International Association for the Evaluation of Educational Achievement

Teacher Education Study in Mathematics (TEDS-M) 2008

Released Items

Future Teacher Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge - Secondary

Prepared by:

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- **Revised July 2009**

TEDS-M Secondary Level Released Items

The goal for selecting the released set of test items was to have approximately 25% of each of the full item sets for Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) that would represent the full range of difficulty, content, and item format.

The initial step in the selection was to take a stratified random sample of the items, stratifying on both proportion correct and sub-domains of MCK and MPCK. The next step was to consider if the selected items were part of item sets. If they were, an attempt was made to use the full set so that the full context of the items would be made public. Representation of the Anchor Points was another important consideration as was balance of item formats (MC, CR, CMC). Ownership of the items was also considered. The test items were then reviewed to determine if they efficiently and accurately represented the full item set.

The set of secondary released items consists of:

- 24 MCK items (7 from the Algebra sub-domain, 9 from Geometry, 8 from Number and 1 from Data) including samples of the Cognitive sub-domains of Knowing (6), Applying (13) and Reasoning (6); and

- 9 MPCK items (5 from the Algebra domain, 0 from Geometry, 3 from Number, and 1 from Data) illustrating the two sub-domains of Curriculum/Planning (4) and Enacting (5).

Main Study ID	Outcome	Content Domain	Sub-domain	Label	Item Format	Key	Max Points
MFC604A1	MCK	Algebra	Applying	Solve a word problem with underlying linear relations	CR	SG[1]	1
MFC604A2	MCK	Algebra	Applying	Solve a word problems with underlying linear relations	CR	SG	1
MFC604B	MPCK	Algebra	Enacting	Analyze why one word problem is more difficult than another.	CR	SG	1
MFC610A	MCK	Number	Knowing	Determine whether a given concept or procedure results in an irrational number.	CMC	1	1
MFC610C	MCK	Number	Knowing	Determine whether a given concept or procedure results in an irrational number.	CMC	1	1
MFC610D	MCK	Number	Knowing	Determine whether a given concept or procedure results in an irrational number.	CMC	3	1
MFC704	MCK	Geometry	Applying	Determine lengths of segments in a figure.	CR	SG	2
MFC705A	MCK	Geometry	Knowing	Determine correct representation for a solution to an equation in a plane	CMC	2	1
MFC705B	MCK	Geometry	Knowing	Determine correct representation for a solution to an equation in space.	CMC	3	1
MFC709A	MPCK	Number	Enacting	Determine whether students' responses are valid proofs.	CMC	1	1
MFC709B	MPCK	Number	Enacting	Determine whether students' responses are valid proofs.	CMC	2	1
MFC709C	MPCK	Number	Enacting	Determine whether students' responses are valid proofs.	CMC	2	1
MFC710A	MCK	Algebra	Applying	Determine whether a situation can be modeled by an exponential function.	CMC	2	1
MFC710B	MCK	Algebra	Applying	Determine whether a situation can be modeled by an exponential function.	CMC	2	1
MFC710C	MCK	Algebra	Applying	Determine whether a situation can be modeled by an exponential function.	CMC	1	1
MFC711	MCK	Algebra	Reasoning	Write a proof about the sum of two functions.	CR	SG	2
MFC712A	MPCK	Algebra	Curriculum & Planning	Determine types of knowledge needed to prove the quadratic formula.	CMC	1	1
MFC712B	MPCK	Algebra	Curriculum & Planning	Determine types of knowledge needed to prove the quadratic formula.	CMC	1	1

[1] SG – See Scoring Guide

Main Study ID	Outcome	Content Domain	Sub-domain	Label	Item Format	Key	Max Points
MFC712C	MPCK	Algebra	Curriculum & Planning	Determine types of knowledge needed to prove the quadratic formula.	CMC	1	1
MFC712D	MPCK	Algebra	Curriculum & Planning	Determine types of knowledge needed to prove the quadratic formula.	CMC	2	1
MFC802A	MCK	Number	Reasoning	Decide whether argument is a proof of a statement about the quotient of the square of natural number and 3.	CMC	2	1
MFC802B	MCK	Number	Reasoning	Decide whether argument is a proof of a statement about the quotient of the square of natural number and 3.	CMC	1	1
MFC802C	MCK	Number	Reasoning	Decide whether argument is a proof of a statement about the quotient of the square of natural number and 3.	CMC	2	1
MFC802D	MCK	Number	Reasoning	Decide whether argument is a proof of a statement about the quotient of the square of natural number and 3.	CMC	2	1
MFC804	MCK	Number	Knowing	Combinatorics – Find number of ways to choose 2 students from 10, and 8 students from 10.	MC	3	1
MFC806A	MCK	Data	Applying	Determine whether student's interpretation of histogram is right or wrong.	MC	2	1
MFC806B	MPCK	Data	Enacting	Explain student's thinking about histogram.	CR	SG	1
MFC808A1	MCK	Geometry	Applying	Correct student's answer about lines of symmetry in a regular hexagon.	CMC	1	1
MFC808A2	MCK	Geometry	Applying	Correct student's answer about lines of symmetry in a regular hexagon.	CMC	2	1
MFC808B1	MCK	Geometry	Applying	Correct student's answer about lines of symmetry in a regular pentagon.	CMC	1	1
MFC808B2	MCK	Geometry	Applying	Correct student's answer about lines of symmetry in a regular pentagon.	CMC	2	1
MFC808C1	MCK	Geometry	Applying	Correct student's answer about lines of symmetry in a rhombus.	CMC	2	1
MFC808C2	MCK	Geometry	Applying	Correct student's answer about lines of symmetry in a rhombus.	CMC	1	1
MFC814	MCK	Algebra	Reasoning	Determine whether a statement about an operation with matrices is correct, and justify response.	CR	SG	2

ID: MFC604A1 MFC604A2	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CR	Max Points: 2
Outcome: MCK	Domain: Algebra		Sub-domain: Applying	

The following problems appear in a mathematics textbook for <lower secondary school>.

1. [Peter], [David], and [James] play a game with marbles. They have 198 marbles altogether. [Peter] has 6 times as many marbles as [David], and [James] has 2 times as many marbles as [David]. How many marbles does each boy have?

2. Three children [Wendy], [Joyce] and [Gabriela] have 198 zeds altogether. [Wendy] has 6 times as much money as [Joyce], and 3 times as much as [Gabriela]. How many zeds does each child have?

(a) Solve each problem.

MFC604A1

Solution to Problem 1:

MFC604A2

Solution to Problem 2:

Note: The correct answers to MFC604A1 and MFC604A2 follow:

Problem 1: David has 22 marbles, Peter has 132 marbles, and James has 44.

Problem 2: Wendy has 132 zeds, Joyce has 22 zeds, and Gabriela has 44 zeds.

The following methods are considered in the scoring guide:

1) Using *one variable*, setting up *one equation* and solving.
   Example (Problem 1): Let m = the number of marbles that David has. Then Peter has $6m$ and James has $2m$. Therefore, $6m + 2m + m = 198$, and $m = 22$.
2) Using *more than one variable*, establishing a *system of equations*, performing substitutions, and solving.
   Example (Problem 1): Let p = the number of marbles that Peter has, d = the number of marbles that David has, and j = the number of marbles that James has $p = 6d$ and $j = 2d$, $p + d + j = 198$.
3) Trial and error or guess and check
4) Ratio or other arithmetic methods
5) Representation/diagram

Code:	**Response**	**Item ID: MFC604A1**
11	Response uses Method 1 correctly to solve Problem 1 and get the correct answers.	
12	Response uses Method 2 correctly to solve Problem 1 and get the correct answers.	
13	Response uses Method 3 correctly to solve Problem 1 and get the correct answers.	
14	Response uses Method 4 correctly to solve Problem 1 and get the correct answers.	
15	Response uses Method 5 correctly to solve Problem 1 and get the correct answers and get the correct answers.	
19	Response uses a valid but different method from the list above to solve Problem 1 and get the correct answers.	
	Incorrect Response	
70	Response uses one of Methods 1 - 5 to start Problem 1, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
71	Response uses a correct but different method from the list above to solve Problem 1, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

Code	Response	Item ID: MFC604A2
	Correct Response	
11	Response uses Method 1 to solve Problem 2.	
12	Response uses Method 2 to solve Problem 2.	
13	Response uses Method 3 to solve Problem 2.	
14	Response uses Method 4 to solve Problem 2.	
15	Response uses Method 5 to solve Problem 2.	
19	Responses use a correct but different method from the list above to solve Problem 2 and get the correct answers.	
	Incorrect Response	
70	Response uses one of Methods 1 - 5 to start Problem 2, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
71	Response uses a correct but different method from this list to solve Problem 2, but arrives at an incorrect answer or cannot complete the solution because of a computation or algebra error.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC604B	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CR	Max Points: 1
Outcome: MPCK	Domain: Algebra		Sub-domain: Enacting	

MFC604B

(b) Typically Problem 2 is more difficult than Problem 1 for <lower secondary> students. Give one reason that might account for the difference in difficulty level.

Code	Response	Item ID: MFC604B
	Correct Response	
10	Reason clearly expresses a difference in the mathematical or cognitive complexity of the two problems. *Examples:* *1) In Problem 1 it is easier (in comparison to Problem 2) to choose the base variable, and see the relations between the variables. In Problem 1, the number of marbles that both Peter and James have is in direct relationship to the number of marbles that David has. However, in Problem 2, the relation between the number of zeds that Joyce and Gabriela have is not directly stated.* *2) Problem 2 is phrased in such a way that the respondent seems more likely to use fractional equations than whole number equations. Fractional equations can be more challenging to solve, making calculations more prone to error.*	
	Incorrect Response	
79	Incorrect reason (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC610A MFC610C MFC610D	MS Booklet: SM1, SM3	MS Block: B1SM	Item Format: CMC	Max Points: 3
Outcome: MCK	Domain: Number		Sub-domain: Knowing	

Determine whether each of the following is an irrational number always, sometimes or never.

Check one box in each row.

			Always	Sometimes	Never
MFC610A	A.	The result of dividing the circumference of a circle by its diameter.	☐ 1	☐ 2	☐ 3
MFC610C	C.	The diagonal of a square with side of length 1.	☐ 1	☐ 2	☐ 3
MFC610D	D.	Result of dividing 22 by 7.	☐ 1	☐ 2	☐ 3

ID: MFC704	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Outcome: MCK	Domain: Geometry		Sub-domain: Applying	

On the figure, *ABCD* is a parallelogram, [ ], *AM* and *BM* are angle bisectors of angles *BAD* and *ABC* respectively. If the perimeter of *ABCD* is 6 cm, find the sides of triangle *ABM*.

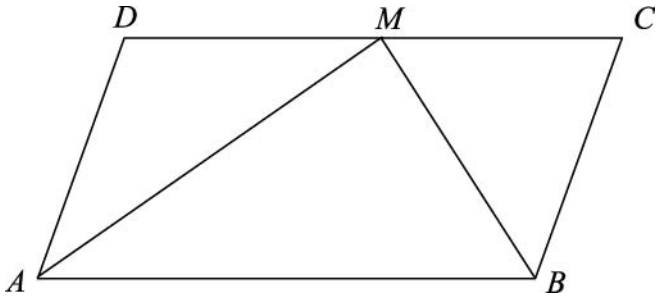
Write your answers on the lines below.

MFC704

AB = _____cm

AM = _____cm

BM = _____cm



Code	Response	Item ID: MFC704
	Correct Response	
20	Responses that indicate all three correct entries below: *AB* = 2 cm *AM* = √3 cm or equivalent *BM* = 1 cm	
	Partially Correct Response	
10	Any two entries correct and one incorrect (or blank).	
11	Any one entry correct and two incorrect (or blank).	
	Incorrect Response	
79	Incorrect mathematical statements or statement of no value (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	

ID: MFC705A MFC705B	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 2
Outcome: MCK	Domain: Geometry		Sub-domain: Knowing	

We know that there is only one point on the real line that satisfies the equation [ ], namely [ ].

Suppose now that we consider this same equation in the plane, with coordinates x and y, and then in space with coordinates x, y, and z. What does the set of points that satisfy the equation [ ] look like in these settings?

Check one box in each row.

			One point	One line	One plane	Other
MFC705A	A.	The solution to [ ] in the plane	☐ 1	☐ 2	☐ 3	☐ 4
MFC705B	B.	The solution to [ ] in space	☐ 1	☐ 2	☐ 3	☐ 4

ID: MFC709A MFC709B MFC709C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Outcome: MPCK	Domain: Number		Sub-domain: Enacting	

Some <lower secondary school> students were asked to prove the following statement:

When you multiply 3 consecutive natural numbers, the product is a multiple of 6.

Below are three responses.

[Kate's] answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3.

Also, at least one number will be even and all even numbers are multiples of 2.

If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

[Leon's] answer

[Maria's] answer

n is any whole number

$$n \times (n+1) \times (n+2) = (n^2 + n) \times (n+2)$$

$$= n^3 + n^2 + 2n^2 + 2n$$

Canceling the *n*'s gives $1 + 1 + 2 + 2 = 6$

Determine whether each proof is valid.

Check one box in each row.

			Valid	Not valid
MFC709A	A.	[Kate's] proof	☐ 1	☐ 2
MFC709B	B.	[Leon's] proof	☐ 1	☐ 2
MFC709C	C.	[Maria's] proof	☐ 1	☐ 2

ID: MFC710A MFC710B MFC710C	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 3
Outcome: MCK	Domain: Algebra		Sub-domain: Applying	

Indicate whether each of the following situations can be modeled by an exponential function.

Check one box in each row.

			Yes	No
MFC710A	A.	The height h of a ball t seconds after it is thrown into the air.	☐ 1	☐ 2
MFC710B	B.	The amount of money A in a bank after w weeks, if each week d zeds are put in the bank.	☐ 1	☐ 2
MFC710C	C.	The value V of a car after t years if it depreciates d % per year.	☐ 1	☐ 2

ID: MFC711	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CR	Max Points: 2
Outcome: MCK	Domain: Algebra		Sub-domain: Reasoning	

Prove the following statement:

If the graphs of linear functions

[ ] and [ ]

intersect at a point *P* on the *x*-axis, the graph of their sum function

[ ]

MFC711 must also go through *P*.

Code	Response	Item ID: MFC711
	Correct Response	
20	Response carefully lays out the steps of the proof in a general way, without using the given formulas of $f(x)$ and $g(x)$. *Example: Suppose f(x) and g(x) intersect at point (p, 0) on the x-axis.* *Then f(p) = 0, g(p) = 0. Then (f + g)(p) = f(p) + g(p) = 0 + 0 = 0.* *Therefore f+g also goes across point (p, 0).*	
21	Response has carefully laid out the steps of the proof using the given formulas of $f(x)$ and $g(x)$. *Example: Suppose f(x) and g(x) intersect at point (p, 0) on the x-axis, then the following inferences can be made:* *(1) f(p) = 0 → ap + b = 0 → p = –b/a;* *(2) g(p) = 0 → cp + d = 0 → p = –d/c;* *(3) f(p) = g(p) → b/a = d/c → ad = bc;* *(4) f(p) = g(p) → ap + b = cp + d → p = –(b + d)/(a + c);* *Since (f + g)(p) = f(p) + g(p), together with two or more of the above inferences, one can show that (f + g) (p) = 0. Therefore (f + g)(x) also goes across point (p, 0).*	
22	Response has carefully laid out the steps of the proof using a graphical argument. *Example: A graph of two lines intersecting on the x-axis is shown. Suppose f(x) and g(x) intersect at point (p, 0) on the x-axis. The value of (f + g)(x) is the sum of f(x) and g(x) for each x. But at x = p, 0 + 0 = 0, so f + g also goes through the point (p, 0).*	
	Partially Correct Response	
10	Response shows evidence of a chain of reasoning about general functions without using the given formulas of $f(x)$ and $g(x)$, but some mistake is made or the response stops before the proof is complete. *Example: Understands f(p) = 0, g(p) = 0, and (f + g)(p) = f(p) + g(p), but doesn't arrive at the fact that (f + g)(p) = 0 and/or the conclusion that (f + g)(x) also goes through (p, 0).*	
11	Response shows evidence of a chain of reasoning using the given formulas of $f(x)$ and $g(x)$, but some mistake is made or the response stops before the proof is complete. *Example: Makes one or more of inferences (1) – (4) under code 21, also states that (f + g)(x) = f(x) + g(x) = (a + c)x + (b + d), even is able to show (f + g)(p) = 0, but there is major flaw in logical reasoning.*	
12	Response shows evidence of a chain of reasoning about general functions using an intuitive/graphical proof, but some mistake is made or the response stops before the proof is complete. *Example: Response is able to show graphically that f(x) and g(x) go through the same point on x-axis, also points out the meaning of the sum function, but isn't able to conclude that the sum function goes through the same point.*	
	Incorrect Response	
79	Incorrect mathematical statement or other incorrect work (including crossed out, erased, stray marks, illegible, or off task)	
	No Response	
99	Blank	

ID: MFC712A MFC712B MFC712C MFC712D	MS Booklet: SM1, SM2	MS Block: B2SM	Item Format: CMC	Max Points: 4
Outcome: MPCK	Domain: Algebra		Sub-domain: Planning	

A mathematics teacher wants to show some students how to prove the quadratic formula.

Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

Check one box in each row.

			Needed	Not needed
MFC712A	A.	How to solve linear equations.	☐ 1	☐ 2
MFC712B	B.	How to solve equations of the form [ ], where [ ].	☐ 1	☐ 2
MFC712C	C.	How to complete the square of a trinomial.	☐ 1	☐ 2
MFC712D	D.	How to add and subtract complex numbers.	☐ 1	☐ 2

ID: MFC802A MFC802B MFC802C MFC802D	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 4
Outcome: MCK	Domain: Number		Sub-domain: Reasoning	

You have to prove the following statement:

If the square of any natural number is divided by 3, then the remainder is only 0 or 1.

State whether each of the following approaches is a mathematically correct proof.

Check one box in each row.

		Yes	No
MFC802A	A. Use the following table:	☐ 1	☐ 2
MFC802B	B. Demonstrate that ☐ is divisible by 3 and for all other numbers, ☐ which always has a remainder of 1 once it has been divided by 3.	☐ 1	☐ 2
MFC802C	C. Choose a natural number *n*, find its square ☐, and then check whether the statement is true or not.	☐ 1	☐ 2
MFC802D	D. Check the statement for the first several prime numbers and then draw a conclusion based on the Fundamental Theorem of Arithmetic.	☐ 1	☐ 2

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100
Remainder when divided by 3	1	1	0	1	1	0	1	1	0	1

ID: MFC804	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Outcome: MCK	Domain: Number		Sub-domain: Knowing	

MFC804

A class has 10 students. If at one time, 2 students are to be chosen, and another time 8 students are to be chosen from the class, which of the following statements is true?

Check one box.

A. There are more ways to choose 2 students than 8 students from the class. ☐1

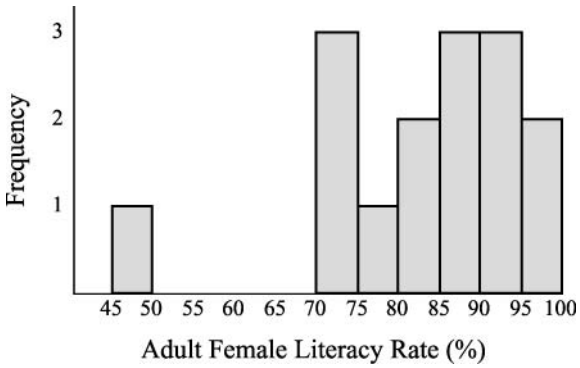
B. There are more ways to choose 8 students than 2 students from the class. ☐2

C. The number of ways to choose 2 students equals the number of ways to choose 8 students. ☐3

D. It is not possible to determine which selection has more possibilities. ☐4

ID: MFC806A	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: MC	Max Points: 1
Outcome: MCK	Domain: Data		Sub-domain: Applying	

The following graph gives information about the adult female literacy rates in Central and South American countries.



Suppose you ask your students to tell you how many countries are represented in the graph. One student says, "There are 7 countries represented."

Check one box.

	Right	Wrong
MFC806A a) Is the student right or wrong?	☐ 1	☐ 2

ID: MFC806B	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 1
Outcome: MPCK	Domain: Data		Sub-domain: Enacting	

MFC806B

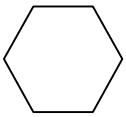
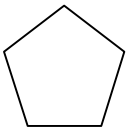
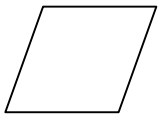
b) In your opinion, what was the student thinking in order to arrive at that conclusion?

Code	Response	Item: MFC806B
	Correct Response	
10	Response indicates that the student thought that each bar represented one country. *Example:* *The student counted the number of bars, and concluded that the answer (7) represented the number of countries.*	
	Incorrect Response	
79	Incorrect response (including crossed out, erased, stray marks, illegible, or off task).	
	No response	
99	Blank	

ID: MFC808A1 & 2 MFC808B1 & 2 MFC808C1 & 2	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CMC	Max Points: 6
Outcome: MCK	Domain: Geometry		Sub-domain: Applying	

Your students have been working on symmetry. They were given the task below requiring them to decide the number of lines of symmetry for three different shapes.

Answers of [Sam] and [Michael] are shown in the table. Correct the answers of each student by checking correct or incorrect.

			Students and their answers about the number of the lines of symmetry	
	Shape	**Shape name**	**[Sam]**	**[Michael]**
MFC808A1 MFC808A2		regular hexagon	6 ☐1 Correct ☐2 Incorrect	12 ☐1 Correct ☐2 Incorrect
MFC808B1 MFC808B2		regular pentagon	5 ☐1 Correct ☐2 Incorrect	10 ☐1 Correct ☐2 Incorrect
MFC808C1 MFC808C2		rhombus	4 ☐1 Correct ☐2 Incorrect	2 ☐1 Correct ☐2 Incorrect

ID: MFC814	MS Booklet: SM2, SM3	MS Block: B3SM	Item Format: CR	Max Points: 2
Outcome: MCK	Domain: Algebra		Sub-domain: Reasoning	

Let $\boxed{}$ and $\boxed{}$. Then $A \otimes B$ is defined to be $\boxed{}$.

MFC814

Is it true that if $A \otimes B = \mathrm{O}$, then either $\boxed{}$ or $\boxed{}$ (where $\boxed{}$ represents the zero matrix)? Justify your answer.

Code	Response	Item ID: MFC814
	Correct Response	
20	Response indicates that the statement is false (or not necessarily true) and provides a correct (and specific) counterexample. *Example: No, it is not true. If A =* $\boxed{}$ *and B =* $\boxed{}$*, then* $A \otimes B =$ $\boxed{}$.	
21	Response indicates that the statement is false (or not necessarily true), and provides a ***general*** description of a counterexample using words. *Example: Let's assume that all elements in the first column of the matrix A is 0, and all elements in the second column of the matrix B is 0. When we apply the operation defined in the question to matrix A and matrix B, we get the 0 matrix at the end.* **Note:** As indicated in the example above, even though the response does not indicate that the second column of matrix *A* and the first column of matrix *B* must have non-zero entries, we code such solutions as correct.	
29	Other correct responses.	
	Partially Correct Response	
10	Response indicates that the statement is false (or not necessarily true), and provides a counterexample that is *not sufficiently* described.	
	Incorrect Response	
70	Response indicates that the statement is false or (not necessarily true), but provides no justification or a justification that is incorrect or irrelevant.	
71	Response indicates that the statement is true.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task).	
	No Response	
99	Blank	