbach et al. [7] and van der Laan and Moes [16] take a substantially different approach by assuming that the cost allocation rule adopted may affect the decision of each region about how much waste to discharge.

On the beneficial side, papers generally analyze water allocation problems and the fair distribution of the welfare resulting from distributing the water of a river among different regions. Based on cooperative game theory, Ambec and Sprumont [2] model this situation by defining a coalitional form game. They analyze how water should be allocated across the agents and propose what monetary transfers should be made. Along these lines, Ambec and Ehlers [1] generalize the aforesaid model by allowing for satiable agents. Wang [19], using a similar model but with a market-based approach, analyzes efficient allocations when trade is restricted to neighboring agents along the river. Khmelnitskaya [10], and van der Brink et al. [18] extend the previous models by considering rivers with multiple springs.[2] Rebille and Richefort [15] analyze the problem of water allocation from a non-cooperative point of view.

Remainder The rest of this paper is organized as follows. Section 2 describes the model and introduces a result that shows that the cost vector can provide useful information worth considering when constructing a cost allocation rule based on responsibility. Section 3 discusses some axioms for cost allocation rules reflecting basic ideas of responsibility. Section 4 defines the Upstream Responsibility rule and provides a characterization of it based on the axioms defined previously. Section 5 concludes, with a discussion of how the basic model presented in the previous sections can be extended to cover more complex situations. The Appendix contains the proofs of the characterization result and the independence of the axioms.

2 The model

Consider a river which is divided into n segments of the same size from upstream to downstream. There is a set of regions, each of which is located in one of the segments, which have discharged waste into the river. This river has a transfer rate t that measures the proportion of waste that is transferred from one segment of the river to the next. This transfer rate may not be exactly known. Consider a general case in which the social planner knows that t is situated within an interval $[\underline{t}, \bar{t}] \subseteq [0, 1]$.[3]

There is a central agency that determines the cost of cleaning the river in each

[2] For more details on the use of cooperative game theory to model water allocation problems, readers are referred to any of the numerous surveys on the matter. See for instance, Béal et al. [5], Beard [4] and Parrachino et al. [14]

[3] The case in which $\underline{t} = \bar{t}$ is the situation in which the social planner knows the actual transfer rate, while the case in which $\underline{t} = 0$ and $\bar{t} = 1$ is the case in which there is no information about t.

segment. We assume that this cost is exactly the amount of waste present in the segment in question.[4] The agency has to allocate the costs of the cleaning process to the different regions in a fair way. Our main objective is to find rules for allocating those costs in a way that reflects the responsibility of each region in the discharging of the waste.

Formally, let $N = \{1, ..., n\} \subset \mathbb{N}$ be a finite set of regions such that i is situated upstream of $i+1$ for all $i \in \{1, \ldots, n-1\}$. Let $C = (c_1, ..., c_n) \in \mathbb{R}^n_+$ be the cleaning cost vector, where c_i represents the cost incurred to clean the river in region i. Then, a *cost allocation problem* is a tuple $(N, C, \underline{t}, \overline{t})$.[5]

A *cost allocation rule* is a mapping x that assigns to each problem $(N, C, \underline{t}, \overline{t})$ a vector $x(N, C, \underline{t}, \overline{t}) = (x_1(N, C, \underline{t}, \overline{t}), \ldots, x_n(N, C, \underline{t}, \overline{t})) \in \mathbb{R}^n_+$ such that $\sum\limits_{i \in N} x_i(N, C, \underline{t}, \overline{t}) = \sum\limits_{i \in N} c_i$, where $x_i(N, C, \underline{t}, \overline{t})$ represents the cost allocated to region i by the rule in this problem.[6]

The solution that the rule x assigns to a problem $(N, C, \underline{t}, \overline{t})$ can be defined also as a matrix of size $n \times n$, $(x_i^j)_{i,j \in \{1,\ldots,n\}}(N, C, \underline{t}, \overline{t})$ such that $\sum\limits_{i \in N} x_i^j(N, C, \underline{t}, \overline{t}) = c_j$ and $\sum\limits_{j \in N} x_i^j(N, C, \underline{t}, \overline{t}) = x_i(N, C, \underline{t}, \overline{t})$. With this interpretation, $x_i^j(N, C, \underline{t}, \overline{t})$ represents the part of the cost of cleaning the river in segment j that region i pays. When there is no risk of confusion about the description of the problem, we will only write $x_i^j(\cdot)$.

Although the transfer rate t cannot be totally known by the social planner, there is some information that can be deduced from the cleaning cost vector because some values of t may be incompatible with C.

[4] This assumption is made for the sake of fluency. We could have assumed instead that the cost of cleaning each segment is an increasing linear function of the amount of waste present in it, without essentially altering the results.

[5] The problem can be defined directly by a triple $(C, \underline{t}, \overline{t})$ given that the information on N is included in C. However, we prefer to maintain both to be consistent with the notation used by Ni and Wang [12].

[6] The condition $\sum\limits_{i \in N} x_i(N, C, \underline{t}, \overline{t}) = \sum\limits_{i \in N} c_i$ is imposed in the studies of Ni and Wang [12] and van der Brink and van der Laan [17] as an axiom called Efficiency. We consider that this property should be included in the definition of a rule.

Proposition 1 *Let $(N, C, \underline{t}, \bar{t})$ be a problem. Then, the transfer rate t satisfies that $t \leq \bar{t}^*(\bar{t}, C)$, where*

$$\bar{t}^*(\bar{t}, C) = \min\left\{\min_{i\in\{2,\ldots n-1\}}\left\{\frac{c_i}{c_{i-1}}\right\}, \frac{c_n}{c_{n-1}+c_n}, \bar{t}\right\}.$$

Additionally, any value of t between $\underline{t}$ and $\bar{t}^*(\bar{t}, C)$ is compatible with C.

Proof. Let $(N, C, \underline{t}, \bar{t})$ be a problem. For any segment $i \in N \setminus \{n\}$, the cost that we observe, c_i, is the difference between all the waste entering the segment, denoted as V_i^*, and the amount transferred to the next segments, given by tV_i^*. Then, $c_i = V_i^* - tV_i^*$ for all $i \in \{1, \ldots, n-1\}$.

Given that the waste cannot be transferred far from the most downstream region[7], we have that $c_n = V_n^*$. Then,

$$V_i^* = \begin{cases} \frac{c_i}{1-t} & \text{if } i \in \{1, \ldots, n-1\} \\ c_i & \text{if } i = n. \end{cases} \tag{1}$$

Let V_i be the amount of waste thrown into the water by region i. It is immediate that $V_i \leq V_i^*$ given that upstream regions may transfer waste to region i. In particular, the amount thrown into the water by region i is the difference between the total amount entered segment i and the amount transferred from its immediate upstream segment. Then, for all $i \in \{2, \ldots, n\}$, $V_i = V_i^* - tV_{i-1}^*$.

However, for $i = 1$, since there is no upstream region, $V_1 = V_1^*$. Then,

$$V_i = \begin{cases} V_i^* & \text{if } i = 1 \\ V_i^* - tV_{i-1}^* & \text{if } i \in \{2, \ldots, n\}. \end{cases} \tag{2}$$

Using expressions (1) and (2), we can obtain an expression of V_i in terms of C and t:

[7]Note that the fact that the region furthest downstream accumulates all the waste that enters it, contrary to what occurs in the other regions, where part of the waste flows on to the next region downstream, introduces a particularity into the treatment of this region. This is compatible with the concept of the river ending in a lake which belongs to a single country. If, however, the river ends in the sea, the model can be easily adapted by dropping this differentiation between regions.

$$V_i = \begin{cases} \frac{c_i}{1-t} & \text{if } i = 1 \\ \frac{c_i}{1-t} - \frac{c_{i-1}}{1-t}t & \text{if } i \in \{2, \ldots, n-1\} \\ c_i - \frac{c_{i-1}}{1-t}t & \text{if } i = n. \end{cases} \tag{3}$$

Given that V_i is, by definition, non-negative and taking into account expression (3), we have that: (i) $\frac{c_i}{1-t} - \frac{c_{i-1}}{1-t}t \geq 0$, that is, $t \leq \frac{c_i}{c_{i-1}}$ for all $i \in \{2, \ldots, n-1\}$, and (ii) $c_n - \frac{c_{n-1}}{1-t}t \geq 0$, so that, $t \leq \frac{c_n}{c_n + c_{n-1}}$. Additionally, it is easy to see from the previous reasoning that any value for t between $\underline{t}$ and $\overline{t}^*(\overline{t}, C)$ is compatible with C. Then we have arrived at the desired result. ■

Then, Proposition 1 allows us to reduce the uncertainty over the transfer rate. In particular, the cost vector C provides, jointly with $\overline{t}$, a maximum limit for this rate that we denote $\overline{t}^*(\overline{t}, C)$. To see the capacity of this result, consider the following example.

Example 1 *Suppose a problem in which $N = \{1, 2, 3, 4\}$, the cost vector is $C = \{10, 16, 8, 24\}$ and the social planner has no information* a priori *about the transfer rate, i.e., $\underline{t} = 0$ and $\overline{t} = 1$. Then, focusing on the costs of cleaning the segments, the information of the transfer of the waste can be improved using the result of Proposition 1. In this case, we obtain that $t^*(\overline{t}, C) = \min\{\frac{8}{5}, \frac{1}{2}, \frac{3}{4}, 1\}$. Therefore, Proposition 1 indicates that the transfer rate is* at most *one half and, then, the information of the transfer rate after observing the cost vector can be adapted.*

Given a problem $(N, C, \underline{t}, \overline{t})$, we will denote by $l_i^j(N, C, \underline{t}, \overline{t})$ the amount of waste present in segment j that has been discharged by region i. When there is no risk of confusion about the description of the problem, we simply write $l_i^j(\cdot)$. The real transfer rate t may be unknown, in which case $l_i^j(\cdot)$ cannot be precisely calculated. However, some limits of this value can be deduced from the information about the transfer rate held by the social planner and from what the planner can infer from the cost vector via Proposition 1. We will denote the lower and higher limits of $l_i^j(\cdot)$ by $\underline{l}_i^j(\cdot)$ and $\overline{l}_i^j(\cdot)$, respectively. The following proposition will provide formulas for $\underline{l}_i^i(\cdot)$ and $\overline{l}_i^i(\cdot)$ for all $i \in N$.[8]

[8]It is also possible, but extremely tedious, to construct formulas for the limits of any $l_i^j(\cdot)$ in a similar way, but these ones are sufficient for our purposes.

Proposition 2 *Let $(N, C, \underline{t}, \overline{t})$ be a problem. Then,*

$$\underline{l}_i^i(\cdot) = \begin{cases} c_i & \text{if } i = 1 \\ c_i - c_{i-1} \cdot \overline{t}^*(\overline{t}, C) & \text{if } i \in \{2, \ldots, n-1\} \\ c_i - \frac{c_{i-1} \cdot \overline{t}^*(\overline{t}, C)}{1 - \overline{t}^*(\overline{t}, C)} & \text{if } i = n. \end{cases}$$

$$\overline{l}_i^i(\cdot) = \begin{cases} c_i & \text{if } i = 1 \\ c_i - c_{i-1} \cdot \underline{t} & \text{if } i \in \{2, \ldots, n-1\} \\ c_i - \frac{c_{i-1} \cdot \underline{t}}{1 - \underline{t}} & \text{if } i = n. \end{cases}$$

Proof. Let $(N, C, \underline{t}, \overline{t})$ be a problem. First, take $i = 1$. Given that this region is the most upstream region in the river, it is straightforward that all the waste in this segment is of its own responsibility. Then, $\underline{l}_1^1(\cdot) = \overline{l}_1^1(\cdot) = c_1$. Take now any $i \in \{2, \ldots, n-1\}$. In this case, we have that $\frac{c_{i-1}}{1-t}$ units of waste entered in region $i-1$. Then, $\frac{c_{i-1} \cdot t}{1-t}$ units of waste entered in region i from the immediate upstream region, $i-1$, and $\frac{c_{i-1} \cdot t^2}{1-t}$ of these units left region i to the immediate downstream region $i+1$. Therefore, $c_{i-1} \cdot t$ units of the waste present in region i are responsibility of the regions situated upstream from i. Then, given that $t \in [\underline{t}, \overline{t}^*(\overline{t}, C)]$, we have that $\underline{l}_i^i(\cdot) = c_i - c_{i-1} \cdot \overline{t}^*(\overline{t}, C)$ and $\overline{l}_i^i(\cdot) = c_i - c_{i-1} \cdot \underline{t}$. Similarly, if $i = n$, we have that $\frac{c_{n-1}}{1-t}$ units of waste entered in region $n-1$. Then, $\frac{c_{n-1} \cdot t}{1-t}$ units of waste entered and remain in region n from its upstream territories and, then, $\underline{l}_n^n(\cdot) = c_n - \frac{c_{n-1} \cdot \overline{t}^*(\overline{t}, C)}{1 - \overline{t}^*(\overline{t}, C)}$ and $\overline{l}_n^n(\cdot) = c_n - \frac{c_{n-1} \cdot \underline{t}}{1 - \underline{t}}$, given that n is the most downstream region. Then, the result is proved. ■

It is natural to require that any rule that seeks to allocate costs in terms of each region's responsibility for producing the waste present in each segment should always respect the limits calculated in Proposition 2 when the costs are allocated. In the rest of this section, we discuss the two rules proposed by Ni and Wang [12] in relation to the fulfilment of this requirement. First, we introduce the formal definitions of those rules.[9]

[9] Although their formal definitions in Ni and Wang [12] only describe the total cost that each region pays for cleaning the whole river, we prefer to formulate them in terms of the exact distributions across the different segments.

Definition 1 *The* Local Responsibility Sharing *rule, α, is given by*

$$\alpha_i^j(\cdot) = \begin{cases} 0 & \text{if } i \neq j \\ c_i & \text{if } i = j. \end{cases}$$

Definition 2 *The* Upstream Equal Sharing *rule, β, is given by*

$$\beta_i^j(\cdot) = \begin{cases} 0 & \text{if } i > j \\ \frac{c_j}{j} & \text{if } i \leq j. \end{cases}$$

On the one hand, the Local Responsibility Sharing rule meets the aforesaid requirement of responsibility only when $\underline{t} = 0$. However, it can only be accepted as a rule that allocates costs taking responsibilities into account if the real transfer rate, t, is 0 in all rivers. Nevertheless, this literature only makes sense when waste is transferred from one region to another, an idea that is totally realistic. On the other hand, the following example shows that, independently of the information about t ($\underline{t}$ and $\overline{t}$), the Upstream Equal Sharing rule does not satisfy the requirement of allocating costs within the intervals of responsibility defined in Proposition 2.

Example 2 *Consider the family of cost allocation problems $(N, C, \underline{t}, \overline{t})$ such that $N = \{1, 2, 3, 4\}$ and $C = \{10, 16, 8, 24\}$. In all these problems, the Upstream Equal Sharing rule assigns to region 2 only half of the cost of cleaning its own segment; i.e., $\beta_2^2(\cdot) = 8$. However, given that $\overline{t}^*(\overline{t}, C) \leq \frac{1}{2}$, it is easy to calculate from Proposition 2 that $\underline{l}_2^2(\cdot) \geq 11$ for all possible values of $\underline{t}$ and $\overline{t}$. Hence, region 2 should pay at least 11 if responsibilities are considered.*

3 Axioms

The axioms that we present for a rule are based on basic ideas about responsibility for the waste present in the river channel. The first axiom, Limits of Responsibility, seeks to avoid the problem found in the rules of Ni and Wang [12] studied in the previous section. To that end, the property requires that the cost paid by each region for cleaning its own segment should always be within the limits calculated in Proposition 2.

Limits of Responsibility (LR): For all problems $(N, C, \underline{t}, \overline{t})$, and for all $i \in N$, $x_i^i(\cdot) \in [\underline{l}_i^i(\cdot), \overline{l}_i^i(\cdot)]$.

The second axiom, No Downstream Responsibility, states that a region j located downstream from another region i has no responsibility for the waste present in i, and should therefore not pay any part of the cost of cleaning it up.

No Downstream Responsibility (NDR): For all problems $(N, C, \underline{t}, \overline{t})$ and all $i, j \in N$ such that $i < j$, $x_j^i(\cdot) = 0$.

To introduce the next property, Consistent Responsibility, assume three regions i, j and k such that i is located upstream from j and j upstream from k. A rule decides how the cost of cleaning the river in region j should be divided among all the regions depending on their responsibility for the waste present in this region. In particular, it establishes the responsibility of region i relative to the responsibility of region j for producing that waste $\left(\frac{x_i^j(N,C,\underline{t},\overline{t})}{x_j^j(N,C,\underline{t},\overline{t})}\right)$. Thus, given that regions i and j do not produce any waste other than that which arrives at some time at j, the axiom states that the rule must establish the same degree of responsibility of region i relative to the responsibility of region j for the waste present in region k $\left(\frac{x_i^k(N,C,\underline{t},\overline{t})}{x_j^k(N,C,\underline{t},\overline{t})}\right)$.[10]

Consistent Responsibility (CR): For all problems $(N, C, \underline{t}, \overline{t})$ and all $i, j, k \in N$ such that $i < j < k$,

$$x_j^j(\cdot) \cdot x_i^k(\cdot) = x_j^k(\cdot) \cdot x_i^j(\cdot).$$

The last property, Monotonicity with respect to Information on the Transfer Rate, refers to situations in which, *ceteris paribus*, the information on the transfer rate improves. Given a problem $(N, C, \underline{t}, \overline{t})$, it is known from Proposition 1 that the transfer rate t is within the interval $[\underline{t}, \overline{t}^*(\overline{t}, C)]$. Assume that information on the transfer rate becomes more precise in such a way that some previous possible values of t can now be ruled out; that is, consider a new problem $(N, C, \underline{u}, \overline{u})$ such that $[\underline{u}, \overline{u}^*(\overline{u}, C)] \subset [\underline{t}, \overline{t}^*(\overline{t}, C)]$. If this informational improvement is such that the values discarded are mainly from the lower (higher) part of

[10]The axiom is not expressed in terms of these quotients but in terms of products so as to avoid indeterminate forms.

the interval $[\underline{t}, \overline{t}^*(\bar{t}, C)]$, it would be natural to induce a not lower (not higher) *estimated* value for the real transfer rate. Given that the cost vector is the same, the quantity of waste in any segment for which responsibility lies with all the upstream regions must be no lower (no higher) under the new estimation. Therefore, the axiom requires that for any segment the total amount paid by all its upstream regions for cleaning the segment in question should now be no lower (no higher).

Monotonicity with respect to Information on the transfer rate (MIT): For all problems $(N, C, \underline{t}, \bar{t})$ and $(N, C, \underline{u}, \overline{u})$ such that $[\underline{u}, \overline{u}^*(\overline{u}, C)] \subset [\underline{t}, \overline{t}^*(\bar{t}, C)]$ and for all $i, j \in N$ such that $i < j$,

$$\underline{u} - \underline{t} > \overline{t}^*(\bar{t}, C) - \overline{u}^*(\overline{u}, C) \Rightarrow \sum_{i<j} x_i^j(N, C, \underline{u}, \overline{u}) \geq \sum_{i<j} x_i^j(N, C, \underline{t}, \bar{t})$$

$$\underline{u} - \underline{t} < \overline{t}^*(\bar{t}, C) - \overline{u}^*(\overline{u}, C) \Rightarrow \sum_{i<j} x_i^j(N, C, \underline{u}, \overline{u}) \leq \sum_{i<j} x_i^j(N, C, \underline{t}, \bar{t}).$$

Observe that the above list of axioms includes, on the one hand, a basic principle of fairness in this context (LR) and, on the other hand, a set of three very weak properties (NDR, CR and MIT) that are satisfied by many possible rules which are very different one from another (for example, both α and β satisfy them). However, as shown below, the addition of LR to these three axioms isolates one particular rule.

4 The Upstream Responsibility rule and a characterization

This section presents a new cost allocation rule based on the responsibility of the agents involved in discharging waste into a river. We then characterize it using the four axioms from Section 3.

We begin by presenting our new rule, the Upstream Responsibility rule, in an intuitive way. To assign the total cost of cleaning each segment i, this rule first imputes to the region situated in that segment the mid-point in the interval between the lower and upper limits of its responsibility, obtained from Proposition 2. The remaining cost, if any, is allocated to the upstream regions in line with the proportions applied in the allocation of the cost of the previous segment. The formal definition of the rule is a little more complex.

Definition 3 *The* Upstream Responsibility *rule, γ, is given by:*

$$\gamma_i^j(N, C, \underline{t}, \bar{t}) = \begin{cases} 0 & \text{if } i > j, \\ c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i} & \text{if } i \leq j < n, \\ c_i - \frac{c_{i-1} \cdot s}{1-s} & \text{if } i = j = n, \\ \frac{c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i}}{1-s} & \text{if } i < j = n, \end{cases}$$

where $s = \frac{\underline{t} + \bar{t}^*(\bar{t}, C)}{2}$ and c_0 is set to 0.

In order to illustrate the solution proposed by our rule in comparison with those proposed in Ni and Wang [12], it is applied below to a particular cost allocation problem.

Example 3 *Consider again the cost allocation problem defined in Example 1. The solutions proposed to this problem by each of the rules in Ni and Wang [12] are given by the following matrices:*

$$\alpha(\cdot) = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 24 \end{pmatrix}$$

$$\beta(\cdot) = \begin{pmatrix} 10 & 8 & \frac{8}{3} & 6 \\ 0 & 8 & \frac{8}{3} & 6 \\ 0 & 0 & \frac{8}{3} & 6 \\ 0 & 0 & 0 & 6 \end{pmatrix}.$$

In contrast, the Upstream Responsibility rule assigns to this problem the following solution:

$$\gamma(\cdot) = \begin{pmatrix} 10 & \frac{5}{2} & \frac{5}{8} & \frac{5}{24} \\ 0 & \frac{27}{2} & \frac{27}{8} & \frac{9}{8} \\ 0 & 0 & 4 & \frac{4}{3} \\ 0 & 0 & 0 & \frac{64}{3} \end{pmatrix}.$$

The following result states that the Upstream Responsibility rule is characterized by the combination of the four axioms introduced above.

Theorem 1 *A rule satisfies LR, NDR, CR and MIT if and only if it is the Upstream Responsibility rule* γ.

We also show that this characterization is tight.

Proposition 3 *Axioms LR, NDR, CR and MIT are independent.*

5 Concluding remarks

In this paper, we have shown that given a transboundary river with waste transfer, the costs of cleaning the various regions provides information about the responsibility of each of them in producing the waste. This information can be useful for a social planner who has to allocate those costs to the different regions fairly. According to the information on responsibilities inferred from the cost vector, we have proposed a new property, LR, which can be combined with three more basic properties (NDR, CR and MIT) to characterize a unique cost allocation rule: the Upstream Responsibility rule.

In order to infer this information about the responsibility of the various regions, we have followed a simplified model that enables us to obtain the results in a simpler manner. Although the model, as described, may seem too simple to be applied to real cost allocation problems, it is not difficult to extend it to more general situations. Some of those extensions are discussed below.

One of the basic assumptions of our model is that a river is a segment divided into subsegments of the same size. We could posit a river with subsegments of different sizes to reflect that in reality regions occupy different extensions along a river. This case can easily be analyzed from our framework by dividing all regions into smaller subsegments of the same size (by using the maximum common divisor) and dividing the cost of cleaning each region proportionally among all those subsegments. Another natural extension of our model is to consider a river which is not a segment but a network divided into segments. This could be useful in modeling a river with tributaries and/or forks. In that case, all the results of the paper can be easily adapted by incorporating the number of outlets on each fork into the calculation of the limits of the transfer rate and extending the rule as Dong et al. [6] extend the rules of Ni and Wang [12].

Another implicit assumption of our model is that the uncertainty of the social planner on the transfer rate takes the form of a random variable with uniform distribution, so the mean value between $\underline{t}$ and $\overline{t}^*(\bar{t}, C)$ is always a good *estimator* of t. However, different distributions may be assumed *a priori* and, for those cases, a reformulation of axiom MIT would be necessary. To be more precise, we would have to consider a modified version of MIT in which the changes in the intervals are evaluated on the basis not of their lengths but of their masses of probability of the specific random variable assumed. As a result, the rule characterized would change to one in which the transfer rate considered in allocating costs would be the expected value of the random variable after truncating it at $\overline{t}^*(\bar{t}, C)$.

Finally, another basic assumption of our model is that the transfer rate is constant along the river. A natural extension of the model in this regard would be to consider that the transfer rate changes in some areas of the river. This could be useful in modeling rivers that run through changing terrain. In that case, the model could be adapted by dividing the problem into subproblems with homogeneous terrain. By applying Proposition 1 to each of them, different limits can be deduced for each particular transfer rate.

Appendix

Proof of Theorem 1

First, it is easy to see that the Upstream Responsibility rule γ satisfies LR, NDR, CR and MIT. To prove the other implication, consider a problem $(N, C, \underline{t}, \overline{t})$ and its corresponding $\overline{t}^*(\overline{t}, C)$ inferred from Proposition 1. Let x be a rule satisfying LR, NDR, CR and MIT. We are going to show that x has to correspond to γ.

We will calculate the assignment given by x in n steps. In the $j-$th step, we calculate the values of $x_i^j(\cdot)$ for all $i \in \{1, \ldots, n\}$.

- Step 1: We distribute the cost c_1. In this case, by NDR, $x_i^1(\cdot) = 0$ for all $i > 1$. Then, by definition of a rule, $x_1^1(\cdot) = c_1$. If $n = 1$, the proof is finished. If $n > 1$, go to step 2.

- Step 2: We distribute the cost c_2. By NDR, $x_i^2(N, C, \underline{t}, \overline{t}) = 0$ for all $i > 2$. Consider other problem (N, C, s, s), where $s = \frac{\underline{t}+\overline{t}^*(\overline{t},C)}{2}$. Now, we have two cases:

  - If $n = 2$, we have by LR that $x_2^2(N, C, s, s) = c_2 - \frac{c_1 \cdot s}{1-s}$. We are going to prove that $x_2^2(N, C, s, s) = x_2^2(N, C, \underline{t}, \overline{t})$. If $\underline{t} = s = \overline{t}^*$, it is straightforward that they are equal. For the rest of the cases, consider all problems (N, C, r, r) such that $r \in [\underline{t}, s)$. Then, by LR we have that $x_2^2(N, C, r, r) = c_2 - \frac{c_1 \cdot r}{1-r}$. Given that $r - \underline{t} < \overline{t}^* - r$, we have by MIT that $x_1^2(N, C, r, r) \leq x_1^2(N, C, \underline{t}, \overline{t})$ and then, by definition, $x_2^2(N, C, r, r) \geq x_2^2(N, C, \underline{t}, \overline{t})$. Therefore, $x_2^2(N, C, \underline{t}, \overline{t}) \leq c_2 - \frac{c_1 \cdot (s-\varepsilon)}{1-(s-\varepsilon)}$ for all $\varepsilon \geq 0$. Similarly, we can deduce that $x_2^2(N, C, u, u) \leq x_2^2(N, C, \underline{t}, \overline{t})$ for all $u \in (s, \overline{t}^*]$ and, then, $x_2^2(N, C, \underline{t}, \overline{t}) \geq c_2 - \frac{c_1 \cdot (s+\varepsilon)}{1-(s+\varepsilon)}$ for all $\varepsilon \geq 0$. Then, the unique possibility is that $x_2^2(N, C, \underline{t}, \overline{t}) = x_2^2(N, C, s, s)$. Therefore, $x_2^2(N, C, \underline{t}, \overline{t}) = c_2 - \frac{c_1 \cdot s}{1-s}$ and, then, by definition, $x_1^2(N, C, \underline{t}, \overline{t}) = \frac{c_1 \cdot s}{1-s}$.
  - If $n > 2$, we have by LR that $x_2^2(N, C, s, s) = c_2 - c_1 \cdot s$. We are going to prove that $x_2^2(N, C, s, s) = x_2^2(N, C, \underline{t}, \overline{t})$. If $\underline{t} = s = \overline{t}^*$, it is straightforward that they are equal. For the rest of the cases, consider all problems (N, C, r, r) such that $r \in [\underline{t}, s)$. Then, by LR we have that $x_2^2(N, C, r, r) = c_2 - c_1 \cdot r$. Given that $r - \underline{t} < \overline{t}^* - r$, we have

by MIT that $x_1^2(N,C,r,r) \leq x_1^2(N,C,\underline{t},\overline{t})$ and then, by definition, $x_2^2(N,C,r,r) \geq x_2^2(N,C,\underline{t},\overline{t})$. Therefore, $x_2^2(N,C,\underline{t},\overline{t}) \leq c_2 - c_1 \cdot (s-\varepsilon)$ for all $\varepsilon \geq 0$. Similarly, we can deduce that $x_2^2(N,C,u,u) \leq x_2^2(N,C,\underline{t},\overline{t})$ for all $u \in (s,\overline{t}^*]$ and, then, $x_2^2(N,C,\underline{t},\overline{t}) \geq c_2 - c_1 \cdot (s+\varepsilon)$ for all $\varepsilon \geq 0$. Then, the unique possibility is that $x_2^2(N,C,\underline{t},\overline{t}) = x_2^2(N,C,s,s)$. Therefore, $x_2^2(N,C,\underline{t},\overline{t}) = c_2 - c_1 \cdot s$ and, then, by definition, $x_1^2(N,C,\underline{t},\overline{t}) = c_1 \cdot s$. Now, go to step 3.

- Step j, with $j \in \{3,\ldots,n\}$: We distribute the cost c_j. By the application of NDR, $x_i^j(N,C,\underline{t},\overline{t}) = 0$ for all $i > j$. Consider other problem (N,C,s,s), where $s = \frac{\underline{t}+\overline{t}^*(\overline{t},C)}{2}$. Now, we have two cases:

  - If $n = j$, we have by LR that $x_n^n(N,C,s,s) = c_n - \frac{c_{n-1}\cdot s}{1-s}$. We are going to prove that $x_n^n(N,C,s,s) = x_n^n(N,C,\underline{t},\overline{t})$. If $\underline{t} = s = \overline{t}^*$, it is straightforward that they are equal. For the rest of the cases, consider all problems (N,C,r,r) such that $r \in [\underline{t},s)$. Then, by LR we have that $x_n^n(N,C,r,r) = c_n - \frac{c_{n-1}\cdot r}{1-r}$. Given that $r - \underline{t} < \overline{t}^* - r$, we have by MIT that $\sum\limits_{i<n} x_i^n(N,C,r,r) \leq \sum\limits_{i<n} x_i^n(N,C,\underline{t},\overline{t})$ and then, by definition, $x_n^n(N,C,r,r) \geq x_n^n(N,C,\underline{t},\overline{t})$. Therefore, $x_n^n(N,C,\underline{t},\overline{t}) \leq c_n - \frac{c_{n-1}\cdot(s-\varepsilon)}{1-(s-\varepsilon)}$ for all $\varepsilon \geq 0$. Similarly, we can deduce that $x_n^n(N,C,u,u) \leq x_n^n(N,C,\underline{t},\overline{t})$ for all $u \in (s,\overline{t}^*]$ and, then, $x_n^n(N,C,\underline{t},\overline{t}) \geq c_n - \frac{c_{n-1}\cdot(s+\varepsilon)}{1-(s+\varepsilon)}$ for all $\varepsilon \geq 0$. Then, the unique possibility is that $x_n^n(N,C,\underline{t},\overline{t}) = x_n^n(N,C,s,s)$. Therefore, $x_n^n(N,C,\underline{t},\overline{t}) = c_n - \frac{c_{n-1}\cdot s}{1-s}$ and, by definition, $\sum\limits_{i=1}^{n-1} x_i^n(N,C,\underline{t},\overline{t}) = \frac{c_{n-1}\cdot s}{1-s}$. By CR, $x_i^n(N,C,\underline{t},\overline{t})\cdot x_k^{n-1}(N,C,\underline{t},\overline{t}) = x_k^n(N,C,\underline{t},\overline{t})\cdot x_i^{n-1}(N,C,\underline{t},\overline{t})$ for all $i,k \in \{1,\ldots,n-1\}$ Or, equivalently, $x_i^n(N,C,\underline{t},\overline{t})\cdot \sum\limits_{i=1}^{n-1} x_i^{n-1}(N,C,\underline{t},\overline{t}) = x_i^{n-1}(N,C,\underline{t},\overline{t}) \cdot \sum\limits_{i=1}^{n-1} x_i^n(N,C,\underline{t},\overline{t})$ for all $i \in \{1,\ldots,n-1\}$.

    Given that $\sum\limits_{i=1}^{n-1} x_i^{n-1}(N,C,\underline{t},\overline{t}) = c_{n-1}$ and that we also know from step $j-1$ that $x_i^{n-1}(N,C,\underline{t},\overline{t}) = c_i \cdot s^{n-1-i} - c_{i-1}\cdot s^{n-i}$, we have that for all $i \in \{1,\ldots,n-1\}$,

    $$x_i^n(N,C,\underline{t},\overline{t}) = \frac{c_i \cdot s^{n-i-1} - c_{i-1}\cdot s^{n-i}}{c_{n-1}} \cdot \frac{c_{n-1}\cdot s}{1-s}.$$

    Therefore, for all $i \in \{1,\ldots,n-1\}$,

$$x_i^n(N, C, \underline{t}, \bar{t}) = \frac{c_i \cdot s^{n-i} - c_{i-1} \cdot s^{n-i+1}}{1-s}.$$

– If $n > j$, we have by LR that $x_j^j(N, C, s, s) = c_j - c_{j-1} \cdot s$. As before, we can prove by MIT that $x_j^j(N, C, s, s) = x_j^j(N, C, \underline{t}, \bar{t})$ and, then, $x_j^j(N, C, \underline{t}, \bar{t}) = c_j - c_{j-1} \cdot s$. Then, we have that $\sum_{i=1}^{j-1} x_i^j(N, C, \underline{t}, \bar{t}) = c_{j-1} \cdot s$. By CR, $x_i^j(N, C, \underline{t}, \bar{t}) \cdot x_k^{j-1}(N, C, \underline{t}, \bar{t}) = x_k^j(N, C, \underline{t}, \bar{t}) \cdot x_i^{j-1}(N, C, \underline{t}, \bar{t})$ for all $i, k \in \{1, \ldots, j-1\}$. Or, equivalently, $x_i^j(N, C, \underline{t}, \bar{t}) \cdot \sum_{i=1}^{j-1} x_i^{j-1}(N, C, \underline{t}, \bar{t}) = x_i^{j-1}(N, C, \underline{t}, \bar{t}) \cdot \sum_{i=1}^{j-1} x_i^j(N, C, \underline{t}, \bar{t})$ for all $i \in \{1, \ldots, j-1\}$.

Given that $\sum_{i=1}^{j-1} x_i^{j-1}(N, C, \underline{t}, \bar{t}) = c_{j-1}$ and that we also know from step $j-1$ that $x_i^{j-1}(N, C, \underline{t}, \bar{t}) = c_i \cdot s^{j-1-i} - c_{i-1} \cdot s^{j-i}$, we have that for all $i \in \{1, \ldots, j-1\}$,

$$x_i^j(N, C, \underline{t}, \bar{t}) = \frac{c_i \cdot s^{j-1-i} - c_{i-1} \cdot s^{j-i}}{c_{j-1}} \cdot c_{j-1} \cdot s.$$

Therefore, for all $i \in \{1, \ldots, j-1\}$,

$$x_i^j(N, C, \underline{t}, \bar{t}) = c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i}.$$

Now, go to step $j+1$. ■

Proof of Proposition 3

The following examples prove that the axioms are independent.

Limits of Responsibility: The Upstream Equal Sharing rule, β, introduced by Ni and Wang [12] satisfies NDR, CR and MIT. However, it does not satisfy LR as we have shown in Example 2.

No Downstream Responsibility: Let ω be the following rule:

$$\omega_i^j(N, C, \underline{t}, \bar{t}) = \begin{cases} c_i - c_{i-1} \cdot s & \text{if } i = j < n, \\ c_i - \frac{c_{i-1} \cdot s}{1-s} & \text{if } i = j = n, \\ c_{i-2} \cdot s & \text{if } i = j + 1, \\ \frac{c_i \cdot s}{1-s} & \text{if } i + 1 = j = n, \\ 0 & \text{otherwise}, \end{cases}$$

where $s = \frac{\underline{t} + \bar{t}^*(\bar{t}, C)}{2}$ and c_0 is set to 0.

It is easy to see that this rule ω satisfies MIT, LR and CR. However, the following example shows that it does not satisfy NDR. Let $N = \{1, 2, 3\}$, $C = \{10, 10, 10\}$, $\underline{t} = 0$ and $\bar{t} = 1$ be a cost allocation problem. We have that $\omega_3^2(N, C, \underline{t}, \bar{t}) = \frac{5}{2}$, while NDR states that $\omega_3^2(N, C, \underline{t}, \bar{t}) = 0$.

Consistent Responsibility: Let φ be the following rule:

$$\varphi_i^j(N, C, \underline{t}, \bar{t}) = \begin{cases} 0 & \text{if } i > j, \\ c_i - c_{i-1} \cdot s & \text{if } i = j < n, \\ c_i - \frac{c_{i-1} \cdot s}{1-s} & \text{if } i = j = n, \\ \frac{c_{j-1} \cdot s}{j-1} & \text{if } i < j < n, \\ \frac{c_{j-1}}{j-1} \cdot \frac{s}{1-s} & \text{if } i < j = n, \end{cases}$$

where $s = \frac{\underline{t}+\bar{t}^*(\bar{t},C)}{2}$ and c_0 is set to 0.

It is easy to see that φ satisfies LR, NDR and MIT. However, the following example shows that it does not satisfy CR. Let $N = \{1,2,3\}$, $C = \{10,10,5\}$, $\underline{t} = 0$ and $\bar{t} = 1$ be a cost allocation problem. We have that $\varphi_2^2(\cdot) = \frac{25}{3}$, $\varphi_1^3(\cdot) = 1$, $\varphi_2^3(\cdot) = 1$ and $\varphi_1^2(\cdot) = \frac{5}{3}$. Then, $\varphi_2^2(\cdot)\cdot\varphi_1^3(\cdot) = \frac{25}{3} \neq \frac{5}{3} = \varphi_1^2(\cdot)\cdot\varphi_2^3(\cdot)$, while CR would imply that $\varphi_2^2(\cdot)\cdot\varphi_1^3(\cdot) = \varphi_1^2(\cdot)\cdot\varphi_2^3(\cdot)$.

Monotonicity with respect to Information on the transfer rate: Let ρ be the following rule:

$$\rho_i^j(N,C,\underline{t},\bar{t}) = \begin{cases} 0 & \text{if } i > j, \\ c_i \cdot \underline{t}^{j-i} - c_{i-1}\cdot \underline{t}^{j+1-i} & \text{if } i \leq j < n, \\ c_i - \frac{c_{i-1}\cdot \underline{t}}{1-\underline{t}} & \text{if } i = j = n, \\ \frac{c_i\cdot \underline{t}^{j-i} - c_{i-1}\cdot \underline{t}^{j-i+1}}{1-\underline{t}} & \text{if } i < j = n, \end{cases}$$

where c_0 is set to 0.

It is easy to see that ρ satisfies LR, NDR and CR. However, the following example shows that it does not satisfy MIT. Let $(N,C,\underline{t},\bar{t})$ and $(N,C,\underline{u},\overline{u})$ be two cost allocation problems, with $N = \{1,2\}$, $C = \{10,20\}$, $\underline{t} = 0$, $\bar{t} = 1$ and $\underline{u} = \overline{u} = \frac{1}{4}$. We have that $\rho_1^2(N,C,\underline{t},\bar{t}) = 0$ and $\rho_1^2(N,C,\underline{u},\overline{u}) = \frac{10}{3}$, although MIT would imply that $\rho_1^2(N,C,\underline{u},\overline{u}) \leq \rho_1^2(N,C,\underline{t},\bar{t})$.

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