Abstract

This paper attempts to explain several empirical findings regarding religion. The main one is between religion and the fear of death. Some empirical evidence indicates moderately religious individuals fear death more than either atheists or extremely religious individuals. The model also explains the positive relationship often found between religious activity (e.g. church attendance) and age. It also provides an explanation of the positive relationship between education and religious activity despite a negative relationship between education and religious belief.

JEL Classifications: Z12

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1 Introduction

Several studies have found that moderately religious individuals fear death more than either atheists or extremely religious individuals. This paper provides a rational choice explanation of this empirical finding. Depending on functional forms, it is also consistent with studies that find a positive relationship between religion and the fear of death. Furthermore, it predicts changes in the fear of death over the life cycle. Moreover, it examines possible effects of religious competition on both religious behaviour and the fear of death. In addition, it provides an explanation for increased religious activities later in life. It also offers a possible explanation of the empirical finding that the relationship between education and measures of religiosity based on reported beliefs is negative, while the relationship between education and religiosity based on reported behaviour (e.g. church attendance) is positive.

This paper fills a gap in not only the economics literature but also the literature on attitudes towards death in general. In their review of the literature, Neimeyer, Wittkowski and Moser (2004) state:

In general, research on death attitudes has tended to follow an atheoretical “statistical dragnet” method, as investigators simply report significant associations between variables, or at most test interesting but isolated hypotheses that have little or no relation to broader theories of human functioning. (p. 327)

Neimeyer et al cite terror management theory as an exception to this general rule. This paper contributes to further filling this gap by offering a rational choice explanation.

A central feature of many religions is a belief in an afterlife. Indeed, Carl Jung has argued that most religions can be viewed as “complicated systems of preparation for
death” (as quoted by Ardelt and Koenig, 2006). This paper argues that this feature of religion is a key to understanding why moderately religious individuals may fear death more than very religious individuals. Thus, it uses what the economics literature sometimes calls the afterlife capital model of religion.

This paper assumes a three period model. In period 3, an individual is dead. When dead, an individual faces three possible outcomes: (1) the end of existence, (2) heaven, or its equivalent and (3) damnation. In periods 1 and 2, the individual may make costly investments in religious capital. These investments may take many forms including church attendance, dietary restrictions, prayer, reading religious writings, refraining from blood transfusions and tattoos, etc.

One point that many religious and nonreligious scholars of religion agree on is that the existence of God and an afterlife cannot be known with certainty. For example, Dawkins (2006) argues that the spectrum of probabilities individuals place on God’s existence is continuous and that unlike some religious followers, few atheists would take a position at an endpoint of the spectrum. If God’s existence is a necessary condition for an afterlife, this implies that few atheists would place zero probability on the existence of an afterlife. At the other extreme, in defending religion against criticism from authors such as Dawkins, Gray (2008, D1) states: “Doubt has been an integral part of religion at least since the Book of Job, while science has often gone with credulity”. Although each side argues that they are unique in recognizing uncertainty, this paper will accept the argument of each that in the absence of empirical evidence, the probability of an afterlife is a subjective probability. It will follow Dawkins and assume a continuous distribution of probabilities and represent Gray’s doubt as the subjective probability that no afterlife
exists. This probability is considered to be exogenously determined and differs for different individuals. However, unlike Dawkins, for simplicity this paper will assume that a pure atheist considers the probability of the existence of an afterlife to be zero.

The probability of ending up in heaven is a positive function of a person’s investment in religious capital. The probability of ending up in hell is a negative function of an individual’s investment in religious capital. Fear of death is defined as a drop in an individual’s expected utility at death.

A pure atheist makes no investment in religious capital and faces no uncertainty with regard to his expected utility after death. He fears death to the extent his current utility is positive. Those who place a positive probability on the existence of an afterlife may invest in religious capital. This paper argues that those who place a small probability on the existence of an afterlife will rationally make small investments in religious capital. This results in a small subjective probability of going to heaven relative to the probability of going to hell, increasing their fear of death, relative to atheists. Those who perceive a larger probability of an afterlife, make larger investments in religious capital. The result is a higher subjective probability of going to heaven, relative to the moderately religious. This decreases their fear of death relative to the moderately religious and may even decrease it relative to atheists. It may even result in an expected gain in utility at death.

In addition, by assuming individuals discount the future, this paper finds that individuals should invest more in religious capital in the second period of the model. This agrees with empirical findings that age and church attendance is positively correlated (Brañas-Garza and Neuman, 2004; Glaeser and Sacerdote, 2001; Chatters,
Taylor and Lincoln 1999; Miller and Nakamura, 1996; Azzi and Ehrenberg, 1975). It also finds that an increase in an individual’s discount factor (i.e. a decrease in his discount rate) has ambiguous effects on religious investment in period 2 but unambiguously increases religious investment in period 1. Moreover, the total effect of an increase in the discount factor is to increase total net investment in religious capital.

This paper argues that the positive relationship between religious investment and the discount factor explains another empirical finding of the literature. Empirically, there is a negative relationship between education and measures of religiosity based on reported beliefs. However, with the exception of a few East European countries, there is a positive relationship between education and religiosity based on reported behaviour. The first relationship can be interpreted as the correlation between education and the probability that an afterlife exists. However, educated individuals tend to have lower discount rates (Harrison, Lau and Williams, 2002). Thus, it is not surprising that religious activities can increase with education despite the lower number of believers. Just as successful students are likely to attend lectures and fulfill other academic requirements, if they are also religious, it is not surprising that they also attend church and fulfill religious requirements.

1.1 Review of some relevant literature

Azzi and Ehrenberg (1975) were the first to use the afterlife capital model of religion. Their paper models church attendance as being an investment in an afterlife. However, it does not address the relationship between religion and the fear of death. Moreover, they

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1 Several East European countries do not exhibit a positive relationship between education and church attendance. Moreover, other countries tend not to exhibit a positive relationship as statistically significant as the one in the United States. For more details and a discussion on this point, see Glaeser and Sacerdote (2001).
model a continuous distribution of afterlives, determined by religious activity during life. It is unclear whether the afterlife of many religions are well represented by such an approach. In the case of religions involving reincarnation as different life forms, it may be the case that the quality of the next life can be modelled as a continuous function of behaviour in this life. However, few traditional religions offer such a large number of outcomes. In addition, Azzi and Ehrenberg assume that the church attendance of either partner in a marriage is a substitute for the other’s attendance. Ulbrich and Wallace (1983) point out that there is little theological basis in this assumption. Therefore, although the authors deserve a great deal of credit for being the first to address these issues, it is not clear that their approach represents most common religious beliefs.

Blomberg, DeLeire and Hess (2006) develop a model where financial religious contributions are motivated by both current consumption and afterlife considerations. They estimate their model and conclude that current consumption is the dominant consideration for the level of contributions over a lifetime. However, contributions do respond positively to increases in the probability of death. Thus, they argue that afterlife considerations are an important determinant in explaining the life cycle of financial religious contributions.

Wink and Scott (2005), using longitudinal data, find that people who score high on a scale of religiosity fear death less in late adulthood than those who score low. However, the moderately religious fear death the most. Moreover, this relationship between the fear of death in late adulthood and religiosity holds whether religiosity is measured in late adulthood or middle adulthood.
Smith, Nehemkis and Charter (1983) find a similar curvilinear relationship between having an afterlife-as-reward perspective of death and the fear of death.² Both those who were most and least likely to take this view of death, feared death less than those between the extremes. Somewhat similar to this paper, the authors hypothesize that it is uncertainty regarding an afterlife that creates a greater fear of death for those less certain about the existence of an afterlife. Smith et al also find a positive relationship between age and church attendance and a negative relationship between age and the fear of death.

Earlier research that also found a curvilinear relationship between religion and the fear of death includes Downey (1984), McMordie (1981) and Nelson and Cantrell (1980).

It should be noted that not all studies have found such a nonlinear relationship between religion and fear of death. For example, Power and Smith (2008) using a sample of Atlantic Canadian university students, found no evidence of a curvilinear relationship. Instead, they find a positive relationship between religiosity and some measures of death anxiety (fear of the dead, fear of being destroyed, fear of conscience death) and a negative relationship between religiosity and fear of the unknown. In a study of Jewish men, Florian and Kravetz (1983) also find different linear results for different measures of the fear of death. For example, religious men had a greater fear of punishment in the hereafter and significantly less fear to self-annihilation than less religious men. However, their methods did not test for nonlinear relationships.

Iannoccone, Stark and Finke (1998) find, with varying degrees of significance and insignificance, that professors, scientists and graduate schooled survey respondents are

² The empirical literature on the fear of death typically refers to this inverted U-shape relationship as quasi-linear.
less likely to be religious, believe in the bible, or describe themselves as “close to god”. However, they are slightly (statistically insignificant) more likely to attend church regularly. In addition, in separate regressions, they find that those in “hard” (natural) sciences are more religious than those in social sciences.

Brown and Taylor (2007) find a positive relationship between education and church attendance, even when endogeneity bias is controlled for. However, they find a negative (but not always significant) relationship between science education and church attendance.

Glaeser and Sacerdote (2001) explain the positive American relationship between education and church attendance despite the negative relationship between education and religious belief by arguing that education has two effects. They argue that education increases the proclivity towards social group membership and decreases belief in the returns to religious activity. They provide compelling evidence to support their argument. However, this paper argues that even without the socializing effects of education, a positive relationship between education and religious behaviour is not necessarily inconsistent with a negative relationship between education and religious belief.

Hollander, Kahana and Lecker (2003) offer an alternative explanation of the positive relationship between education and religious activity. They do this by assuming religious studies exert a positive externality on utility.

Many religions no longer currently teach that the traditional very unpleasant hell exists. Moreover, in several surveys, many religious people report that they do not believe in an afterlife. However, survey evidence also indicates that significant
proportions of the population do believe in an afterlife and hell (Brañas-Garza, García-Muñoz and Neuman, 2008; Barro and McCleary, 2003; Exline, 2003; Hull and Bold, 1994). In addition, with the exception of the ISSP 1998: Religion II dataset, most surveys ask binary questions, possibly underestimating those that place low but positive probabilities on the existence of an afterlife.

1.2 Outline

Section 2 presents the model. Section 3 examines the comparative statics. Specifically, it finds that for individuals with positive discount rates, investment in religious activities will be higher in the second period. In addition, it finds that an increase in an individual’s discount rate will increase his religious investment in period 1 but have ambiguous effects in his religious investment in period 2. Nonetheless, total lifetime investments in religious activities increase. Section 4 uses the findings of section 3 to argue that the positive empirical relationship between education and religious activities, despite a negative relationship between education and religious beliefs’ is consistent with the model. Section 5 examines the implications of the model for an individual’s fear of death. The first subsection finds that the model is consistent with the often-found curvilinear relationship between religiosity and the fear of death. The second subsection finds that religious individuals, who find they will die early and not be able make planned religious investments in period 2, should fear death more than those who live to period 2. This is consistent with many empirical findings. Moreover, it finds that this negative relationship between age and the fear of death should be stronger for religious individuals than atheists, providing a testable implication of the model. Subsection 5.3 discusses possible effects of religious choice on the fear of death and religious investment. In cases
were only the correct religion leads to an afterlife, such choice decreases religious
investment and increases the fear of death. Section 6 concludes the paper.

2 The model

Consider a three period model. In the last period, an individual is dead. Let \( p_E \) be the
exogenously given probability with which an individual believes that death is the end of
existence. In this case, his payoff is \( E \). If death is not the end of existence, the individual
believes that he will either go to heaven and receive a payoff of \( B \) (Bliss) or go to hell and
receive a payoff of \( D \) (Damnation). The subjective probability he places on going to
heaven is given by \( p_B(a_1 + a_2) \). It is assumed that \( p_B \) is a positive function of his total
investment in religious capital \( a = a_1 + a_2 \), where \( a_i (i = 1, 2) \) is his investment in period \( i \).
These investments can take whatever form his religion encourages or requires. Further
assume that if the individual makes no investment in religious capital, the probability of
going to heaven is zero: \( p_B(0) = 0 \).

The subjective probability he places on going to hell is given by \( p_D(a_1 + a_2) \). It is
a negative function of religious investments. Given the lack of empirical evidence of an
afterlife, it should be emphasised that all these probabilities are subjective.

The sum of these probabilities must equal 1:

\[
p_B(a_1 + a_2) + p_E + p_D(a_1 + a_2) = 1
\]  \( \text{(1)} \)

Since \( p_B(0) = 0 \), \( p_D(0) = 1 - p_E \). An individual who makes no investment in religious
capital either goes to hell or ceases to exist.

For the special case of a pure atheist: \( p_B(a_1 + a_2) = p_D(a_1 + a_2) = 0 \). At the other
end of the spectrum, the probabilities facing a purely religious person are such that \( p_B(a_1 + a_2) = 1 - p_D(a_1 + a_2) \).
Given equation (1), it follows that for a given probability of an afterlife, a
marginal increase in \( p_B \) must equal a marginal decrease in \( p_D \):

\[
\frac{dp_B}{da}(a_1 + a_2) = -\frac{dp_D}{da}(a_1 + a_2) > 0
\]  

(2)

Assume that increases in \( a \), increase the probability of going to heaven at a decreasing
rate. Therefore, \( \frac{d^2 p_B}{da^2}(a_1 + a_2) < 0 \) and \( \frac{d^2 p_D}{da^2}(a_1 + a_2) > 0 \).

The agent’s preferences over outcomes at death are represented by a von
Neumann-Morgenstern utility function \( U(L) \), where \( L \) is a payoff value. An individual is
assumed to be risk adverse: \( \frac{d^2 U(L)}{dL^2} < 0 \).

The individual’s utility from the payoff of going to heaven is greater than the
utility from nothingness which is greater than his utility from hell: \( U(B) > U(E) > U(D) \).

If death is the end of existence, a natural normalization for its payoff and resulting utility
level is \( E = U(E) = 0 \). Thus, \( U(B) > 0 \) and \( U(D) < 0 \). Assume these utility levels are
finite.

Utility in period \( i \) \( (i = 1, 2) \) is given by \( U_i(L_i - a_i) \) where \( L_i \) is the payoff he would
receive in the normal course of life. Let \( \delta \) represent the individual’s discount factor. At
the beginning of period 1, the present discounted value of his expected net payoff from
his investment in religious capital is given by \( \pi \):

\[
\pi = U(L_1-a_1)+\delta U(L_2-a_2)+\delta^2 p_B(a_1+a_2) U(B)+\delta^2 p_E U(E)+\delta^2 p_D(a_1+a_2)U(D)
\]  

(3)

Assume \( a_1 \) and \( a_2 \) are constrained to be nonnegative. The first order conditions for his
maximization problem are

\[
\frac{\partial \pi}{\partial a_1} = \delta^2 \frac{dp_B}{da}(a_1 + a_2)U(B) + \delta^2 \frac{dp_D}{da}(a_1 + a_2)U(D) - \frac{dU}{dL_1}(L_1-a_1) + \mu_1 = 0
\]  

(4)
\[
\frac{\partial \pi}{\partial a_2} = \delta^2 \frac{d p}{da} (a_1 + a_2)U(B) + \delta^2 \frac{d p}{da} (a_1 + a_2)U(D) - \delta \frac{d U}{d L_2} (L_2 - a_2) + \mu_2 = 0 \tag{5}
\]

The multipliers (\(\mu_1\) and \(\mu_2\)) on the nonnegativity constraints may be positive for atheists and those with a low \(p_E\). The result is that these individuals will not make investments in religious capital. The multipliers will be zero for all others, resulting in positive investments in religious capital. Using equation (2), \(U(E) = 0\) and assuming an interior solution, these conditions can be rewritten:

\[
\delta^2 \frac{d p}{da} (a_1 + a_2) [U(B) - U(D)] = \frac{d U}{d L_1} (L_1 - a_1) \tag{6}
\]

\[
\delta^2 \frac{d p}{da} (a_1 + a_2) [U(B) - U(D)] = \delta \frac{d U}{d L_2} (L_2 - a_2) \tag{7}
\]

Equations (6) and (7) indicate that the agent will set the present value of the marginal expected benefit of his investment equal to the present value of its marginal cost in each period.

### 3 Comparative Statics

This section will first examine how the discount factor affects investment in religious capital. It will then show that an increase in the doubt of the existence of an afterlife will decrease investment in religious capital.

First, note that the left side of equations (6) and (7) are identical. Thus,

\[
\frac{d U}{d L_1} (L_1 - a_1) = \delta \frac{d U}{d L_2} (L_2 - a_2) \tag{8}
\]

\[
\frac{d^2 U}{d L_1^2} (L_1 - a_1) = \delta \frac{d^2 U}{d L_2^2} (L_2 - a_2) \tag{9}
\]
Given the previous assumptions made, if $L_1 \leq L_2$ and $\delta < 1$, we have $a_2 > a_1$. Therefore, we should expect individuals who discount the future to invest in religious activities more in the second period of their lives. This agrees with several empirical studies. For example, Ulbrich and Wallace (1983) find a positive relationship between age and church attendance. Argue, Johnson and White (1999) find that religiosity (measured by its effects on daily life, which is strongly correlated with variables such as church attendance) increases with age. Stolzenberg, Blair-Loy and Waite (1995) find that female participation in religious organizations increases with age during young adulthood. Brañas-Garza and Neuman (2004) find a major increase in attendance at Mass in Spain at age 60.

This paper will now examine how changes in the discount factor affect investment in religious capital. Rather than directly differentiating the first order conditions, it is more convenient to differentiate the objective function, evaluated at the optimum, and make use of the envelope theorem:

$$\pi = U(L_1-a_1)+\delta U(L_2-a_2)+\delta^2 p_B(a_1+a_2) U(B)+\delta^2 p_E U(E)+\delta^2 p_D(a_1+a_2) U(D) \quad (3)$$

Using the envelope theorem, $U(E) = 0$ and equations (2) and (7):

$$\frac{\partial \pi}{\partial \delta} = U(L_2-a_2) + 2\delta p_B (a_1 + a_2) u(B) + 2\delta p_D (a_1 + a_2) u(D) \quad (10)$$

$$\frac{\partial \pi}{\partial a, \partial \delta} = 2\delta \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] > 0 \quad (11)$$

---

3 Yang (2008) provides empirical evidence that “happiness” increases with age. If $L_2 > L_1$, the difference between the optimal levels of $a_1$ and $a_2$ is reinforced. However, this paper does not formally consider the possibility that if religious investments involve time, the opportunity cost of religious investments may increase as older individuals tend to have higher wage rates (assuming they are not retired). For a discussion of this consideration, see Azzi and Ehrenberg (1975).

4 They find that a similar relationship for men is due to changes in their characteristics (e.g. childlessness) rather than age itself.
\[
\frac{\partial \pi}{\partial a_i \partial \delta} = -\frac{dU}{dL_2} (L_2 - a_z) + 2\delta \frac{dp_a}{da} (a_i + a_z) [U(B) - U(D)] \\
= -\delta \frac{dp_a}{da} (a_i + a_z) [U(B) - U(D)] + 2\delta \frac{dp_a}{da} (a_i + a_z) [U(B) - U(D)] \\
= \delta \frac{dp_a}{da} (a_i + a_z) [U(B) - U(D)] > 0 \quad (12)
\]

Differentiating the first order conditions:

\[
\frac{\partial^2 \pi}{\partial a_i^2} = \delta \frac{d^2 p_a}{da^2} (a_i + a_z) [U(B) - U(D)] + \frac{d^2 U}{dL_1^2} (L_1 - a_1) < 0 \quad (13)
\]

\[
\frac{\partial^2 \pi}{\partial a_z^2} = \delta \frac{d^2 p_a}{da^2} (a_i + a_z) [U(B) - U(D)] + \frac{d^2 U}{dL_2^2} (L_2 - a_2) < 0 \quad (14)
\]

\[
\frac{\partial^3 \pi}{\partial a_1\partial a_z} = \frac{\partial^2 \pi}{\partial a_i\partial a_z} = \delta \frac{d^2 p_a}{da^2} (a_i + a_z, p_2) [U(B) - U(D)] < 0 \quad (15)
\]

The signs of the above partial derivatives were determined using previously stated assumptions.

Using (10)-(15) for the total derivatives of the first order conditions and putting the result in matrix form:

\[
\begin{bmatrix}
\frac{\partial^2 \pi}{\partial a_i^2} & \frac{\partial^2 \pi}{\partial a_i \partial a_z} \\
\frac{\partial^2 \pi}{\partial a_z \partial a_i} & \frac{\partial^2 \pi}{\partial a_z^2}
\end{bmatrix}
\begin{bmatrix}
da_i \\
da_z
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial^2 \pi}{\partial \delta \partial a_i} \\
\frac{\partial^2 \pi}{\partial \delta \partial a_z}
\end{bmatrix}
d\delta \quad (16)
\]

Using Cramer’s rule to solve for \(da_i/d\delta\):

\[
da_i = \frac{\begin{vmatrix}
\frac{\partial \pi}{\partial \delta} & \frac{\partial \pi}{\partial a_i} \\
\frac{\partial \pi}{\partial a_1} & \frac{\partial \pi}{\partial a_z}
\end{vmatrix}}{egin{vmatrix}
\frac{\partial^2 \pi}{\partial a_1^2} & \frac{\partial^2 \pi}{\partial a_1 \partial a_z} \\
\frac{\partial^2 \pi}{\partial a_z \partial a_1} & \frac{\partial^2 \pi}{\partial a_z^2}
\end{vmatrix}} = \frac{\begin{vmatrix}
\frac{\partial \pi}{\partial \delta} & \frac{\partial \pi}{\partial a_i} \\
\frac{\partial \pi}{\partial a_1} & \frac{\partial \pi}{\partial a_z}
\end{vmatrix}}{egin{vmatrix}
\frac{\partial^2 \pi}{\partial a_1^2} & \frac{\partial^2 \pi}{\partial a_1 \partial a_z} \\
\frac{\partial^2 \pi}{\partial a_z \partial a_1} & \frac{\partial^2 \pi}{\partial a_z^2}
\end{vmatrix}} \quad (17)
\]
For the second order conditions to hold, the denominator of (17) must be positive. The numerator is given by

\[-2\delta^3 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da^2} - 2\delta^2 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 U}{dL_2^2} (L_2 - a_2) \]

\[+ \delta \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] d^2 p_B (a_1 + a_2) \]

\[= -\delta^3 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da^2} - 2\delta^2 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 U}{dL_2^2} (L_2 - a_2) > 0 \]

Thus, \(\frac{da_1}{d\delta} > 0\).

Doing the same for \(\frac{da_2}{d\delta}\):

\[\frac{da_2}{d\delta} = \begin{vmatrix}
\frac{\partial^2 \pi}{\partial a_1^2} & -\frac{\partial \pi}{\partial a_1} \\
\frac{\partial^2 \pi}{\partial a_1 \partial a_2} & -\frac{\partial \pi}{\partial a_2} \\
\frac{\partial^2 \pi}{\partial a_2^2} & -\frac{\partial \pi}{\partial a_2}
\end{vmatrix}
\]

(18)

The value of the numerator is given by

\[-\delta^3 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da^2} (a_1 + a_2) - \delta \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 U}{dL_1} (L_1 - a_1) \]

\[+ 2\delta^3 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da^2} (a_1 + a_2) \]

\[= \delta^3 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da^2} (a_1 + a_2) - \delta \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 U}{dL_1} (L_1 - a_1) \]

The first term is negative and the second term is positive. Thus, the sign of \(\frac{da_2}{d\delta}\) is unclear. This result is intuitive. An increase in the discount rate increases the present value of the expected benefit in period 3. However, it also reduces the incentive to make investments in period 2 rather than period 1, leading to ambiguous results. Nonetheless,
it is possible to determine the total effect of a change in the discount factor. The total
effect of an increase in the discount factor is given by

\[ \frac{da}{d\delta} = \frac{da_1}{d\delta} + \frac{da_2}{d\delta} = \begin{vmatrix}
\frac{\partial^2 \pi}{\partial a_1 \partial a_1} & \frac{\partial \pi}{\partial a_2} \\
\frac{\partial \pi}{\partial a_1} & \frac{\partial^2 \pi}{\partial a_2 \partial a_2}
\end{vmatrix} \frac{\partial^2 \pi}{\partial \delta \partial a_1} - \begin{vmatrix}
\frac{\partial \pi}{\partial a_1} & \frac{\partial \pi}{\partial a_2} \\
\frac{\partial \pi}{\partial a_1} & \frac{\partial \pi}{\partial a_2}
\end{vmatrix} \frac{\partial^2 \pi}{\partial \delta \partial a_2} \]  \tag{19}

The summation of the numerators is given by

\[ -\delta^3 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da_1^2} - 2\delta^2 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 U}{dL_2^2} (L_2 - a_2) \]

\[ + \delta^2 \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \frac{d^2 p_B}{da_2^2} (a_1 + a_2, pE) - \delta \frac{dp_B}{da} (a_1 + a_2, pE) [U(B) - U(D)] \frac{d^2 U}{dL_1^2} (L_1 - a_1) \]

\[ = -\delta \frac{dp_B}{da} (a_1 + a_2) [U(B) - U(D)] \left[ 2\delta \frac{d^2 U}{dL_2^2} (L_2 - a_2) + \frac{d^2 U}{dL_1^2} (L_1 - a_1) \right] > 0 \]

Thus, the less an individual discounts the future, the greater will be his investments in religious capital.

The same exercise can be preformed for a change in the probability that death is the end of existence. First, totally differentiate the first order conditions of an individual with respect to \( a_1, a_2 \) and \( p_E \):

\[ \frac{\partial^2 \pi}{\partial a_1^2} - \frac{\partial \pi}{\partial a_2} \frac{\partial \pi}{\partial a_1} dp_E = 0 \]  \tag{20}

\[ \frac{\partial \pi}{\partial a_1} \frac{\partial^2 \pi}{\partial a_2} + \frac{\partial \pi}{\partial a_2} \frac{\partial \pi}{\partial a_2} \frac{\partial \pi}{\partial p_E} dp_E = 0 \]  \tag{21}

where

\[ \frac{\partial^2 \pi}{\partial a_1^2} = \delta^3 \frac{d^2 p_B}{da_1^2} (a_1 + a_2) [U(B) - U(D)] + \frac{d^2 U}{dL_1} (L_1 - a_1) < 0 \]  \tag{22}
\[
\frac{\partial^2 \pi}{\partial a_i^2} = \delta \frac{\partial}{\partial a_i} \frac{d^2 p_B}{da_i^2} (a_i + a_2) \left[ U(B) - U(D) \right] + \delta \frac{d^2 U}{dL_2^2} (L_2 - a_2) < 0
\] (23)

\[
\frac{\partial^2 \pi}{\partial a_2 \partial a_i} = \frac{\partial^2 \pi}{\partial a_i \partial a_2} = \delta \frac{\partial}{\partial a_i} \frac{d^2 p_B}{da_i^2} (a_i + a_2) \left[ U(B) - U(D) \right] < 0
\] (24)

\[
\frac{\partial^2 \pi}{\partial p_E \partial a_i} = \delta^2 \left[ \frac{d^2 p_B}{da_i^2} (a_i + a_2) \right] \left[ U(B) - U(D) \right] + \delta \frac{d^2 U}{dL_2^2} (L_2 - a_2) < 0
\] (25)

\[
\frac{\partial^2 \pi}{\partial p_E \partial a_2} = \delta^2 \left[ \frac{d^2 p_B}{da_2^2} (a_i + a_2) \right] \left[ U(B) - U(D) \right] + \delta \frac{d^2 U}{dL_2^2} (L_2 - a_2) < 0
\] (26)

Using equations (22) to (26):

\[
\begin{vmatrix}
-\frac{\partial^2 \pi}{\partial p_E \partial a_i} & -\frac{\partial^2 \pi}{\partial p_E \partial a_2} & \frac{\partial^2 \pi}{\partial a_2 \partial a_i} & \frac{\partial^2 \pi}{\partial a_i \partial a_2}
\end{vmatrix}
\] (27)

Putting (20) and (21) into matrix form:

\[
\begin{bmatrix}
\frac{\partial^2 \pi}{\partial a_i^2} & \frac{\partial^2 \pi}{\partial a_i \partial a_1} & \frac{\partial^2 \pi}{\partial a_2 \partial a_1} & \frac{\partial^2 \pi}{\partial a_1 \partial a_2}
\end{bmatrix}
\begin{bmatrix}
da_1 \\
da_2 
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial^2 \pi}{\partial p_E \partial a_i} \\
\frac{\partial^2 \pi}{\partial p_E \partial a_2}
\end{bmatrix}
\]

Using Cramer’s rule and (27):

\[
\begin{vmatrix}
\frac{\partial \pi}{\partial p_E \partial a_i} & \frac{\partial \pi}{\partial a_2 \partial a_i} \\
\frac{\partial \pi}{\partial a_1 \partial a_i} & \frac{\partial \pi}{\partial a_i \partial a_2}
\end{vmatrix}
da_1
\]

\[
\begin{vmatrix}
\frac{\partial^2 \pi}{\partial a_i^2} & \frac{\partial^2 \pi}{\partial a_i \partial a_1} & \frac{\partial^2 \pi}{\partial a_2 \partial a_1} & \frac{\partial^2 \pi}{\partial a_1 \partial a_2} \\
\frac{\partial^2 \pi}{\partial a_2 \partial a_1} & \frac{\partial^2 \pi}{\partial a_1 \partial a_2}
\end{vmatrix}
da_2
\] (28)

Thus,

\[
\frac{da}{dp_E} = 2 \frac{da_1}{dp_E} = 2 \frac{da_2}{dp_E} < 0
\] (29)
Thus, an increase in doubt reduces religious investment in both periods by an equal amount (provided the nonnegativity constraints are not binding). Since the beginning of the section established the likelihood of $a_1 < a_2$, those most doubtful of the existence of an afterlife are most likely to only invest in religious capital in period 2.

Empirical evidence supports this predicted positive relationship between a belief in an afterlife and religious investment. Ulbrich and Wallace (1983) and Brañas-Garza, García-Muñoz and Neuman (2008) find church attendance positively related to a belief in an afterlife. Brañas-Garza et al also find that the frequency of prayer is positively related to a belief in an afterlife.

4 Education and Religion

The results of section 3 imply that it is unsurprising that the empirical evidence indicates a positive relationship between education and church attendance despite a negative relationship between education and religious beliefs. Educated individuals tend to have lower discount rates (Harrison, Lau and Williams, 2002). Thus, as (19) implies, it is not surprising that they are more likely to attend church and fulfill other religious obligations. The negative relationship between education and religious beliefs would work through (28).

5 Fear of Death

Section 5.1 will examine the fear of death for those who make religious investments on the assumption they will live for two periods. Section 5.2 will examine the fear of death for individuals who make the same plans but unexpectedly find they will die at end of period 1. Section 5.3 will consider the case of individuals who have made a choice between religions that offer exclusive access to a desirable afterlife.
5.1 Fear of death in second period

This paper defines fear as an expected drop in utility at death. Given that $U(E) = 0$, an individual should fear death in periods 1 and 2 if inequalities (30) and (31) hold:

\begin{align*}
U(L_1 - a_1) &> \delta^2 p_B(a_1 + a_2) U(B) + \delta^2 p_D(a_1 + a_2) U(D) \\
U(L_2 - a_2) &> \delta p_B(a_1 + a_2) U(B) + \delta p_D(a_1 + a_2) U(D)
\end{align*}

(30)  (31)

Given that for a pure atheist, $p_E = 1$ and $p_B(a_1 + a_2) = p_D(a_1 + a_2) = a_1 = a_2 = 0$, an atheist should fear death in period $i$ if $U(L_i) > 0$. In other words, only atheists with negative payoffs from living will not fear death.

The case of those who believe $p_E < 1$, is more interesting. Individuals who place a positive but low probability on the existence of an afterlife will have low investments in religious capital. Therefore, for these individuals, $p_B$ will be low relative to $p_D$ and it is likely that $|p_B(a_1 + a_2) U(B)| < |p_D(a_1 + a_2) U(D)|$. For given values of $L_i$, these individuals will fear death more than atheists will. As $p_E$ increases, more is invested in religious capital. It would not be surprising if at some point $|p_D(a_1 + a_2) U(B)| > |p_B(a_1 + a_2) U(D)|$.

An individual with a very high expectation of an afterlife may even look forward to death. Individuals with a low $L_i$ are more likely to look forward to death.

More formally, in period 2, the drop in expected utility from death is given by the following value function:

\begin{align*}
F(p_E) &= U(L_2 - a^*(p_E)) - \delta p_B(a^*(p_E)) U(B) - p_E U(E) - \delta p_D(a^*(p_E)) U(D) \\
&= U(L_2 - a^2(p_E)) - \delta p_B(a^*(p_E)) U(B) - \delta [1 - p_E - p_B(a^*(p_E))] U(D)
\end{align*}

(32)
At first, this mathematical representation of fear appears to be new to the economics literature. However, equation (32) can also be looked at as simply the equivalent variation of the change from life to death.\(^5\)

For an atheist, \(p_E=1\) and \(a_2 = p_B(0) = p_D(0) = 0\). Thus,

\[
F(p_E=1) = U(L_2). 
\]  

(33)

An atheist should fear death when \(L_2\) is positive. The only time an atheist should not fear death is when \(L_2 \leq 0\).

Now consider an individual for whom \(p_E < 1\) but \(\mu_1 > 0\) and \(\mu_2 > 0\) (the nonnegativity constraints are binding). In this case, \(a_1 = a_2 = p_B(0) = 0\) but \(p_D(0) = 1 - p_E\). Thus,

\[
F(p_E < 1) = U(L_2) - \delta (1 - p_E)U(D) > V(p_E = 1)
\]  

(34)

The effect of the lower value of \(p_E\) is to decrease the probability of a zero payoff at death and increase the chance of a negative payoff from hell. Thus, such a moderately religious person will fear death more than an atheist. Moreover, as \(p_E\) decreases (but not enough for \(\mu_1 = \mu_2 = 0\)), fear of death increases:

\[
\frac{dF(p_E)}{dp_E} = U(D) < 0, \text{ for } p_E < 1, \mu_1 \neq 0, \mu_2 \neq 0
\]  

(35)

Thus, when \(p_E\) is low enough for the nonnegativity constraints to be binding, fear of death increases as \(p_E\) falls.

Differentiating equation (32) gives the change in the fear of death as \(p_E\) changes for individuals for whom the nonnegativity constraint is not binding:

\[
\frac{dF}{dp_E}(p_E) = -\frac{\partial U}{\partial L} \left( L_2 - a_2^*(p_E) \right) \frac{da_2}{dp_E} - \delta \frac{\partial p_B(a^*(p_E))}{\partial a} \frac{da}{dp_E} \left[ U(B) - U(D) \right] + \delta U(D)
\]  

(36)

\(^5\) Daniel Hungerman made the author aware of this point. As he pointed out, \(F(p_E)\) is the amount a person would be willing to give up to avoid death.
Using equations (7) and (29):

\[
\frac{dF}{dp_E}(p_E) = -3 \frac{dU}{dL}(L_2 - a^*_2(p_E)) \frac{da_2}{dp_E} + \delta U(D) \tag{37}
\]

The first term on the right hand side is positive and the second term (\(\delta U(D)\)) is negative. As \(p_E\) increases, \(a_2\) decreases. Given our assumption of risk aversion, this will reduce the magnitude of the first term. Thus, for individuals with a high \(p_E\), increases in \(p_E\) are likely to reduce their fear of death as the \(\delta U(D)\) term dominates equation (37). However, it is possible that individuals with a low \(p_E\) will find their fear of death decrease as \(p_E\) decreases. This is likely when their religious investment reduces their utility during life enough to cause a high enough marginal utility for the first term to outweigh the second term in equation (37).

-Figure 1 illustrates this result. At \(p_E = 1\), the individual’s fear of death is equal to \(U(L_2)\): The drop in utility at death for an atheist who makes no religious investments. Here, it is assumed that \(U(L_2) > 0\). As \(p_E\) decreases, an individual’s fear of death initially increases. At some point it may decrease. It may eventually decrease enough for a very religious person to experience a lower fear of death than an atheist. This is the case illustrated here. It is even possible that the individual’s fear of death could decrease to the point of being negative (not illustrated here).

5.2 Early Death

Several (but not all) studies have found that fear of death decreases with age (Tang, Wu and Kwok, 2002; Tang, Wu and Yan, 2002; Cicirelli, 1998; Rasmussen and Brems,
In this case, a rational individual may have been postponing his investment in religious capital until later in life. Equations (30) and (31) can be rewritten to reflect this situation:

\[ U(L_1 - a_1) > \delta p_B(a_1) U(B) + \delta p_D(a_1) U(D) \]  
\[ U(L_2 - a_2) > \delta p_B(a_1 + a_2) U(B) + \delta p_D(a_1 + a_2) U(D) \] 

Assume \( L_1 = L_2 \). Even if an individual assumes it is his average investment per time period during life that effects the probability of going to heaven, the fact that he was rationally postponing his major investment to the second period, would result in a lower \( p_B \) than if he had lived for another period.

This explanation provides a testable prediction of the model. Atheists do not face the same problem of an unexpected early death being inconsistent with their investment in religious capital. Thus, if the explanation of this paper is correct, one should not observe the same intertemporal difference in the fear of death with atheists.

5.3 Wrong religion

The preceding analysis has assumed that when investing in religious capital, only one religion is available. However, in reality, many religions are available. To make matters more difficult, the beliefs of each often don’t allow an individual to diversify his risk by making investments in the religious capital of other religions. Thus, there is a positive probability that the individual is investing in the wrong religion. A perceptive individual should recognize this possibility.

Assume that the individual has already made his choice of religion. There is some subjective probability \( p_C \) that he has selected the correct religion. Lacking

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6 This possibility could be more formally modelled by incorporating an exogenous probability \( q \) of the individual dying after period 1 and going straight to his afterlife. However, this would significantly complicate the model and for small \( q \) would not significantly affect the results.
empirical evidence on which religion is correct, assume this probability is exogenous.

Consequently, the probability that he has selected the wrong religion is \( 1 - p_C \). If he has selected the wrong religion, and an afterlife exists, his payoff is \( U(D) \), regardless of religions contributions. For simplicity, it will be assumed that the probability functions \( p_B(a) \) and \( p_D(a) \) are identical for different religions. Thus, his payoff function is

\[
\pi = U(L_1-a_1) + \delta U(L_2-a_2) + p_C \delta^2 p_B(a_1+a_2) U(B) + \delta^2 p_E U(E) + p_C \delta^2 p_D(a_1+a_2) U(D) \\
+ (1-p_C)(1-p_E) \delta^2 U(D)
\]  

(40)

The first order conditions are now

\[
p_C \delta^2 \frac{dp_B}{da}(a_1+a_2)[U(B)-U(D)] = \frac{dU}{dL_1}(L_1-a_1) 
\]  

(41)

\[
p_C \delta^2 \frac{dp_B}{da}(a_1+a_2)[U(B)-U(D)] = \delta \frac{dU}{dL_2}(L_2-a_2)
\]  

(42)

Thus, the marginal benefits of religious investments have been reduced. Qualitatively, the effect is the same as a reduction in the discount factor. Therefore, the effect is to reduce investments in religious capital.

The effect of \( p_C \) on the fear of death can be seen by rewriting equations (30) and (31) to take into account this possibility:

\[
U(L_1-a_1) > p_C \delta^2 [p_B(a_1+a_2)U(B) + p_D(a_1+a_2)U(D)] + (1-p_C)(p_B+p_D)\delta^2 U(D) 
\]  

(43)

\[
U(L_2-a_2) > p_C \delta [p_B(a_1+a_2)U(B) + p_D(a_1+a_2)U(D)] + (1-p_C)(p_B+p_D)p_C \delta U(D)
\]  

(44)

One direct effect of \( p_C \) is to increase the expected drop in utility at death by introducing an additional reason why religious investments may be pointless. This is because the expected benefit of investing in the religion selected may be zero even if an afterlife exists. A second direct effect is introducing a probability of going to hell that is independent of religious investments. A probability of choosing the wrong religion also
has indirect effects. One is that \( p_c \) decreases \( a_1 \). This increases utility during life. It also decreases the probability of going to heaven and consequently the expected utility of the afterlife. Both indirect effects also increase the fear of death.

This raises a complication for supply side considerations of religious participation. Typically, papers such as Iannaccone, Finke and Stark (1997) imply that competition between religions should increase religious participation. A full discussion of the empirical evidence is beyond the scope of this paper. However, the analysis of this section does imply that the relationship may not be as simplistic as is often argued. If the religious competition is between churches that don’t claim to provide exclusive access to heaven, the theory is reasonable. This would likely be the case for many protestant denominations in rural parts of the United States. It would also be true of those who believe there is no right religion and everyone might go to hell. However, in cases such as the Catholic and Islamic faiths, this is unlikely to be the case. Indeed, those in multicultural urban centers may find that a close exposure to people of such differing faiths reminds them that \( p_c \) is unlikely to be equal to 1, reducing their investments in religious capital. Moreover, individuals in such centres may fear death more than those in less religiously diverse populations.

6 Discussion and Conclusion

Several empirical papers have found a quasilinear relationship between religiosity and the fear of death. This paper has shown that such a relationship is consistent with rational choice theory. It did so by assuming that individuals place different subjective probabilities on the existence of an afterlife. Fear of death is defined as an expected drop in utility at death. Those who believe that the probability of death being the end of
existence is less than 1 can invest in religious capital to increase the subjective probability of going to heaven rather than hell. Heaven is assumed to give positive utility, hell negative utility and nonexistence, zero utility. Atheists have no incentive to invest in religious capital and fear death to the extent that life gives them positive utility. Those who place a small probability on the existence of an afterlife, rationally invest little in religious capital. This results in a large subjective probability of going to hell rather than heaven. Consequently, they fear death more than pure atheists who only have to worry about the zero utility of nonexistence. Individuals who place a higher probability on the existence of an afterlife will rationally invest more in religious capital. This increases the subjective probability of going to heaven rather than hell. This may result in a lower fear of death than less religious individuals and perhaps even lower than atheists. However, depending on the functional forms involved, this framework is also consistent with research that finds a positive relationship between the fear of death and religiosity.

This paper also examined other implications of the model. For example, it shows that a positive relationship between education and religious behaviour is not inconsistent with a negative relationship between education and religious beliefs. The model also predicts a negative relationship between the fear of death and age. Moreover, it examines the effects of different discount factors. It also finds that in certain circumstances, religious competition may reduce, rather than increase, religious participation.

A potential criticism of this model are the assumptions regarding $p_B(a)$. Specifically, the assumptions that $p_B(0) = 0$ and that the probability of heaven is positive function of investments in religious activities for any $p_E$ less than 1. This ignores
religious beliefs of salvation by faith alone. Such beliefs would seem to imply that those with $p_E$ above a given level, may not have enough faith to get into heaven, regardless of religious contributions. However, this would simply reinforce the result that moderately religious individuals have the most to fear from death. A less straightforward issue would be the way this might effect religious investments of the most religious individuals. To the extent they believe their low $p_E$ increases their probability of going to heaven, it might reduce their need to make religious contributions. However, even if this is true, it seems reasonable to assume that it should not reduce their investment enough to result in a lower ex-anti afterlife payoff than the less religious.

An interesting extension of this research would be to examine the implications for suicide bombers and others considering martyrdom. Their revealed preferences indicate that they feel an early death increases their expected utility. A detailed analysis is beyond the immediate scope of this paper. However, such an extension of this research could produce useful policy prescriptions.

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7 The author is grateful to Ronald Horst for making him aware of this view of salvation.
References


Figure 1: Period 2 fear of death as a function of the subjective probability of no afterlife

$F(P_E)$
Fear of Death

$U(L_2)$

$F(p_E)$

$p_E$
Probability of nonexistence