

An electoral system for a Parliament: Mirror and Governability

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0. Abstract

We show a procedure for determining the total number of representatives for each political party in proportional representation getting high proportionality and governability. This procedure is very flexible for the size of the Parliament, the number of electoral constituencies and his size. The electoral system is based on the allocation of seats at several levels. Firstly, at least, one half of the seats are allocated in the current constituencies. Afterwards, total votes of the political parties are considered in order to allot the remaining seats in two steps. A small (or even zero) continuous threshold is applied to allocate seats according to the total number of votes in the first step; and in the second one, a convex and increase adjustment function is used to transform the quotas. An application to the Congress of Spain and to the Camera of Italy is shown.

1. Introduction

In electoral systems for proportional representation, the constituency sizes, thresholds, electoral formulae and Parliament size all play important roles.

Sometimes allotments are made separately in each constituency. Therefore, it is possible that total votes to the political parties and total seats do not follow the same order. Any one of the nine elections to Spain's Congress of Deputies (celebrated between 1977 and 2004) can be used to exemplify this contradiction.

When the constituencies are small, the third and the next political parties do not obtain representation. Their votes are lost and, therefore, low proportionality is obtained.

When the constituencies are large, or in the extreme case when there is only one constituency, high proportionality is obtained, but because a large number of political parties can obtain representation in the Parliament, it may prove difficult to govern. In this case, so as to avoid impediments to governability, a bonus (of seats) for the winning party or coalition of parties can be established. The 2006 Italian Electoral System to the Camera offers one example of such a situation.

This paper presents a model of an electoral system based on the allotment of seats to the political parties in several steps, providing for high proportionality as well as

governability. Firstly we take historical constituencies, and then one half (or more) of the total seats of the Parliament are allocated to the political parties, in proportion to their votes in each constituency.

Next, the total votes of the parties and total seats obtained in the first allotment are considered to carry out a complementary apportionment with minimum requirements. In this step high proportionality is obtained. Finally, an increasing and convex adjustment function is applied to transform the quotas, to benefit the biggest party. We illustrate this with an application to the Spanish Congress and the Italian Camera.

Section 2 evokes a version of the Jefferson and Webster methods to be applied in the context of proportional apportionments with minimal requirements. In sections 3 we show paradoxes between total votes and total seats in elections celebrated in Italy and Spain. In section 4 we apply a complementary allotment to the political parties to obtain high proportionality and, usually, to avoid paradoxes (Parliament being a mirror of the votes).

In section 5, we define properties for an adjustment function to favor governability through a new apportionment in proportion to the adjusted quotas. Then the *potential function model* is analyzed. To apply, the three previous sets of allotment data from the 2004 Congress election in Spain and from the 2006 camera Election in Italy are used. Finally, the section 6 includes an application of the bi-proportional allotment to Spain.

2. The problem of proportional allotment

2.1. Jefferson and Webster methods

Given a vector of votes $\mathbf{v} = (v_1, v_2, \dots, v_n) > 0$ obtained by n political parties in a constituency whose size is h , an allotment of the h seats of the constituency is a vector of integer non-negative numbers $\mathbf{a} = (a_1, a_2, \dots, a_n) \geq 0$, such that $\sum_{i=1}^n a_i = h$.

The vector of quotas, $\mathbf{q} = (q_1, \dots, q_n)$, contains the exact proportions. It is obtained by

$$q_i = \frac{h \cdot v_i}{\sum_{j=1}^n v_j}, \quad i = 1, \dots, n.$$

For proportional representation, the divisor methods are very important, because they are consistent and monotonous [1]. The Jefferson and Webster methods are particularly well known. Webster's is impartial and Jefferson's verifies the quota, that is, each

political party receives, at least, the integer part of its quota. For this reason, Jefferson's method is frequently used.

To apply Jefferson's method [1], we must find a real number x such that $\sum_{i=1}^n \lfloor v_i / x \rfloor = h$.

If r is a positive number $r = e + f$, with $f \in (0,1)$, and e is an integer, then $\lfloor r \rfloor = e$, and if $r = e$ then $\lfloor r \rfloor \in \{e-1, e\}$. When x has been found, the allotment is $a_i = \lfloor v_i / x \rfloor$, $i = 1, \dots, n$.

On the other hand, to obtain proportional allotment using Webster's method, we must find a real number x such that $\sum_{i=1}^n \lceil v_i / x \rceil = h$. Where, if r is a positive number, then $\lceil r \rceil$

is the closest integer to r (if $r = e + \frac{1}{2}$ then $\lceil r \rceil \in \{e, e+1\}$). When x has been found, the allotment is $a_i = \lceil v_i / x \rceil$, $i = 1, \dots, n$.

2.2 Proportional allotment with minimum requirements

Sometimes minimum requirements for an allotment problem are established. The minimum requirements can be represented by a vector $\mathbf{min} = (m_1, m_2, \dots, m_n) \geq 0$.

The problem is noted by $(\mathbf{v}, \mathbf{min}; h)$, and it is feasible if $\sum_{i=1}^n m_i \leq h$.

Now the allotment \mathbf{a} must verify $m_i \leq a_i$, $i = 1, 2, \dots, n$, and $\sum_{i=1}^n a_i = h$.

To obtain proportional allotment using the Jefferson method, when minimum requirements exist, we must find a real number x such that $\sum_{i=1}^n \max(\lfloor v_i / x \rfloor, m_i) = h$.

Then $a_i = \max(\lfloor v_i / x \rfloor, m_i)$, $i = 1, \dots, n$ is the allotment (and similar when the Webster method is used).

Example. If $\mathbf{v} = (240, 150, 120, 72)$; $\mathbf{min} = (3, 3, 1, 0)$ and $h = 9$ we can observe that for $x = 60$,

$$\begin{aligned}
& \sum_{i=1}^4 \max(\lfloor v_i / x \rfloor, m_i) = \\
& = \max(\lfloor 240 / 60 \rfloor, 3) + \max(\lfloor 150 / 60 \rfloor, 3) + \max(\lfloor 120 / 60 \rfloor, 1) + \max(\lfloor 72 / 60 \rfloor, 0) = \\
& = \max(\lfloor 4 \rfloor, 3) + \max(\lfloor 2.5 \rfloor, 3) + \max(\lfloor 2 \rfloor, 1) + \max(1.2, 0) = \\
& = \lfloor 4 \rfloor + 3 + \lfloor 2 \rfloor + 1
\end{aligned}$$

There are two fractions (exactly, $\lfloor 4 \rfloor$ and $\lfloor 2 \rfloor$) such that each one can be rounded to two different integer numbers (in this example there are ties), $\lfloor 4 \rfloor \in \{3, 4\}$ and $\lfloor 2 \rfloor \in \{1, 2\}$. Therefore, there are two solutions $\mathbf{a}_1 = (3, 3, 2, 1)$ and $\mathbf{a}_2 = (4, 3, 1, 1)$.

For the same vectors of votes, \mathbf{v} , and minimum requirements \mathbf{min} , but changing $h = 9$ to $h = 10$, all values of x belonging to the interval $[48, 60]$ give the same allotment, $\mathbf{a}_3 = (4, 3, 2, 1)$, and this is the only apportionment for the problem $(\mathbf{v}, \mathbf{min}, h) = ((240, 150, 120, 72), (3, 3, 1, 0); 10)$.

3. Total votes and total seats

Usually, an electoral system allots the seats in several constituencies. When the number of constituencies is high, many of them can be small. Then, if there is not a complementary allotment taking into account the total number of votes of the political parties, the proportionality may be low. And so, paradoxically, a party with fewer total votes receiving more total seats appears often. In all nine elections to the Spanish Congress to date, celebrated between 1977 and 2004, important paradoxes come to light when total votes and total seats to the political parties are compared. We can clearly see this paradox in Table I for the last election (celebrated in 2004); and the same paradox appears in the last election to the Italian Camera, celebrated in 2006, as shown in Table II.

3.1 The Spanish Congressional election of 2004

The total votes obtained by the political parties in the election to the Congress in 2004 in Spain are given in the second column of Table I (listed in decreasing order), and the corresponding allocations to the political parties are shown in the fourth column.

Table I. Total votes and seats obtained in the 2004 Congress election in Spain

Party	Votes	Quotas	Current Allotment	Ordered
PSOE	11,026,163	155.3	164	Yes
PP	9,763,144	137.5	148	Yes
IU	1,284,081	18.1	5	NO
CiU	835,471	11.8	10	Yes
ERC	652,196	9.2	8	Yes
PNV	420,980	5.9	7	Yes
CC	235,221	3.3	3	Yes
BNG	208,688	2.9	2	Yes
PA	181,868	2.7	0	NO
CHA	94,252	1.3	1	Yes
EA	80,905	1.1	1	Yes
NA-BAI	61,045	0.9	1	Yes
Totals	24,844,014	350	350	

In this case, the national left-wing party IU obtains more total votes than the regional parties CiU, ERC and PNV (three times more than PNV), but IU obtains fewer seats than each one of them. On the other hand, PA obtains more total votes than CHA, EA or NA-BAI, yet obtains fewer total seats than each.

The electoral method used to allot the seats in the constituencies of Spain is the Jefferson method. It can be considered the most favorable method for the party that wins in each constituency; but, as there are different winners in the 52 constituencies of the Congress, the total bonus (difference between total seats and the quota, for a given party) obtained by the first political party can be less than the bonus obtained by the second political party. This phenomenon was seen in the election of 2004, because the PSOE (first party) obtained a bonus of $164 - 155.3 = 8.7$ seats, whereas the PP (second party) obtained a bonus of 10.5 seats. Such behavior does not facilitate governability.

3.2 The Italian Camera election in 2006

Similar to the case of Spain, and shown in Table II, are the total votes and seats obtained by the Italian political parties in the election to the Italian Camera in 2006.

Table II. Total votes and seats obtained in the 2006 Camera election in Italy

Party	Votes	Quotas	Current Allotment	Ordered
L' Olivo	11,930,983	195.8	220	Yes
Forza Italia	9,048,976	148.5	137	Yes
Alleanza Naz.	4,707,126	77.2	71	Yes
UDC	2,580,190	42.3	39	NO
Rif. Comun.	2,229,464	36.6	41	Yes
Lega Nord	1,747,730	28.7	26	Yes
La Rosa nel P.	990,694	16.3	18	Yes
Comunisti It.	884,127	14.5	16	Yes
Di Pietro It.	877,052	14.4	16	Yes
Verdi	784,803	12.9	15	Yes
Udeur Popol.	534,088	8.8	10	Yes
P. Pensionati	333,983	5.5	0	NO
Dc-Nuovo Psi	285,474	4.7	4	Yes
Alt. Soc. Mus.	255,410	4.2	0	NO
Fiamma Tric..	231,313	3.8	0	NO
Svp	182,704	3.0	4	Yes
Total	37,604,117	617.0	617	

In this case, UDC obtains 350,000 votes more than Rinfondazione Comunista but UDC obtains two seats less. Also, Pensionati has more votes than Dc-Nouvo Psi and than Svp, but Pensionati ends up with no representation whereas Svp obtains four seats, etc. The Italian electoral system (for the Camera) guarantees 55% of the seats to the winning coalition. In 2006 the coalition of center-left obtained approximately 24,000 votes more than the right-wing coalition (over 38 million in total). Therefore the center-left coalition received 340 seats as opposed to the 277 of the right (so, the center-left coalition receives a bonus of 63 seats). Nevertheless, the winning political party, l'Olivo, obtains only $220 - 195.8 = 24.2$ seats over its quota. Many political parties benefited from belonging to the winning coalition —nine of them, to be exact. It is not logical to favor so many parties.

4. An electoral system obtaining proportionality through a complementary allotment with minimum requirements

A distribution, as either of the two shown above, can be followed by another complementary allotment (that we denote **b**) in which the total number of obtained seats and total number of votes are considered [2]. In such a case, high proportionality can be achieved (the Bundestag election in Germany is a very good example).

The number P of seats for this complementary allotment, to obtain high proportionality depends on the initial distribution of H seats in the constituencies, and this distribution depends in turn on the constituency sizes. When the constituencies are small (in Germany they are uninominal districts), P must be greater than when they are large (as in Italy). The electoral system for the German Bundestag uses $P = H$ and obtains very high proportionality. Values of P much smaller than H can prove sufficient. For example, for Italy and Spain it is possible to use $P \leq \frac{H}{10}$ and obtain high proportionality (near 95%).

4.1 How must the complementary allotment be done?

In the first allotment, there are n constituencies and m political parties. Thus we have a table of votes as seen below in Table III.

Table III. Votes in the constituencies

Parties → ↓ Constituencies	P ₁	P ₂	...	P _m
C ₁	v ₁₁	v ₁₂	...	v _{1m}
C ₂	v ₂₁	v ₂₂	...	v _{2m}
⋮	⋮	⋮	v _{ij}	⋮
C _n	v _{n1}	v _{n2}	...	v _{nm}

where v_{ij} is the number of votes obtained in the constituency i by the political party j . Next we apply, in each constituency C_i , a proportional method to allot h_i seats to the

votes $(v_{i1}, v_{i2}, \dots, v_{im})$ and we obtain the table of seats shown in the following Table IV:

Table IV: First allotment

Parties → ↓ Constituencies	P ₁	P ₂	...	P _m
C ₁	s_{11}	s_{12}	...	s_{1m}
C ₂	s_{21}	s_{22}	...	s_{2m}
⋮	⋮	⋮	s_{ij}	⋮
C _n	s_{n1}	s_{n2}	...	s_{nm}

where s_{ij} is the number of seats allocated in the constituency i to the political party j .

The total number of votes, v_j , obtained by the political party j is:

$$v_j = \sum_{i=1}^n v_{ij}, \quad j = 1, \dots, m.$$

And the total number of seats, s_j obtained by the political party j is:

$$s_j = \sum_{i=1}^n s_{ij}, \quad j = 1, \dots, m.$$

Trivially, if H is the total number of seats allocated in the first allotment:

$$s_1 + s_2 + \dots + s_m = H = h_1 + h_2 + \dots + h_n.$$

Then, the complementary allotment of P seats to increase proportionality can be established as a proportional apportionment problem with minimum requirements.

In this case the votes for the allotment are the vector $\mathbf{v} = (v_1, \dots, v_m)$; the minimum requirements are $\mathbf{min} = (s_1, \dots, s_m)$ and the number H_1 of seats to be allocated is

$$H_1 = H + P.$$

4.2 Application to Spain's 2004 Congressional election

In this case we consider that the first allotment is the one of 2004. Then, the vector of votes is that of Table I (second column) and the vector of the minimum requirements is also as seen in Table I (fourth column, the total number of seats obtained by the political parties using current allotment). We are going to use a number of additional seats for the complementary distribution equal to 30, applying the Jefferson method to obtain the new allotment. Hence, Table V shows the quotas for $H_1 = H + P = 350 + 30 = 380$ Seats, the minimum requirements (first allotment) and the complementary allotment (last column).

Table V. Proportionality with 2004 Congress data in Spain

Party	Votes	Quotas for 380	Allotment in 52 constituencies	Com. Allotment (proportionality)
PSOE	11,026,163	168.7	164	170
PP	9,763,144	149.4	148	151
IU	1,284,081	19.6	5	19
CIU	835,471	12.8	10	12
ERC	652,196	10.0	8	10
PNV	420,980	6.4	7	7
CC	235,221	3.6	3	3
BNG	208,688	3.2	2	3
PA	181,868	2.8	0	2
CHA	94,252	1.4	1	1
EA	80,905	1.2	1	1
NA-BAI	61,045	0.9	1	1
Totals	24,844,014	380	350	380

4.3 Application to Italia's 2006 Camera election

We consider only the allotment in 26 constituencies. That is, the allotment in Valle d'Aosta (one seat) and Estero (12 seats) are not included here. Currently in all of the 26 constituencies 617 seats are allocated. Then, firstly we are going allocating 540 seats to the 26 constituencies using Greatest Remainder method and, after that, we use the d'Hondt method in each constituency in order to allocating seats to the political parties

(as in the current Italian electoral system). The total number of seats, obtained by the political parties, are shown in the fourth column (they are the minimum requirements for the complementary distribution. For the complementary distribution we use 30 seats. That is $H = 540$, $P = 30$ and $H_1 = H + P = 540 + 30 = 570$. The other 47 (to the total 617) are used for governability in the next section.

Table VI shows the quotas for $H_1 = 570$ seats, the total number of seats received by the parties in the constituencies (first allotment, fourth column) and the complementary allotment (last column).

Table VI. Proportionality with 2006 Camera data in Italy

Party	Votes	Quotas-570	Allotment in 26 constituencies	Com. Allotment (proportionality)
L' Olivo	11,930,983	180.8	204	204
Forza Italia	9,048,976	137.1	153	153
Alleanza Naz.	4,707,126	71.3	72	72
UDC	2,580,190	39.1	35	35
Rif. Comun.	2,229,464	33.8	31	31
Lega Nord	1,747,730	26.5	23	23
La Rosa nel P.	990,694	15.1	5	10
Comunisti It.	884,127	13.4	3	9
Di Pietro It.	877,052	13.3	3	8
Verdi	784,803	11.9	3	7
Udeur Popol.	534,088	8.1	4	5
P. Pensionati	333,983	5.1	0	3
Dc-Nuovo Psi	285,474	4.3	0	2
Alt. Soc. Mus.	255,410	3.9	0	2
Fiamma Tric..	231,313	3.5	0	2
Svp	182,704	2.8	4	4
Total	37,604,117	570.0	540	570

5. Governability

5.1 Adjustment function of quotas to favour the biggest political parties

Politicians may wish for an electoral system that enhances governability. To this end, a bonus to the biggest political party (or coalition) is proposed. The Italian electoral law (approved in December 2005) is an example. Yet the method underlying this law is not logical.

We can use part of the seats of the Congress to favour governability, and assigning this portion of seats, let us say G seats, as a second complementary allotment. Nonetheless, in this case the distribution is not in proportion to the quotas, but rather in proportion to a power of the quotas, that is, in proportion to q^α , choosing $\alpha > 1$, for example, $\alpha = 2$ (or $\alpha = 3$) with minimum requirements (the minimum will be the result of the allotment of $H+P$ previous seats).

5.2 What candidates receive the $P+G$ seats allocated in proportion to the total votes and in proportion to squares total votes?

There are several possibilities. We state two of them here.

a. The parties present a national list. Then each party receives its new seats according to this list.

b. Biproportional allotment is attained with minimal requirements, [2], (BAZI-program [4] is useful to compute the solution).

5.3.1 Application of model to the 2004 Congress election in Spain.

In this case we consider that the first allotment is the current one; and the minimum requirements are the complementary allotment shown in the last column of Table V. We use $\alpha = 2$ as the adjustment function for the governability, and we add 40 seats ($G = 40$) to the 380 previous ones to assign a total of 420 in the final allotment.

The corresponding results are given in fifth column of Table VII. In the last column we show the allotment corresponding to $\alpha = 3$.

Table VII. Proportionality and Governability, 2004 Congress election in Spain

Party	Quotas for 420	Minimum	Allotment $\alpha = 2$ (Governability)	Allotment $\alpha = 3$ (Governability)
PSOE	186.4	170	202	210
PP	165.1	151	159	151
IU	21.7	19	19	19
CIU	14.1	12	12	12
ERC	11.0	10	10	10
PNV	7.1	7	7	7
CC	4.0	3	3	3
BNG	3.5	3	3	3
PA	3.1	2	2	2
CHA	1.6	1	1	1
EA	1.4	1	1	1
NA-B	1.0	1	1	1
Total	420.0	380	420	420

When there are clearly two major parties, as in the Spanish case, it is very simple to verify that if the first party obtains over 45% of votes and 45% of seats (that is, 171 seats) in the complementary allotment, and the second one obtains less than 36.5% of votes and 36.5% of seats (139 seats), then the first party receives the 40 seats for governability and therefore its number of representatives is greater than 50% of seats in the Congress.

But if $\alpha = 3$ and the same data for the first party is given, then the second party can obtain 40.5% of votes and 40.5% of seats in the complementary allotment (that is, 154 seats) and the first party receives the 40 seats for governability.

5.3.2 Application model to the 2006 Camera election in Italy.

In this case the minimum requirements for the allotment we consider that the first allotment is the current one; and the minimum requirements are the complementary allotment shown in the last column of Table V. We add 47 seats ($G = 47$) to the 570 previous one (for the governability), to assign a total of 617 in the final allotment.

Table VII. Proportionality and Governability, 2004 Camera election in Italy.

Party	Quotas <i>H = 617</i>	Minimum, (proportionality)	Allotment, $\alpha = 2$ (Governability)
L' Olivo	195.8	204	251
Forza Italia	148.5	153	153
Alleanza Naz.	77.2	72	72
UDC	42.3	35	35
Rif. Comun.	36.6	31	31
Lega Nord	28.7	23	23
La Rosa nel P.	16.3	10	10
Comunisti It.	14.5	9	9
Di Pietro It.	14.4	8	8
Verdi	12.9	7	7
Udeur Popol.	8.8	5	5
P. Pensionati	5.5	3	3
Dc-Nuovo Psi	4.7	2	2
Alt. Soc. Mus.	4.2	2	2
Fiamma Tric..	3.8	2	2
Svp	3.0	4	4
Total	617.0	570	617

In this case the total of the 47 seats are assigned to the first party, l'Olivo.

We can observe that, in this case the first party (l'Olivo) can obtain a winner coalition easier with the governability bonus G than without it (present electoral system, fourth column in Table II). On the other hand, votes and seats are ordered for all parties excepting for Svp. The obtained allotment is more proportional than the current one. No thresholds have been used in our model.

6. Biproportional allotment

We have given a method to obtain the total number of seats for the political parties when high proportionality and governability are required. Both, the Italian and Spanish

Constitutions establish the sizes of its constituencies. Then for the two examples, Spanish Congress and Italian Camera we have two marginal (the representation of the political parties and the representation of the voters), now it is possible to apply biproportional allotment to obtain how many seats must be allocated to each political party in each constituency. For example, for the Spanish 2004 election the Table VIII shows the votes the parties in every constituency and the corresponding biproportional allotment (the second column is the marginal corresponding to the constituencies size and the first row is the marginal for the representation of the political parties.)

Province/City	Size	PSOE: ⇒ 158	PP: ⇒ 139	IU: ⇒ 18	Aut-1	Aut-2
A Coruña	9	287,324 ⇒ 4	329,389 ⇒ 4	14,125 ⇒ 0	86,459 ⇒ 1	
Alava	4	561,374 ⇒ 1	48,992 ⇒ 1	14,181 ⇒ 1	47,090 ⇒ 1	7,838 ⇒ 0
Albacete	4	108,715 ⇒ 2	110,338 ⇒ 2	9,145 ⇒ 0		
Alicante	11	374,631 ⇒ 5	434,812 ⇒ 6	34,774 ⇒ 0		
Almería	5	145,868 ⇒ 3	135,434 ⇒ 2	9,522 ⇒ 0	7,190 ⇒ 0	
Asturias	8	305,240 ⇒ 4	307,977 ⇒ 3	59,253 ⇒ 1		
Avila	3	38,640 ⇒ 1	67,622 ⇒ 2	3,598 ⇒ 0		
Badajoz	6	219,172 ⇒ 3	176,699 ⇒ 3	16,589 ⇒ 0		
Barcelona	31	1,268,028 ⇒ 12	485,504 ⇒ 4	198,116 ⇒ 4	586,854 ⇒ 6	428,986 ⇒ 5
Burgos	4	91,727 ⇒ 2	122,415 ⇒ 2	7,703 ⇒ 0		
Cáceres	4	137,654 ⇒ 2	118,627 ⇒ 2	7,569 ⇒ 0		
Cádiz	9	326,152 ⇒ 4	216,416 ⇒ 3	38,611 ⇒ 1	33,592 ⇒ 1	
Cantabria	5	149,906 ⇒ 2	190,383 ⇒ 3	12,146 ⇒ 0		
Castellón	5	139,236 ⇒ 3	142,462 ⇒ 2	10,322 ⇒ 0		
Ceuta	1	12,769 ⇒ 0	21,142 ⇒ 1	218 ⇒ 0		
Ciudad Real	5	147,271 ⇒ 3	142,508 ⇒ 2	8,581 ⇒ 0		
Córdoba	7	246,324 ⇒ 4	166,665 ⇒ 2	47,908 ⇒ 1	19,648 ⇒ 0	
Cuenca	3	60,697 ⇒ 1	66,515 ⇒ 2	3,258 ⇒ 0		
Girona	6	113,089 ⇒ 2	40,959 ⇒ 0	15,070 ⇒ 0	96,928 ⇒ 2	83,482 ⇒ 2
Granada	7	268,870 ⇒ 4	193,484 ⇒ 2	31,227 ⇒ 1	14,030 ⇒ 0	
Guadalajara	3	52,915 ⇒ 1	57,078 ⇒ 2	5,310 ⇒ 0		
Guipúzcoa	6	98,100 ⇒ 1	56904 ⇒ 1	28,668 ⇒ 1	115,402 ⇒ 2	42,971 ⇒ 1
Huelva	5	154,579 ⇒ 3	84,173 ⇒ 2	15,097 ⇒ 0	14,542 ⇒ 0	
Huesca	3	61,500 ⇒ 2	50,493 ⇒ 1	3,650 ⇒ 0	8,629 ⇒ 0	
I. Baleares	8	185,623 ⇒ 4	215,737 ⇒ 4	0 ⇒ 0		
Jaén	6	228,611 ⇒ 4	143,288 ⇒ 2	24,483 ⇒ 0	15,493 ⇒ 0	
La Rioja	4	81,390 ⇒ 2	92,441 ⇒ 2	5,115 ⇒ 0		
Las Palmas	8	167,926 ⇒ 3	208,995 ⇒ 4	9,876 ⇒ 0	89,420 ⇒ 1	
León	5	156,786 ⇒ 3	150,688 ⇒ 2	7,160 ⇒ 0		
Lleida	4	68,971 ⇒ 1	34,116 ⇒ 0	6,910 ⇒ 0	68,735 ⇒ 2	50,104 ⇒ 1
Lugo	4	92,708 ⇒ 2	123,986 ⇒ 2	2,570 ⇒ 0	25,313 ⇒ 0	
Madrid	35	1,544,676 ⇒ 16	1,576,636 ⇒ 15	225,109 ⇒ 4		
Málaga	10	367,758 ⇒ 5	269,063 ⇒ 4	47,182 ⇒ 1	32,368 ⇒ 0	
Melilla	1	11,273 ⇒ 0	14,856 ⇒ 1	229 ⇒ 0		
Murcia	9	252,246 ⇒ 3	413,902 ⇒ 6	30,787 ⇒ 0		
Navarra	5	113,906 ⇒ 2	127,653 ⇒ 3	19,899 ⇒ 0		
Ourense	4	74,636 ⇒ 1	132,631 ⇒ 3	2,055 ⇒ 0	26,153 ⇒ 0	
Palencia	3	51,824 ⇒ 1	60,449 ⇒ 2	3,415 ⇒ 0		
Pontevedra	7	228,016 ⇒ 3	279,454 ⇒ 3	13,158 ⇒ 0	70,763 ⇒ 1	
Salamanca	4	94,655 ⇒ 2	128,932 ⇒ 2	4,713 ⇒ 0		
Sta. C. Tenerife	7	165,158 ⇒ 3	133,677 ⇒ 2	8,736 ⇒ 0	145,801 ⇒ 2	
Segovia	3	39,976 ⇒ 1	52,500 ⇒ 2	3,470 ⇒ 0		
Sevilla	12	639,293 ⇒ 7	306,464 ⇒ 3	73,344 ⇒ 1	45,005 ⇒ 1	
Soria	3	22,287 ⇒ 1	29,187 ⇒ 2	1,230 ⇒ 0		
Tarragona	6	136,660 ⇒ 3	65,528 ⇒ 1	14,694 ⇒ 0	82,954 ⇒ 1	76,330 ⇒ 1
Teruel	3	36,152 ⇒ 2	35,920 ⇒ 1	2,514 ⇒ 0	4,463 ⇒ 0	
Toledo	5	167,807 ⇒ 3	171,325 ⇒ 2	12,707 ⇒ 0		
Valencia	16	613,833 ⇒ 7	665,526 ⇒ 8	78,515 ⇒ 1		
Valladolid	5	155,401 ⇒ 3	163,009 ⇒ 2	13,029 ⇒ 0		
Vizcaya	9	185,514 ⇒ 3	129,889 ⇒ 2	59,493 ⇒ 1	258,488 ⇒ 3	30,096 ⇒ 0
Zamora	3	53,757 ⇒ 1	71,821 ⇒ 2	3,375 ⇒ 0		
Zaragoza	7	224,776 ⇒ 3	198,480 ⇒ 3	15,672 ⇒ 0	81,160 ⇒ 1	

Table VIII: Bi-proportional-D'Hondt allotment. Spanish Congress 2004.

In the Table, Aut-1 means BNG in Galicia (A Coruña, Lugo, Ourense and Pontevedra), PNV in the Basque Country (Álava, Guipúzcoa and Vizcaya), PA in Andalucía (Almería, Cádiz, Córdoba, Granada, Huelva, Jaén, Málaga and Sevilla), CiU in Catalunya (Barcelona, Girona, Lleida and Tarragona), CHA in Aragón (Huesca, Teruel and Zaragoza). Aut-2 means ERC in Catalunya and EA in the Basque Country.

In [6] the bi-proportional method is explained and applied to the Spanish Congress election.

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