Income Redistribution and Public Good Provision in a Diverse Society*

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Abstract

I analyze the post-electoral coalition formation process in a two dimensional political environment where one of the dimensions is the degree of a proportional tax rate and the other dimension represents the degree of a group specific public good. There are three parties who are office motivated and care only instrumentally about policy. The collected taxes are distributed between income redistribution and the group-specific public good. I consider a case where elections already have taken place and the electoral outcome forces parties to form a coalition government. I analyze when stable coalitions would exist and obtain that for that to occur the benefits of being in office should exceed a certain level. This critical level increases as the income of the rich or the degree of diversity in the society increases. I also analyze how the set of policies implemented by those coalitions are affected by the income levels and the degree of diversity and obtain that the higher the level of income of the rich the smaller the set of policies that can be implemented. The degree of diversity has the same effect whenever the party who obtains a positive utility from the public good forms part of the government. The set of policies that can be implemented increases as the income level of the poor increases if the degree of taxation is the only issue of conflict between the parties who form the government. I also consider both office and policy motivated parties and obtain that as before the benefits of being in office should exceed a certain level but in this case it is more likely that stable coalitions form.

KEYWORDS: Electoral competition, coalition formation, public goods, income redistribution

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1 Introduction

In many countries governments are formed by multiparty coalitions. As stated by Laver and Schofield (1998), many Western European countries have been governed by coalitions for most of the twentieth century. It is the heterogeneity of preferences in the society that leads to the formation of different parties who take different positions on the existing issues. Therefore, the heterogeneity of preferences leads to a multi-dimensional political competition. This paper aims to combine these two aspects, heterogeneity and coalition formation by analyzing which stable governing coalitions can be formed in such an environment and the policies that would be implemented.

Although preference heterogeneity can lead to a multi-dimensional political competition, in a certain electoral competition some of these dimensions might be much more relevant than others. That is, some issues where voters show greater degree of heterogeneity would be the more salient issues where parties would intent to clearly signal their position and their difference from others. Taking these arguments into account, I analyze a situation in which voters' preferences show heterogeneity on only two dimensions perceived as the most salient ones and parties compete on these two dimensions. In particular, I consider an environment where one of the dimensions is the degree of income taxation and the other dimension the provision of a certain public good.

In many countries around the world, a certain part of the heterogeneity of preferences of the society might be due to differences in ethnicity and/or religious beliefs. Those different ethnic or religious groups are represented by parties who make promises and policy demands in line with the interest of those groups. In this paper I analyze a situation where one of the policy dimensions is a result of a ethnic or religious diversity. More specifically, I consider a society whose members differ both in their level of income and also in their ethnicity or religion[1].

Each different group in the society is represented by a party who has the same ideal policy point as its potential voters. The government decides on the proportional tax rate and on the amount of public good provided. The amount of public good provided directly affects the amount of redistribution as the total tax revenue has to be divided between redistribution and public good provision.

This two dimensional framework can cause preference heterogeneity in two directions: On the one side people who prefer a high level of taxation and people who prefer no income taxation, and on the other side people who prefer to have a certain level of public good and others who prefer that no public good is provided. As the public good can only be provided if there exists a positive income tax, it would be impossible to satisfy the desire of a group who prefers to have no income taxation and a positive amount of public good. Therefore, I consider a society which consists of three groups: the rich group

[1] From now on I will denote this difference as an ethnic one.

from the ethnic majority, the poor group from the ethnic majority and the poor group from the ethnic minority. The poor group from the ethnic majority benefits from an income redistribution and the poor minority group in addition to income redistribution also obtains utility from a public good whereas the rich group is negatively effected by the income redistribution and obtains no utility from the public good. Educational facilities such as schools which use as the instruction language a language used only by the ethnic minority, or worship places such as churches or mosques which are only used by the ethnic or religious minority could be considered as examples where only the minority group would benefit from the provision of such a public good.

I analyze a coalition formation game where first parties make a policy announcement. Then, voters vote to the party who makes the announcement they like the most. Then, the votes are transformed into seats according to ideal proportionality. That is, the seat share of a party equals its vote share. I consider a case where the first two steps have already taken place, that is, the election has already taken place and the seats in the parliament are distributed. More specifically, I assume that no party obtains a majority and some parties have to form a coalition to obtain a majority government.

As in Aragones (2007 a,b), I consider parties who are mainly office motivated and care only instrumentally about policy. As in these two papers I assume that voters care about the policies implemented. Thus the parties who represent the voters would be concerned about the policy that would implemented whenever they form part of the government as compromising their policy position might affect negatively those parties' future voter support. In terms of the utility obtained from being in office, I consider the case where the utility of office benefits a party obtains depends on its seat share in the parliament and the seat share in the parliament of its coalition member(s). I assume that parties share spoils of office proportionally to their seat share. I characterize the set of stable coalitions that can be formed and obtain that when benefits of office are sufficiently large, a stable coalition always exists and is formed by the two smallest parties. That is, for a stable coalition to exist, the value of the office should be higher than a certain threshold which depends on the seat share of parties, the income levels of the groups in the society and the degree of diversity in the society.

I obtain that, the higher the degree of diversity the higher should be the benefit of being in office in order to guarantee the existence of a stable coalition. The level of income of the rich group has the same effect. The effect of the level of income of the two poor groups on the threshold of the benefits of office depends on which parties would form the government. If the government is formed by the ethnic majority, the minimum level of office spoils needed to guarantee the existence of a stable coalition decreases as the income of the poor group increases whereas if the ethnic minority group forms part of the government, the effect of the level of income on the minimum level of office spoils needed to guarantee the existence of a stable coalition would be in the other direction. Moreover, as the seat shares of parties become more equal the minimum level of office spoils needed to guarantee the existence of a stable coalition increases. I also obtain that the higher the level of income of the rich the smaller is the set of policies that can be implemented. An increase in the degree of diversity has the same effect whenever the ethnic minority forms part of the government.

I extend the above described analysis to find the conditions under which a stable government would be formed if parties care both about benefits of office and policy. I obtain that, as in the case of parties who care only instrumentally about policy, if the benefits of office exceed a certain level stable coalitions would exist. However, in this case it is more likely that stable coalitions exist as the party outside the government would be willing to give up less to join a coalition in terms of policy compromise.

In a similar framework, Bandiera and Levy (2007) analyze a political game considering endogenous parties who are solely policy motivated and who might form pre-electoral coalitions and might decide not to run. The electoral rule they use is plurality, that is the party who obtains the highest amount of votes is the winner. They obtain that the only stable coalition that can be formed is the one formed by the rich group from the ethnic majority and the poor group of the ethnic minority whereas I obtain that a stable coalition would be formed by the two smallest parties whoever those are. In their analysis the fact that the largest party would win the elections and form the government alone makes it possible to have a stable coalition when parties are only policy motivated. They also obtain that as the public good becomes more valuable for the ethnic minority, the tax rate decreases, the public good expenditures increases and hence the outcome becomes less favorable for the poor majority and more favorable for the rich group; thus the effect of democracy is diminishing as diversity increases. On the other hand, I obtain that whenever the ethnic minority forms part of the government an increase in the degree of diversity would make both the rich and the poor from the ethnic majority worse-off in the sense that the set of policies that could be implemented would decrease in detriment of those parties but it would make the poor ethnic minority better-off.

2 Model

In this section I first describe the timing of the electoral game. There exist a certain number of parties denoted by i where $i \in I$. First, each party i makes its policy promise. Then, voters who observe these promises cast their votes. Then these votes are transformed into seats. The electoral rule is proportional representation. More specifically, I assume that the electoral formula is such that "ideal proportionality" applies. That is, the seat share of a party equals its vote share. A government can only be formed if it obtains the support of more than half of the parliamentary members. That is, a majority single party government can only be formed if one party obtains more than half of the votes. If no party obtains an absolute majority of seats, then the government should be formed by a majoritarian coalition.

If no party obtains the majority of seats, they engage in bargaining on which coalition to form and which policy to implement. In this process, parties' aim is to maximize their utility. Parties obtain utility from being part of the government and from the policy implemented only if they form part of the government. The utility function of a party i takes the following form:

$$V_i(C,X) = \begin{cases} B_i(C) + U_i(X) & \text{if party } i \text{ forms part of the government} \\ 0 & \text{if party } i \text{ does not form part of the government} \end{cases}$$

where C denotes the government coalition formed; $B_i(C)$ is the utility that party i obtains from being part of coalition C and $U_i(X)$ is the utility that party i obtains from the policy implemented by the government given that it forms part of the government and X is the set of policy variables. In what follows, I first describe the preferences of the voters and then the political parties.

2.1 Voters

The society consists of a certain number of citizens, N. Each citizen belongs to one of three different groups: the rich from the ethnic majority (R), the poor from the ethnic majority (P) and the poor from the ethnic minority (E). The size of these three groups are n_R, n_P, and n_E respectively where $n_R + n_P + n_E = N$. The income of the poor (both P and E) is y_P and the income of R is y_R. These three groups have the following utility functions:

$$U_R(t,T,g) = y_R(1-t) + T$$

$$U_P(t,T,g) = y_P(1-t) + T$$

$$U_E(t,T,g) = y_P(1-t) + T + kv(g)$$

In these utility functions t denotes a proportional tax rate, T is the per capita redistribution and g denotes the level of public good which is only enjoyed by the ethnic minority, E. $v(g)$ denotes the utility obtained from this group specific public good. $v(g)$ is assumed to be concave with $v'(0) = \infty$ and $v'(\infty) = 0$ such that the ideal point of E forms part of the policy set. The parameter k ($k > 0$) measures the degree of diversity in the society: the higher is k the more valued is the public good by the ethnic minority. The two groups from the ethnic majority obtain only a utility from redistribution and the ethnic majority obtains utility from both redistribution and the public good.

Voters vote sincerely, that is, they cast their vote to the party that proposes the policy that would give them the highest utility among all the policy proposals.

2.2 Parties

In the political environment there are three parties who are competing in the elections. A party is denoted by i where $i \in I = \{P, R, E\}$. Each of these three parties represents one of the groups in the society. Parties care about office spoils and only instrumentally about the policy implemented. As stated before, the utility function of each party takes the following form:

$$V_i(C, X) = \begin{cases} B_i(C) + U_i(X) & \text{if party } i \text{ forms part of the government} \\ 0 & \text{if party } i \text{ does not form part of the government} \end{cases}$$

where C denotes the government coalition formed; $B_i(C)$ is the utility that party i obtains from being part of coalition C and $U_i(X)$ is the utility that party i obtains from the policy implemented by the government given that it forms part of the government and X is the set of policy variables.

As each party represents a certain group of the society, I assume that when a party forms part of the government it obtains the same utility from the policy implemented as the group it represents. Therefore the utility that obtains a party which forms part of the government from the policy implemented is:

$$U_R(t, T, g) = y_R(1 - t) + T$$

$$U_P(t, T, g) = y_P(1 - t) + T$$

$$U_E(t, T, g) = y_P(1 - t) + T + kv(g)$$

The government that is formed decides on the level of a proportional tax rate, t where $1 \geq t \geq 0$, the level of a public good provision and the per capita income redistribution. Therefore, the budget constraint of a government is as follows:

$$tNy = NT + g$$

where y is the average income. That is, $y = \frac{y_R n_R + y_P(n_P + n_E)}{N}$. Thus, the government has to decide on the value of two policy parameters. So, X can be written as $X = (t, g)$.

Using the budget constraint we can define the indifference curves of the three parties (and groups) in terms of t and g. As $T = ty - \frac{g}{N}$, we obtain that the indifference curves of P and R have the following form:

$$y_i + t(y - y_i) - \frac{g}{N} = u$$

where $i \in \{P, R\}$. Therefore, $\frac{\triangle g}{\triangle t} = N(y - y_i)$ for both groups which implies that these two groups have linear indifference curves with a positive slope for P as $y > y_P$ and with a negative slope for R as $y_R > y$. Therefore, the ideal policy of P would be $(t, g) = (1, 0)$ as it obtains more utility the higher is t and the lower is g; and the ideal policy of R would be $(t, g) = (0, 0)$ as it obtains more utility the lower is t and g.

Similarly, we can obtain the indifference curve of E:

$$y_P + t(y - y_P) - \frac{g}{N} + kv(g) = u$$

As $v(g)$ is concave and all other terms in the indifference curve are linear, the indifference curves of E are quasi-concave and the ideal policy would be $t^* = 1$ and g^* such that $\frac{1}{N} = kv'(g^*)$. So, the policy space and the ideal points of each group (P^*, R^*, E^*) can be depicted as follows:

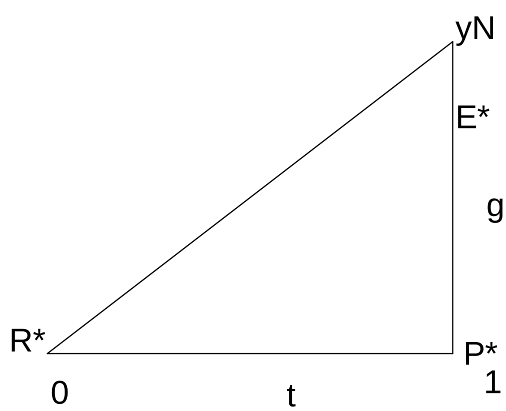

Figure 1

Notice that the maximum amount of g depends on the level of t. Therefore, the maximum amount of public good that can be provided would correspond to $t = 1$ and $T = 0$ i.e. $g = Ny$. At the ideal point of E, $t^* = 1$ and $\frac{1}{Nk} = v'(g^*)$. Therefore, in order to guarantee that g^* belongs to the policy set we need that $v'(Ny) \leq 1/Nk$ as $v(g)$ is a concave function.

Now I define the second element of the utility functions of the parties: $B(C)$. As stated before, I consider the case where $B(C)$ is 0 for a party who does not form part of the government and $B(C) = B * \frac{s_i}{\sum\limits_{j \in C} s_j}$ where B is a positive constant and s_i is the seat share of party i. That is, parties share a fixed amount of office spoils proportionally to their seat shares. As stated before, I assume ideal proportionality. Thus, the seat share of a party simply equals its vote share.

As parties represent a specific group each party can only propose the ideal policy of the group it belongs to. Therefore, as voters vote sincerely, $s_R = n_R/N$, $s_P = n_P/N$ and $s_E = n_E/N$ which implies that $B(C) = B * \frac{n_i}{\sum_{j \in C} n_j}$.

3 Equilibrium Stable Coalitions

In the analysis that follows the key question is that from the whole set of possible majoritarian coalitions that could be formed which ones would be stable and what would be the policy choice of these coalitions? A coalition of parties would only implement policies that would lie on the Pareto Set of this coalition as for any policy outside their Pareto Set they could agree on a different one which would increase the utility of at least one of the parties forming part of the government without decreasing the utility of any of them.

In this section, I first analyze which stable coalitions might be formed and which set of policies might be implemented for given parameter values. Then, I show how the set of policies that could be implemented by a stable coalition would be affected by a change in the degree of diversity, k, the income level of the rich group, y_R, and the income level of the two poor groups, y_P.

Therefore, first of all, we should find the Pareto Set for each of the possible coalitions. For the coalition PR, the Pareto Set would be $t \in [0, 1]$ and $g = 0$ as they have linear indifference curves with opposite signed slopes and g affects their utility negatively. For the coalition PE the Pareto Set would be $t = 1$ and $g \in [0, g^*]$ as their utility increases linearly in t. In order to find the the Pareto Set of the coalition ER the following maximization problem has to solved:

$$\max_{\{t,g\}} y_R + t(y - y_R) - \frac{g}{N} \qquad (1)$$

$$s.to\ y_P + t(y - y_P) - \frac{g}{N} + kv(g) \geq u_E$$

Solving this maximization problem we obtain that in the Pareto Set t can take any value and g should satisfy the following equation:

$$kv'(g) = \frac{y_P - y_R}{N(y - y_R)} \qquad (2)$$

That is, the Pareto Set contains policy points with a fixed level of g as long as this level is feasible for a given $t < 1$. If not, the Pareto Set would be the pairs of t with the highest level of g possible. Notice that $kv'(g) = \frac{y_P - y_R}{N(y - y_R)}$ corresponds to a level of g below the ideal point of E as $\frac{y_P - y_R}{N(y - y_R)} > \frac{1}{N}$ and $v(g)$

is concave. In addition, when $t = 1$, the Pareto Set includes all points where g takes values between the level satisfying $kv'(g) = \frac{y_P - y_R}{N(y - y_R)}$ and $kv'(g)$. The figure below depicts the Pareto sets for all two party coalitions. If we consider the consensus government PER, one can easily see that for any possible policy proposition of this coalition government, there exists another policy proposition that can be be proposed by a coalition of two parties which would give them a higher or at least the same level of utility.

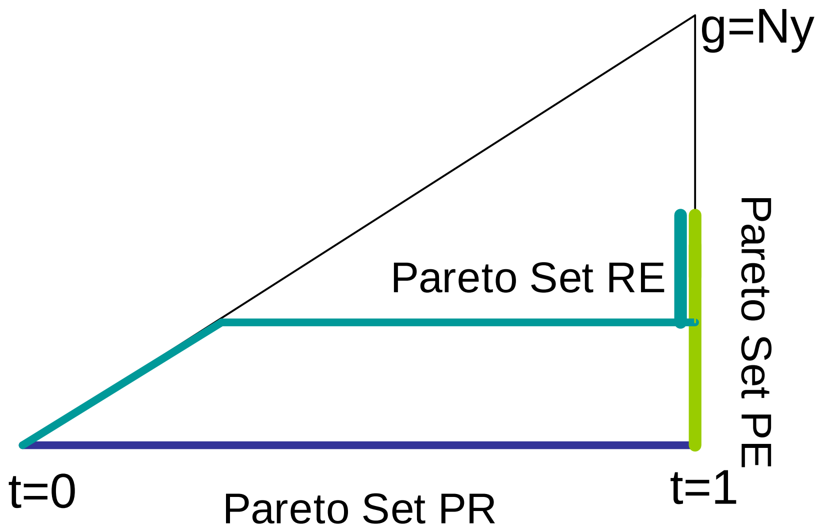

Figure 2

As I described above, I assume that parties who form the government obtain a benefit from being in office $B(C)$ and that they share a fixed amount of utility among themselves. Moreover, I assume that parties share the benefits of office proportionally to their seat share. As the electoral rule is proportional representation and I consider ideal proportionality the seat share of a party equals its vote share which is simply the size of the group it represents. Therefore, if two parties i and j form a government the amount of office spoils received by party i is: $\frac{n_i}{n_i + n_j} B$ where B is a positive constant. Therefore, if a party forms part of the government its total utility is the utility obtained from the policy implemented plus the utility obtained from office spoils. If a party does not enter the coalition, and its two rivals form the coalition then this party would not obtain any utility neither from office spoils nor from the policy implemented. As stated before, I assume that no group constitutes more than half of the society.

In order to be able to obtain explicit results $v(g)$ should take a specific form. For technical reasons I assume that $v(g)$ takes a linear form. More specifically, I take $v(g) = g$, $k > \frac{1}{N}$ and $(y_R - y)N > \frac{y - y_P}{k - \frac{1}{N}}$. The first inequality indicates that the ideal point of E is $(t, g) = (1, Ny)$ and the second one guarantees that the Pareto Set of RE would be the line connecting their ideal points. The second inequality can be rewritten as $(y_R - y) > \frac{y - y_P}{Nk - 1}$. Finding the conditions under which a stable government can be formed and what policies this government would implement for this specific case, as a next step I obtain implications for the more general case where $v(g)$ could be any concave function. The next figure shows the Pareto Sets for the three groups of the society (or parties) when $v(g)$ is linear and the above stated conditions are satisfied:

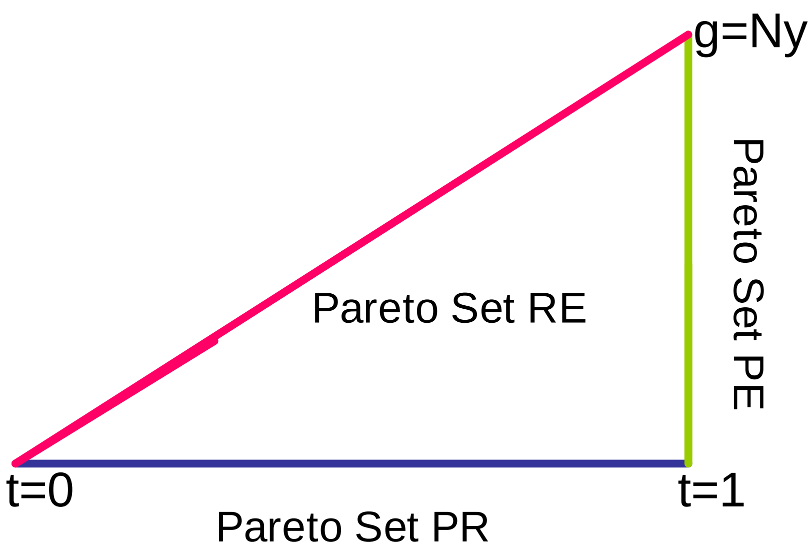

Figure 3

In this situation, all policies on the Pareto Set of two parties would be acceptable policies, yet when deciding which coalition to form a party does not only take the policy into account but also the amount of office spoils it would obtain. Notice that, a consensus government consisting of all three parties would never form as for any possible policy proposal any coalition of two parties would be better-off by deviating as they would increase their utility obtained from office spoils and could agree on a policy point that at least wouldn't decrease their utility from the implemented policy. Therefore, I focus simply on two party coalitions.

I first define under which conditions parties P and R would form a stable coalition. The result is as follows[2]:

Proposition 1: Coalition PR forms if and only if $\frac{B(\frac{n_R}{n_P+n_R}-\frac{n_R}{n_E+n_R})}{y_R-y} > \frac{B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R})}{y-y_P} + 1$ and E is the largest group in the society.

[2] For a formal proof of all propositions and lemmas see the appendix.

From the proposition above we can see that the necessary condition for PR to form is that E should be the largest group in the society. Moreover, for B large enough $\frac{B(\frac{n_R}{n_P+n_R}-\frac{n_R}{n_E+n_R})}{y_R-y} > \frac{B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R})+y-y_P}{y-y_P}$ will always be satisfied. The set of policies that could be implemented by coalition PR would be $g=0$ and $t \in [0,1]$ such that $\frac{B(\frac{n_R}{n_P+n_R}-\frac{n_R}{n_E+n_R})}{y_R-y} > t > \frac{B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R})}{y-y_P}+1$. Moreover, we can observe that the set of policies that coalition PR could implement does not depend on the degree on diversity, k. As B affects the left hand side of the policy set positively and the right hand side negatively, as B increases the set of policies that could be implemented would increase in both directions.

We can also check how the set of implementable policies by PR changes when y_R or y_P changes. As y depends on y_R and y_P we should substitute y with $y = \frac{y_R n_R + y_P(n_P+n_E)}{n_P+n_E+n_R}$. Therefore, the set of implementable policies becomes $\frac{B(\frac{n_R}{n_P+n_R}-\frac{n_R}{n_E+n_R})}{\frac{y_R(n_E+n_P)-y_P(n_P+n_E)}{n_P+n_E+n_R}} > t > \frac{B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R})}{\frac{y_R n_R - y_P n_R}{n_P+n_E+n_R}}+1$. Therefore, as y_R increases the left hand side decreases and the right hand increases. Therefore, the set of possible policies becomes smaller as R's loss from redistribution would be higher and P would loose more from giving up on redistribution as the income transfer it would get by forming coalition with E would be higher. On the other hand, as y_P increases the left hand side increases and the right hand side decreases. Therefore, the set of possible policies becomes larger. The intuition behind this finding is just the reverse of the one described above for an increase in y_R.

With a similar analysis as in Proposition 1 we can find when parties R and E would form a stable coalition. The result is as follows:

Proposition 2: Coalition RE forms iff $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > \frac{B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})+kNy-y_P}{kNy-y_P}$ and P is the largest group in the society.

From the proposition above we can see that the necessary condition for RE to form is that P should be the largest group in the society. For B large enough $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > \frac{B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})+kNy-y_P}{kNy-y_P}$ will always be satisfied. The set of policies that could be implemented by this coalition would be $g=tNy$ and any $t \in [0,1]$ such that $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > t > \frac{B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})+kNy-y_P}{kNy-y_P}$. As B affects the left hand side of the policy set positively and the right hand side negatively, as B increases, the set of policies that could be implemented would increase in both directions.

To see how the value of k affects the set of policies that could be implemented we should take the derivative of the right hand side of the set of policies that could be implemented which gives us: $\frac{-B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})}{(kNy-y_P)^2}$ which is always positive. Notice that k does not affect the left hand side as k does not affect the maximum utility that R can get from P. Therefore, if k increases, the set of policies that can be implemented decreases, in the sense that the minimum value of t that can be implemented increases. The intuition behind this fact is that as k increases, the maximum amount of utility that E can obtain from P increases which forces R to give up more in favor of E.

If y_R increases, the left hand side of $\frac{B(\frac{n_R}{n_R+n_E}-\frac{n_R}{n_R+n_P})}{y_R} > t > \frac{B(\frac{n_E}{n_E+n_P}-\frac{n_E}{n_R+n_E})+kNy-y_P}{kNy-y_P}$ decreases, and the right hand side which can be written as $\frac{B(\frac{n_E}{n_E+n_P}-\frac{n_E}{n_R+n_E})}{kNy-y_P}+1$ increases as y increases. Therefore, the set of possible policies becomes smaller as the maximum utility in terms of policy they would get from P would increase.

If y_P increases, the left hand side is not affected as y_P does not affect the maximum utility that R can get from P. To see how the right hand side is affected some steps of calculations are needed. The result is as follows:

Lemma 1: When RE form a stable coalition and y_P increases, $\frac{B(\frac{n_E}{n_E+n_P}-\frac{n_E}{n_R+n_E})+kNy-y_P}{kNy-y_P}$ also increases.

Therefore, as y_P increases, the set of possible policies becomes smaller in detriment of R. The intuition behind this fact is that as y_P increases, the maximum amount of utility that E can obtain from P increases which forces R to give up more in favor of E.

Finally, we can also find under which conditions PE would be formed as a stable coalition. The result is as follows:

Proposition 3: Coalition PE forms iff $BN(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > \frac{B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})+Ny(k-\frac{1}{N})}{k-\frac{1}{N}}$ and R is the largest group in the society.

From the proposition above we see that the necessary condition for PE to form is that R should be the largest group in the society. Besides, for B large enough $B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > \frac{B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})+Ny(k-\frac{1}{N})}{Nk-1}$ will always be satisfied. The set of policies that could be implemented by this coalition would be $t=1$ and any $g \in [0, Ny]$ such that $BN(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > g > \frac{B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})+Ny(k-\frac{1}{N})}{k-\frac{1}{N}}$ i.e. $BN(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > g > \frac{B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})}{k-\frac{1}{N}}+Ny$. As B affects the left hand side of the policy set positively and the right hand side negatively, as B increases, the set of policies that could be implemented would increase in both directions.

From the inequality above we can see that k does not affect the maximum value that can take g. On the other hand, the minimum value that g can take would depends positively on k as $B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})$ is negative. Therefore, if k increases, the set of policies that can be implemented decreases, in the sense that the minimum value of g that can be implemented increases. The intuition behind this fact is the same as in the case of coalition RE.

An increase in y_R doesn't affect the maximum amount of g that P would accept as the loss that P incurs when forming coalition with E derives only from g. On the other hand, the right hand side would increase as y would increase (thus the maximum utility that E can obtain from R) and $kN > 1$. Hence, the set of possible policies would become smaller in detriment of P. If y_P increases the same argument holds.

From the conditions which define when a certain coalition can form, we can see that only minimal winning coalitions in the sense of Riker (1962) form, that is, among all possible coalitions the one with the lowest total seat share possible forms.

From the propositions above, we have obtained that given that one party obtains more seats than its rivals a stable coalition would exist for B large enough. Therefore, as a next step I derive how the minimum value of B necessary for a stable coalitions to exist depends on the incomes and the degree of diversity. From Propositions 1, 2 and 3, we can obtain that a stable coalition would exist if B is larger than $B_{\min}$ where $B_{\min}$ is such that:

$$B_{\min} = \begin{cases} \frac{y_R - y}{\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R} - \frac{y_R - y}{y - y_P}\left(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}\right)} & \text{if } n_E > n_P \text{ and } n_E > n_R \\ \frac{y_R}{\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R} - \frac{y_R}{kNy - y_P}\left(\frac{n_E}{n_P+n_E} - \frac{n_E}{n_R+n_E}\right)} & \text{if } n_P > n_E \text{ and } n_P > n_R \\ \frac{y}{\frac{n_P}{n_E+n_P} - \frac{n_P}{n_P+n_R} - \frac{1}{kN-1}\left(\frac{n_E}{n_R+n_E} - \frac{n_E}{n_P+n_E}\right)} & \text{if } n_R > n_P \text{ and } n_R > n_E \end{cases}$$

Given that, we can find for all three cases how $B_{\min}$ is affected by a change in y_R, y_p and k. I first analyze the case when $n_E > n_P$ and $n_E > n_R$. Then, as we can see from above $B_{\min}$ does not depend on k. To see how the incomes affect this value, I substitute y with $y = \frac{y_R n_R + y_P(n_P+n_E)}{N}$. After some steps of calculations I obtain that $B_{\min} = \frac{(n_E+n_P)(y_R - y_P)/N}{\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R} - \frac{n_E+n_P}{n_R}\left(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}\right)}$. Therefore, as y_R increases $B_{\min}$ increases, and as y_P increases $B_{\min}$ decreases.

Now suppose that $n_P > n_E$ and $n_P > n_R$. Then as k increases $B_{\min}$ increases as $kNy - y_P$ increases and $-\frac{y_R}{kNy-y_P}\left(\frac{n_E}{n_P+n_E} - \frac{n_E}{n_R+n_E}\right)$ is positive. To see how $B_{\min}$ is affected by a change in the incomes as before I substitute y with $y = \frac{y_R n_R + y_P(n_P+n_E)}{N}$. After some steps it can be obtained that $B_{\min} = \frac{y_R}{\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R} - \frac{y_R}{[k(n_P+n_E)-1]y_P + kn_R y_R}\left(\frac{n_E}{n_P+n_E} - \frac{n_E}{n_R+n_E}\right)}$. From Lemma 1 we know that $k(n_P + n_E) > 1$. Thus, as y_P increases, $B_{\min}$ increases. To see how y_R affects $B_{\min}$, we should take its derivative with respect to y_R. Taking $\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R} = c$ (where $c > 0$) and $\frac{n_E}{n_R+n_E} - \frac{n_E}{n_P+n_E} = d$ (where $d > 0$), $B_{\min} = \frac{y_R}{c + d * \frac{y_R}{[k(n_P+n_E)-1]y_P + kn_R y_R}}$. Thus, $\partial B_{\min}/\partial y_R = \frac{c + d * \frac{y_R}{[k(n_P+n_E)-1]y_P + kn_R y_R} - \frac{d y_R [[k(n_P+n_E)-1]y_P}{[[k(n_P+n_E)-1]y_P + kn_R y_R]^2}}{m^2}$ where $m = c + d * \frac{y_R}{[k(n_P+n_E)-1]y_P + kn_R y_R} > 0$. Rearranging it we obtain that $\partial B_{\min}/\partial y_R = \frac{\frac{c + d y_R^2 kn_R}{[[k(n_P+n_E)-1]y_P + kn_R y_R]^2}}{m^2}$ which is always positive. Therefore, as y_R increases, $B_{\min}$ increases.

On the other hand, if $n_R > n_P$ and $n_R > n_E$ then if k increases, then $B_{\min}$ also increases. If y_R or y_P increases, then $B_{\min}$ also increases as y would increase.

From the analysis above we obtain that as the income of the rich group increases, for a stable coalition to exist the utility that parties obtain from being in office should increase. The same is also true if the degree of diversity increases. On the other hand, the effect of the income of the poor on $B_{\min}$ depends on the seat share, i.e. the relative size of the groups.

From the above conditions we can also deduce that the value of $B_{\min}$ depends on how close the relative seat shares of the parties are. As the seat share of all three parties becomes closer, the value of $B_{\min}$ increases. When all three parties have the same vote share a stable coalition fails to exist.

3.1 When the Ethnic Minority has a Quasi-Linear Utility Function

In this part, I discuss the implications of the results above to the case when the ethnic minority has a quasi-linear utility function. That is, individuals of the ethnic minority have the following utility function: $U_E = y_P + t(y - y_P) - \frac{g}{N} + kv(g)$ where $v(g)$ is a concave function and $v'(Ny) < 1/Nk$. When party E forms part of the government, the utility it obtains from the implemented policy takes also the same form. Therefore, the Pareto Set of the parties would be the one depicted in Figure 2.

Notice that for a certain stable coalition to exist, the necessary condition would be the same as before, that is, those parties forming the coalition should be the two smallest ones in terms of their seat shares. By examining Proposition 1, we can see that the results do no depend on the utility function of E. Therefore, the necessary and sufficient conditions for coalition PR to be stable would be the same as in Proposition 1. Moreover, the effect of a change in k, B, y_R or y_P would be the same as before.

Considering the case when coalition PE would be formed, given that party R is the largest party in the society, by reconsidering Proposition 3, the maximum amount of g acceptable by P would not be affected. On the other hand, the minimum amount of g acceptable by party E should satisfy $kv(g) - \frac{g}{N} + \frac{n_E}{n_P + n_E} B > \frac{n_E}{n_R + n_E} B + kv(g^*) - \frac{g^*}{N}$ where g^* is the level of public good at the ideal point of E satisfying $\frac{1}{N} = kv'(g^*)$. As before, if B is large enough there would exist a g which would give both parties a utility higher than the maximum they would obtain from R. The main difference from before is that observing the condition for the minimum g acceptable for E, we can see that it does not depend on y_P or y_R. The reason is that unlike the case of completely linear utilities, an increase in the level of incomes does not affect the maximum utility that party E can get from party R. Therefore, since as before, the maximum amount of g acceptable for P does not depend neither on y_P nor y_R, a change in the level of incomes would have no effect on the set of policies that could be implemented by the coalition PE.

As before, an increase in k would have no effect on the maximum level of g acceptable for party P. On the other hand, if k increases, as $\frac{1}{N} = kv'(g^*)$ and $v(g)$ is concave, g^* would increase which implies that the maximum utility that party E would obtain from party R would increase not only because k

increases but also because g^* does. Therefore, the utility that party E should get from party P should also increase more than the simple increase of k which implies that the minimum level of g acceptable for E should also increase. Thus, as in the case of linear utilities as k increases the set of policies that can be implemented by PE would become smaller in detriment of P and the poor group of the ethnic majority.

Finally, if we consider the case when RE could form a stable coalition, from Figure 2 we can see that the Pareto Set of these two parties is different than when the ethnic minority has linear preferences (Figure 3). The Pareto Set of RE consists of three different parts. I will focus on the case where the set of policies that could be implemented by a stable coalition of RE would only be part of their Pareto Set where g would take a fixed value. (the line connecting points C and D in the figure below)

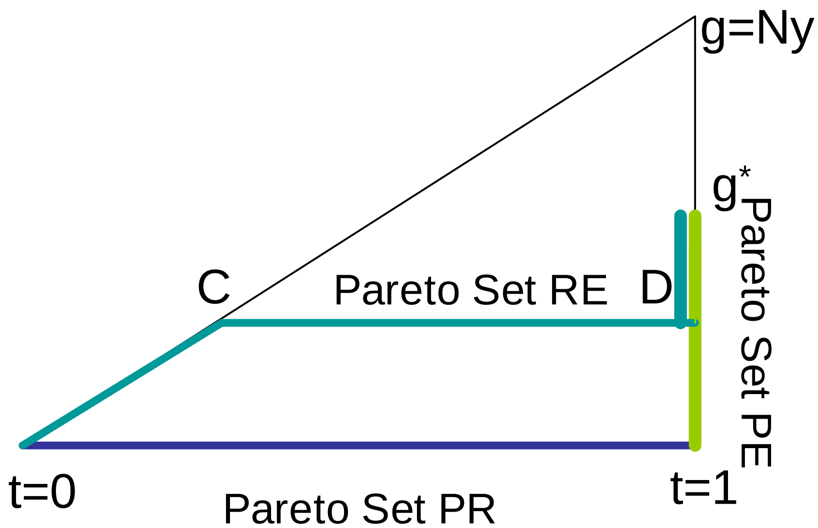

Figure 4

In that case the value of g on their Pareto Set, denoted as g^{PS}, that would be implemented should satisfy equation (2), i.e. $kv'(g^{PS}) = \frac{y_P - y_R}{N(y - y_R)}$. Therefore, from the same argument as in Proposition 2, for RE to form, R should be able to get more utility from E than the maximum it could get from P, i.e. t should be such that $t < \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R}) - \frac{g^{PS}}{N}}{y_R - y}$. Similarly, E should be able to get more utility from R than the maximum it could get from P, i.e. t should be such that $B\frac{n_E}{n_R+n_E} + (1-t)y_P + ty - y + kv(g^{PS}) - \frac{g^{PS}}{N} > B\frac{n_E}{n_P+n_E} - \frac{g^*}{N} + kv(g^*)$. As before, for RE to form, the two inequalities above should have a common solution which implies that P should be the largest group in the society and B should take a value sufficiently large. Moreover, as B increases the set of policies that coalition RE could implement would also increase.

Now, I analyze how a change in y_R or y_P would affect the set of policies that could be implemented by coalition RE. First, I rewrite $kv'(g^{PS}) = \frac{y_P - y_R}{N(y-y_R)}$. Substituting y with $\frac{y_R n_R + y_P(n_P+n_E)}{N}$ we obtain that $kv'(g^{PS}) = \frac{y_P - y_R}{N(\frac{y_R n_R + y_P(n_P+n_E)}{N} - y_R)} = \frac{1}{n_P+n_E}$. Therefore, a change in the levels of income would have no effect on g^{PS}. As $kv'(g^*) = \frac{1}{N}$ it has no effect on g^* neither. If y_R increases, from $t < \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R}) - \frac{g^{PS}}{N}}{y_R - y}$ we can obtain that the maximum t satisfying it would decrease as $y_R - y$ would increase. (The increase in y_R would be more than the increase in y). For E, as y_R increases the minimum amount of t acceptable would increase as y would increase. Therefore, the set of possible policies would become smaller. As the level of incomes has no effect on g^{PS} which takes a constant value, the intuition behind the effect of a change in the income levels on the set of policies that could be implemented is the same as in the case of coalition PR.

If y_P increases, the mean income, y, would increase. Thus, from $t < \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R}) - \frac{g^{PS}}{N}}{y_R - y}$ we can obtain that the maximum t satisfying it would increase. To see how it would affect the minimum value of t acceptable for party E we have to analyze how an increase in y_P affects $(1-t)y_P + ty - y$ which can be written as $(1-t)\frac{n_R y_P - n_R y_R}{n_R + n_P + n_E}$. Therefore, as y_P increases, the minimum value of t would decrease. Thus, as y_P increases, the set of possible policies would become larger.

Finally I analyze how the set of policies that could be implemented by coalition RE would be affected if the degree of diversity, k, increases. Since $kv'(g^{PS}) = \frac{1}{n_P+n_E}$ and $v(g)$ is concave, as k increases, g^{PS} increases (which implies less income redistribution). Thus as for party R we need that $t < \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R}) - \frac{g^{PS}}{N}}{y_R - y}$, as k increases, the maximum t satisfying it would decrease. Since $kv'(g^*) = \frac{1}{N}$, as k increases, g^* also increases. Hence, to find whether the minimum t that party E would accept would increase or decrease as k increases we should find whether $kv(g^*) - g^*/N$ or $kv(g^{PS}) - g^{PS}/N$ would increase more.

In order to achieve this goal, for illustrative reasons, I will take a specific function for $v(g)$: namely $v(g) = g^{\frac{1}{2}}$. Thus, $g^* = \frac{k^2N^2}{4}$ and $g^{PS} = \frac{k^2(n_E+n_P)^2}{4}$. Now suppose that k increases to k_1. Then, $g_1^* = \frac{k_1^2N^2}{4}$ and $g_1^{PS} = \frac{k_1^2(n_E+n_P)^2}{4}$. Thus, the increase in $kv(g^*) - g^*/N$ is $\frac{k_1^2N}{2} - \frac{k_1^2N}{4} - (\frac{k^2N}{2} - \frac{k^2N}{4})$ and the increase in $kv(g^{PS}) - g^{PS}/N$ is $\frac{k_1^2(n_E+n_P)}{2} - \frac{k_1^2(n_E+n_P)^2}{4N} - (\frac{k^2(n_E+n_P)}{2} - \frac{k^2(n_E+n_P)^2}{4N})$. Now I show that $\frac{k_1^2N}{2} - \frac{k_1^2N}{4} - (\frac{k^2N}{2} - \frac{k^2N}{4}) > \frac{k_1^2(n_E+n_P)}{2} - \frac{k_1^2(n_E+n_P)^2}{4N} - (\frac{k^2(n_E+n_P)}{2} - \frac{k^2(n_E+n_P)^2}{4N})$. After some steps, this expression can be reduced as $-\frac{1}{4}(k^2 - k_1^2)(-2(n_P+n_E) + N + (n_P+n_E)^2N > 0$. Notice that $-\frac{1}{4}(k^2-k_1^2) > 0$, so this inequality holds if $-2(n_P+n_E) + N + (n_P+n_E)^2N > 0$. So, we need $N > \frac{2(n_P+n_E)}{(n_P+n_E)^2+1}$ which increases in (n_P+n_E). So, if it holds for the maximum value of (n_P+n_E), i.e. N, it would always hold. If $n_P + n_E = N$, then we have $N^3 - N > 0$ which always holds as $N > 1$. Therefore, as k increases we should find whether $kv(g^*) - g^*/N$ would increase more which implies that as k increases the minimum t that E would accept would increase. Thus, as k increases the set of policies that coalition RE could implement becomes smaller.

Comparing the case of linear and quasi-linear utilities, we can see that it does not affect the analysis for the case where coalition PR forms. When coalition PE forms the only difference is that the income has no effect on the policies that could be implemented, as the maximum amount of utility that party E can get from party R does not depend on it. When RE forms, by focusing on the part of their Pareto Set where the level of g is constant, the effect of the income is as in the case for coalition PR since the income would be the only variable of conflict.

3.2 When Parties are both Office and Policy Motivated

In this section I analyze the conditions for the existence of a stable coalition and the set of policies the stable coalition can be implement when parties obtain a utility from being in office and also from the policy implemented whether they form part of the government or not. That is, I consider the case of both office and policy motivated parties. Therefore, the utility functions of the parties would take the following form:

$$V_i(C,X) = \begin{cases} B * \dfrac{n_i}{\sum\limits_{j\in C} n_j} + U_i(t,g) & \text{if party } i \text{ forms part of the government} \\ U_i(t,g) & \text{if party } i \text{ does not form part of the government} \end{cases}$$

As in the first part of the previous analysis I consider the case where the utility function of the ethnic minority is linear and has the same characteristics as before. That is, $v(g) = g$ and $k > \frac{1}{N}$ and $(y_R - y) > \frac{y - y_P}{Nk-1}$. For a similar reasoning as before, only two party governments would be formed. Given that, I analyze under which conditions each of the three possible two party governments would form.

I first analyze the conditions under which parties P and R would form a stable government. The result is as follows:

Proposition 4: Coalition PR forms if and only if E is the largest group in the society and

i. $\frac{B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y} > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})}{y - y_P} + 1$ when $y - y_P \leq B(\frac{n_P}{n_P+n_R} + \frac{n_E}{n_E+n_R} - \frac{n_P}{n_E+n_P})$

ii. $\frac{y_R[\frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}] + B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y} > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})}{y - y_P} + 1$ when $y - y_P > B(\frac{n_P}{n_P+n_R} + \frac{n_E}{n_E+n_R} - \frac{n_P}{n_E+n_P})$

From the proposition above we can see that the necessary condition for PR to form is that E should be the largest group in the society. Moreover, for B large enough the inequalities stated in Proposition 4i. and ii. will always be satisfied. The set of policies that could be implemented by this coalition would be $g = 0$ and any $t \in [0,1]$ such that $\frac{B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y} > t > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) + y - y_P}{y - y_P}$ if $y - y_P \leq B(\frac{n_P}{n_P+n_R} + \frac{n_E}{n_E+n_R} - \frac{n_P}{n_E+n_P})$ and $g = 0$ and any $t \in [0,1]$ such that $\frac{y_R[\frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}] + B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y} >$

$t > \frac{B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R})}{y-y_P}+1$ if $y-y_P > B(\frac{n_P}{n_P+n_R}+\frac{n_E}{n_E+n_R}-\frac{n_P}{n_E+n_P})$. The most important conclusion we can draw from the result above is that compared to the case where parties only care instrumentally about the policy implemented, when parties care about policy it is more likely that PR could form a stable coalition.

With a similar analysis as in Proposition 4 we can find when parties P and E would form a stable coalition. The result is as follows:

Proposition 5: Coalition PE forms if and only if R is the largest group in the society and
i. $B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > \frac{B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})+Ny(k-\frac{1}{N})}{Nk-1}$ when $y \leq B(\frac{n_R}{n_E+n_R}+\frac{n_P}{n_E+n_P}-\frac{n_P}{n_R+n_P})$
ii. $B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > \frac{y_P-y+(kNy-y_P)[\frac{y_R-y+B(\frac{n_R}{n_E+n_R}+\frac{n_P}{n_E+n_P}-\frac{n_P}{n_R+n_P})}{y_R}]+B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})}{Nk-1}$ when $y > B(\frac{n_R}{n_E+n_R}+\frac{n_P}{n_E+n_P}-\frac{n_P}{n_R+n_P})$

From the proposition above we see that the necessary condition for PE to form is that R should be the largest group in the society. Besides, for B large enough the inequalities stated in Proposition 5i. and ii. will always be satisfied. The set of policies that could be implemented by this coalition would be $t = 1$ and any $g \in [0, Ny]$ such that $BN(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > g > \frac{B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})+Ny(k-\frac{1}{N})}{k-\frac{1}{N}}$ if $y \leq B(\frac{n_R}{n_E+n_R}+\frac{n_P}{n_E+n_P}-\frac{n_P}{n_R+n_P})$; or if $y > B(\frac{n_R}{n_E+n_R}+\frac{n_P}{n_E+n_P}-\frac{n_P}{n_R+n_P})$ then $t = 1$ and $g \in [0, Ny]$ such that $B(\frac{n_P}{n_P+n_E}-\frac{n_P}{n_P+n_R}) > g > \frac{y_P-y+(kNy-y_P)[\frac{y_R-y+B(\frac{n_R}{n_E+n_R}+\frac{n_P}{n_E+n_P}-\frac{n_P}{n_R+n_P})}{y_R}]+B(\frac{n_E}{n_R+n_E}-\frac{n_E}{n_P+n_E})}{Nk-1}$. As before, if parties care about policy it is more likely that parties P and E would form a stable coalition compared to the case where parties care only instrumentally about policy.

Finally, we can also find under which conditions RE would be formed as a stable coalition. The result is as follows:

Proposition 6: Coalition RE forms if and only if P is the largest group in the society and
i. $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > \frac{B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})+kNy-y_P}{kNy-y_P}$ when $B(\frac{n_P}{n_P+n_E}) \geq [y_R - B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$
ii. $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > \frac{y-y_P+(kN-1)[y+B(\frac{n_P}{n_P+n_E})-[1-\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_R+n_P})}{y_R}]y_P]+B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})}{kNy-y_P}$ when $B(\frac{n_P}{n_P+n_E}) < [y_R - B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$

From the proposition above we can see that the necessary condition for RE to form is that P should be the largest group in the society. For B large enough, the inequalities stated in Proposition 6i. and ii. will always be satisfied. The set of policies that could be implemented by this coalition would be $g = tNy$ and $t \in [0, 1]$ such that $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > t > \frac{B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})}{kNy-y_P}+1$ if $B(\frac{n_P}{n_P+n_E}) \geq [y_R - B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$; or if $B(\frac{n_P}{n_P+n_E}) < [y_R - B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$ then $g = tNy$ and any $t \in [0, 1]$ such that: $\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_P+n_R})}{y_R} > t > \frac{y-y_P+(kN-1)[y+B(\frac{n_P}{n_P+n_E})-[1-\frac{B(\frac{n_R}{n_E+n_R}-\frac{n_R}{n_R+n_P})}{y_R}]y_P]+B(\frac{n_E}{n_P+n_E}-\frac{n_E}{n_R+n_E})}{kNy-y_P}$.

As before, if parties care about policy it is more likely that parties R and E would form a stable coalition compared to the case where parties care only instrumentally about policy.

The intuition behind the main result obtained here; that is, the intuition why stable coalitions could form easier when parties care about policy is that the maximum that the party staying outside the government would be willing to give to one of the two parties forming the government might be less, since now staying outside the government gives this party a higher utility than before.

Both for office and instrumentally policy motivated and for office and policy motivated parties we have obtained that for a stable coalition to exist the benefits of office should be large enough. This result implies that if parties are only policy motivated no stable coalition would exist.

On the other hand, if parties were solely office motivated then the smallest parties would form the government and they would implement any policy independent of the level of incomes or the degree of diversity.

4 Discussion and Concluding Remarks

The results of this paper show that in a multi-dimensional political environment, where parties are principally office motivated and care only instrumentally about policy or when parties are both office and policy motivated, for a stable government coalition to exist the benefits of office that the coalition would share should be large enough. Given the distribution of votes, the income levels of the different groups and the degree of diversity, the utility obtained from office spoils should exceed a certain level. As the level of income of the rich or the degree of diversity in terms of the desire for a public good increases this critical value also increases. When parties are both office and policy motivated it is more likely that a stable coalition exists.

While in some countries where no party obtains a majority the duration of governments is short in other countries despite the multidimensionality of the political environment some parties can agree on forming a government which manages to remain in power for a long period. The results obtained in this model sheds light on this difference in the sense that one explanation could be that the higher the level of income of the rich part of the society or the higher the degree of diversity in the society the more difficult would be to have stable a coalition government.

On the other hand, while in some countries as it happened recently in Belgium it might take a long time to form a government in other countries the coalition formation process terminates in a short time. This variation in the time and effort spent to form a government could also be a result of both the difference of the difficulty to have the necessary conditions to form a stable government and the degree of diversity in terms of preferences that would force parties to give up more in terms of preferred policies to reach a compromise.

In the model analyzed in this paper the two dimensions are interrelated in the sense that the maximum amount of public good that can be provided depends on the tax rate implemented. Therefore, the policy set has the shape of a triangle. This framework helps to analyze a situation where the conflict in economic terms in the society does not only arise from the degree of taxation but also from how the collected taxes would be redistributed. On the other hand, the main results of the model do not depend on this specific framework. If we would consider two dimensions which are not interrelated and where the ideal points of the three parties are such that they would form a triangle, as in the model at hand for a stable coalition to exist the benefits of office should be higher than a certain value where this value would depend on the parameters which define the diversity in preferences. The distinctive feature of the framework used in the model at hand is that it allows to analyze the effect of diversity not only on the non-economic dimension but also how diversity would affect the policy compromise on the economic dimension.

In this paper, I have considered that parties obtain a positive utility from being in office and forming a government. Alternatively, we could also consider a case where parties would obtain a negative utility from forming a coalition with another party in top of the possible loss in terms of the implemented policy. This disutility or cost could be interpreted as the time and effort spent to reach an agreement or simply a non-policy disutility on sharing the power with parties who represent another socioeconomic group. The cost that a party would face by forming a coalition with another party could be analyzed in a similar way as I analyze the distribution of office spoils. That is, we could consider that the cost a party faces by forming a coalition with another party increases as the size of the other party increases and decreases as its own size increases. In that case, depending on the degree of disutility each party faces with any of its rival, different stable coalitions might exist. Contrary to the case at hand for a stable coalition to exist the level of costs should be lower than a certain critical level but always positive since with no cost or benefits as this paper shows there would exist no stable coalition. The next step would be to analyze the exact characterization of the conditions that guarantee the existence of stable coalitions and to describe how the income levels and the degree of diversity would affect both the policy outcome and the stable coalitions that would form.

In this paper I have considered voters who vote sincerely, that is, voters cast their vote to the party that makes the policy announcement they like the most. It would be interesting to analyze under the same framework defined in this model whether the set of stable coalitions and the policies they would implement would be affected if voters are forward-looking in the sense that when deciding to which party to cast their vote they would take into account their expectation about which government would be formed.

Appendix

Proof of Proposition 1: A stable coalition can only be formed if all parties forming that coalition could agree on a policy such that they cannot obtain a higher utility by breaking the coalition and forming another coalition with the third party. That is, there should exist a policy supported by PR which gives them at least the same amount of utility they could maximally obtain from forming coalition with E. This policy supported by PR should be on their Pareto Set, i.e., $t \in [0,1]$, $g = 0$. If P and R form a coalition then E would obtain no utility, so E would be willing to form a coalition with P for any policy on their Pareto Set. Thus, the maximum utility that P can get from forming coalition with E is when $t = 1$ and $g = 0$. Then, $V_P = \frac{n_P}{n_P+n_E}B + y$. Thus, the policy offer that R can make P ($t \in [0,1]$, $g = 0$) should satisfy $y_P + t(y - y_P) + \frac{n_P}{n_P+n_R}B > \frac{n_P}{n_P+n_E}B + y$ which would only hold if $n_E > n_R$ and t is such that $t > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) + y - y_P}{y - y_P}$. If $t > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) + y - y_P}{y - y_P}$ and $n_E > n_R$, P would not break the coalition with R, but then, as E would prefer to form a coalition with R to being outside the government, E would be willing to form a coalition with R for any policy on their Pareto Set. Thus, the maximum utility that R can get from forming coalition with E is when $t, g = 0$, i.e. $V_R = \frac{n_R}{n_R+n_E}B + y_R$. So, the policy offer that P would make R ($t \in [0,1]$, $g = 0$) should satisfy $y_R + t(y - y_R) + \frac{n_R}{n_P+n_R}B > \frac{n_R}{n_R+n_E}B + y_R$ which would only hold if $n_E > n_P$ and t is such that $t < \frac{B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_R+n_E})}{y_R - y}$. Therefore, PR would be a stable coalition if and only if $\frac{B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y} > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) + y - y_P}{y - y_P}$ and E is the largest group. #

Proof of Proposition 2: From the same argument as in Proposition 1, coalition RE will form if both R and E cannot increase their utility by breaking the coalition and forming another coalition with P. That is, there should exist a policy supported by RE which gives them at least the same amount of utility they would maximally obtain from forming coalition with P. This policy supported by RE should be on their Pareto Set, i.e., $t \in [0,1]$, $g = tNy$. If E and R form a coalition then P would obtain no utility, so P would be willing to form a coalition with R for any policy on their Pareto Set. Thus, the maximum utility that R can get from forming coalition with P is when $t = 0$ and $g = 0$. Then $V_R = \frac{n_R}{n_R+n_P}B + y_R$. Thus the policy offer that E can make R ($t \in [0,1]$, $g = tNy$) should satisfy $(1-t)y_R + \frac{n_R}{n_R+n_E}B > \frac{n_R}{n_R+n_P}B + y_R$ which would only hold if $n_P > n_E$ and t is such that $t < \frac{B(\frac{n_R}{n_R+n_E} - \frac{n_R}{n_R+n_P})}{y_R}$. If $t < \frac{B(\frac{n_R}{n_R+n_E} - \frac{n_R}{n_R+n_P})}{y_R}$ and $n_P > n_E$, R would not break the coalition with E, but then, as P would prefer to form a coalition with E to being outside the government, P would be willing to form a coalition with E for any policy on their Pareto Set. Thus, the maximum utility that E can get from forming coalition with P is when $t = 1$ and $g = Ny$, i.e. $V_E = \frac{n_E}{n_P+n_E}B + kNy$. So, the policy offer that R should make E ($t \in [0,1]$, $g = tNy$) should satisfy $(1-t)y_P + ktNy + \frac{n_E}{n_R+n_E}B > \frac{n_E}{n_P+n_E}B + kNy$ which would only hold if $n_P > n_R$ and t is such that $t > \frac{B(\frac{n_E}{n_E+n_P} - \frac{n_E}{n_R+n_E}) + kNy - y_P}{kNy - y_P}$. Therefore, RE would be a stable coalition if and only if $\frac{B(\frac{n_R}{n_R+n_E} - \frac{n_R}{n_R+n_P})}{y_R} > \frac{B(\frac{n_E}{n_E+n_P} - \frac{n_E}{n_R+n_E}) + kNy - y_P}{kNy - y_P}$ and P is the largest group. #

Proof of Lemma 1: $\frac{B(\frac{n_E}{n_E+n_P} - \frac{n_E}{n_R+n_E}) + kNy - y_P}{kNy - y_P}$ can be written as $\frac{B(\frac{n_E}{n_E+n_P} - \frac{n_E}{n_R+n_E})}{kNy - y_P} + 1$. So, to see how an increase in y_P affects this expression, we have to find how it affects the denominator, $kNy - y_P$, which can be written as $k[y_R n_R + y_P(n_P + n_E)] - y_P$. So, this expression would increase in y_P if $k(n_P + n_E) > 1$. We had assumed that $(y_R - y) > \frac{y - y_P}{Nk-1}$. As $n_R \leq n_P + n_E$, we should have that $y_R - y \geq y - y_P$ which implies that $\frac{y_R - y}{y - y_P} \geq 1 > \frac{1}{Nk-1}$. Thus, $Nk > 2$. Hence, $k(n_P + n_E) > 1$ as the minimum value that can take $n_P + n_E$ is $\frac{N}{2}$ which implies that $kNy - y_P$ increases as y_P increases. Therefore, as y_P increases, $\frac{B(\frac{n_E}{n_E+n_P} - \frac{n_E}{n_R+n_E}) + kNy - y_P}{kNy - y_P}$ also increases as $B(\frac{n_E}{n_E+n_P} - \frac{n_E}{n_R+n_E}) < 0$. #

Proof of Proposition 3: From the same argument as in Proposition 1, coalition PE will form if both P and E cannot increase their utility by breaking the coalition and forming another coalition with R. That is, there should exist a policy supported by PE which gives them at least the same amount of utility they would maximally obtain from forming coalition with R. This policy supported by PE should be on their Pareto Set, i.e., $t = 1$, $g \in [0, Ny]$. If E and P form a coalition then R would obtain no utility, so R would be willing to form a coalition with P for any policy on their Pareto Set. Thus, the maximum utility that R can get from forming coalition with P is when $t = 1$ and $g = 0$. Then, $V_P = \frac{n_P}{n_R+n_P}B + y$. Thus the policy offer that E can make P ($t = 1$, $g \in [0, Ny]$) should satisfy $y - \frac{g}{N} + \frac{n_P}{n_P+n_E}B > \frac{n_P}{n_R+n_P}B + y$ which would only hold if $n_R > n_E$ and g is such that $g < BN(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_R+n_P})$. If $g < BN(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_R+n_P})$ and $n_R > n_E$, P would not break the coalition with E, but then, as R would prefer to form a coalition with E to being outside the government, R would be willing to form a coalition with E for any policy on their Pareto Set. Thus, the maximum utility that E can get from forming coalition with R is when $t = 1$ and $g = Ny$, i.e. $V_E = \frac{n_E}{n_R+n_E}B + kNy$. So the policy offer that P should make E ($t = 1$, $g = [0, Ny]$) should satisfy $kg + y - \frac{g}{N} + \frac{n_E}{n_P+n_E}B > \frac{n_E}{n_R+n_E}B + kNy$ which would only hold if $n_R > n_P$ and t is such that $g > \frac{B(\frac{n_E}{n_R+n_E} - \frac{n_E}{n_P+n_E}) + kNy - y}{k - \frac{1}{N}}$. Therefore, PE would be a stable coalition if and only if $B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_R+n_P}) > \frac{B(\frac{n_E}{n_R+n_E} - \frac{n_E}{n_P+n_E}) + kNy - y}{k - \frac{1}{N}}$ and R is the largest group. #

Proof of Proposition 4: For the same reason as in the previous propositions for PR to form there should exist a policy supported by PR which gives them at least the same amount of utility they could maximally obtain from forming coalition with E. This policy supported by PR should be on their Pareto Set, i.e. $t \in [0, 1]$, $g = 0$. If P and R form a coalition then E would obtain as utility the policy implemented by PR. However, for any policy implemented by PR (including $t = 1$, $g = 0$), party E would be better off by forming a coalition with P as it would increase its total utility. So E would be willing to form a coalition with P for any policy on their Pareto Set. Thus, the maximum utility that P can get from forming coalition with E is when $t = 1$ and $g = 0$. Then, $V_P = \frac{n_P}{n_P+n_E}B + y$. Thus, the policy offer that R can make P ($t \in [0, 1]$, $g = 0$) should satisfy $y_P + t(y - y_P) + \frac{n_P}{n_P+n_R}B > \frac{n_P}{n_P+n_E}B + y$ which would only hold if $n_E > n_R$ and t is such that $t > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) + y - y_P}{y - y_P}$. Supposing that these two conditions hold P would never break the coalition with R. However, then for any policy that could implement PR, RE could agree on a policy on their Pareto Set which would increase their utility obtained from the policy implemented. Thus for PR to be stable as a necessary condition we need $n_E > n_P$ such that for any such policy the total utility of R would decrease. If this necessary condition also holds then the best policy offer that E would be willing to make R would be the policy point on the Pareto Set of RE ($t \in [0,1]$, $g = tNy$) that would give E the same utility that it would obtain when PR implement $t = \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) + y - y_P}{y - y_P}$ and $g = 0$. For this policy the utility that E would obtain is $y_P + [\frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})}{y - y_P} + 1](y - y_P)$ which can be written as $y + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})$. Thus the best policy point that E would offer R would be the policy point on their Pareto Set satisfying $y_P + t(y - y_P) - ty + ktNy + B\frac{n_E}{n_E+n_R} = y + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})$ which implies that $t = \frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}$. Notice that if $y - y_P \leq B(\frac{n_P}{n_P+n_R} + \frac{n_E}{n_E+n_R} - \frac{n_P}{n_E+n_P})$ then $t \leq 0$ such that the E would be willing to accept the ideal point of R implying that PR would form for exactly the same conditions as in the previous section which is replicated in **i.** On the other hand, if $y - y_P > B(\frac{n_P}{n_P+n_R} + \frac{n_E}{n_E+n_R} - \frac{n_P}{n_E+n_P})$ then $t > 0$; so the maximum that would R get from E is $y_R[1 - \frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}] + B\frac{n_R}{n_E+n_R}$. So, the policy offer that P should make R ($t \in [0,1]$, $g = 0$) should satisfy $y_R + t(y - y_R) + \frac{n_R}{n_P+n_R}B > y_R[1 - \frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}] + B\frac{n_R}{n_E+n_R}$ which implies that t should be such that $t < \frac{y_R[\frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}] + B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y}$. Therefore, when $y - y_P > B(\frac{n_P}{n_P+n_R} + \frac{n_E}{n_E+n_R} - \frac{n_P}{n_E+n_P})$, PR would be a stable coalition if and only if $\frac{y_R[\frac{y - y_P + B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R} - \frac{n_E}{n_E+n_R})}{kNy - y_P}] + B(\frac{n_R}{n_P+n_R} - \frac{n_R}{n_E+n_R})}{y_R - y} > \frac{B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})}{y - y_P} + 1$ and E is the largest group.#

Proof of Proposition 5: For PE to form there should exist a policy supported by PE which gives them at least the same amount of utility they could maximally obtain from forming coalition with R. This policy supported by PE should be on their Pareto Set, i.e., $t = 1$, $g \in [0, Ny]$. If P and E form a coalition then R would obtain as utility the policy implemented by PE. However, for any policy implemented by PE (including $t = 1$, $g = 0$), party R would be better off by forming a coalition with P as it would increase its total utility. So R would be willing to form a coalition with P for any policy on their Pareto Set. Thus, the maximum utility that P can get from forming coalition with R is when $t = 1$ and $g = 0$. Then, $V_P = \frac{n_P}{n_P+n_R}B + y$. Thus, the policy offer that E can make P ($t = 1$, $g \in [0, Ny]$) should satisfy $y + \frac{n_P}{n_P+n_E}B - \frac{g}{N} > \frac{n_P}{n_P+n_R}B + y$ which would only hold if $n_R > n_E$ and g is such that $g < BN(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})$. Supposing that these two conditions hold P would never break the coalition with E. However, then for any policy that could implement PE, RE could agree on a policy on their Pareto Set which would increase their utility obtained from the policy implemented. Thus for PR to be stable as a necessary condition we need $n_R > n_P$ such that for any such policy the total utility of E would decrease. If this necessary condition also holds then the best policy offer that R would be willing to make E would be the policy point on the Pareto Set of RE ($t \in [0,1]$, $g = tNy$) that would give R the same utility that it would obtain when PR implement $g = BN(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})$ and $t = 1$. For this policy the utility that R would obtain is $y - B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})$. Thus the best policy point that R would offer E would be the policy point on their Pareto Set satisfying $y_R + t(y - y_R) - tNy/N + B\frac{n_R}{n_E+n_R} = y - B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R})$ which implies that $t = \frac{y_R - y + B(\frac{n_R}{n_E+n_R} + \frac{n_P}{n_E+n_P} - \frac{n_P}{n_R+n_P})}{y_R}$.

Notice that if $y \leq B(\frac{n_R}{n_E+n_R} + \frac{n_P}{n_E+n_P} - \frac{n_P}{n_R+n_P})$ then $t \geq 1$ such that the R would be willing to accept the ideal point of E implying that PE would form for exactly the same conditions as in the previous section which is replicated in **i.** On the other hand, if $y > B(\frac{n_R}{n_E+n_R} + \frac{n_P}{n_E+n_P} - \frac{n_P}{n_R+n_P})$ then $t < 1$; so the maximum that would E get from R is $y_P + (kNy - y_P)[\frac{y_R - y + B(\frac{n_R}{n_R+n_E} + \frac{n_P}{n_P+n_E} - \frac{n_p}{n_P+n_R})}{y_R}] + B\frac{n_E}{n_E+n_R}$. So, the policy offer that P should make E ($t = 1$, $g \in [0, Ny]$) should satisfy $y + g(k - \frac{1}{N}) + \frac{n_E}{n_P+n_E}B > y_P + (kNy - y_P)[\frac{y_R - y + B(\frac{n_R}{n_R+n_E} + \frac{n_P}{n_P+n_E} - \frac{n_p}{n_P+n_R})}{y_R}] + B\frac{n_E}{n_E+n_R}$ which implies that g should be such that $g > \frac{y_P - y + (kNy - y_P)[\frac{y_R - y + B(\frac{n_R}{n_E+n_R} + \frac{n_P}{n_E+n_P} - \frac{n_P}{n_R+n_P})}{y_R}] + B(\frac{n_E}{n_R+n_E} - \frac{n_E}{n_P+n_E})}{k - 1/N}$. Therefore, when $y > B(\frac{n_R}{n_E+n_R} + \frac{n_P}{n_E+n_P} - \frac{n_P}{n_R+n_P})$, PE would be a stable coalition if and only if $B(\frac{n_P}{n_P+n_E} - \frac{n_P}{n_P+n_R}) > \frac{y_P - y + (kNy - y_P)[\frac{y_R - y + B(\frac{n_R}{n_E+n_R} + \frac{n_P}{n_E+n_P} - \frac{n_P}{n_R+n_P})}{y_R}] + B(\frac{n_E}{n_R+n_E} - \frac{n_E}{n_P+n_E})}{Nk - 1}$ and R is the largest group. #

Proof of Proposition 6: For RE to form there should exist a policy supported by RE which gives them at least the same amount of utility they could maximally obtain from forming coalition with P. This policy supported by RE should be on their Pareto Set, i.e., $t \in [0, 1]$, $g = tNy$. If R and E form a coalition then P would obtain as utility the policy implemented by RE. However, for any policy implemented by RE (including $t = 0$, $g = 0$), party P would be better off by forming a coalition with R as it would increase its total utility. So P would be willing to form a coalition with R for any policy on their Pareto Set. Thus, the maximum utility that R can get from forming coalition with P is when $t, g = 0$. Then, $V_R = \frac{n_R}{n_P+n_R}B + y_R$. Thus, the policy offer that E can make R ($t = 1$, $g \in [0, Ny]$) should satisfy $y_R(1 - t) + \frac{n_R}{n_R+n_E}B > \frac{n_R}{n_P+n_R}B + y_R$ which would only hold if $n_P > n_E$ and t is such that $t < \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}$. Supposing that these two conditions hold R would never break the coalition with E. However, then for any policy that could implement RE, PE could agree on a policy on their Pareto Set which would increase their utility obtained from the policy implemented. Thus for RE to be stable as a necessary condition we need $n_P > n_R$ such that for any such policy the total utility of E would decrease. If this necessary condition also holds then the best policy offer that P would be willing to make E would be the policy point on the Pareto Set of PE ($t = 1$, $g \in [0, Ny]$) that would give P the same utility that it would obtain when RE implement $t = \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}$ and $g = tNy$. For this policy the utility that P would obtain is $[1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}]y_P$. Thus the best policy point that P would offer E would be the policy point on their Pareto Set satisfying $y - g/N + B\frac{n_P}{n_E+n_P} = [1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}]y_P$ which implies that $g = N[y + B(\frac{n_P}{n_P+n_E}) - [1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}]y_P]$. Notice that if $B(\frac{n_P}{n_P+n_E}) \geq [y_R - B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$ then $g \geq Ny$ such that the P would be willing to accept the ideal point of E implying that RE would form for exactly the same conditions as in the previous section which is replicated in **i.** On the other hand, if $B(\frac{n_P}{n_P+n_E}) < [y_R - B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$ then $g < Ny$; so the maximum that would E get from P is $y + (k - 1/N)N[y + B(\frac{n_P}{n_P+n_E}) - [1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}]y_P] + B\frac{n_E}{n_E+n_P}$. So, the policy offer that R should make E ($t \in [0, 1]$, $g = tNy$) should satisfy $y_P + t(kNy - y_P) +$

$\frac{n_E}{n_E+n_R}B > y + (k-1/N)N[y + B(\frac{n_P}{n_P+n_E}) - [1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R}]y_P] + B\frac{n_E}{n_E+n_P}$ which implies that t should be such that $t > \frac{y - y_P + (kN-1)[y + B(\frac{n_P}{n_P+n_E}) - [1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_R+n_P})}{y_R}]y_P] + B(\frac{n_E}{n_P+n_E} - \frac{n_E}{n_R+n_E})}{kNy - y_P}$. Therefore, when $B(\frac{n_P}{n_P+n_E}) < [y_R - B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_R+n_P})]\frac{y_P}{y_R}$, RE would be a stable coalition if and only if $\frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_P+n_R})}{y_R} > \frac{y - y_P + (kN-1)[y + B(\frac{n_P}{n_P+n_E}) - [1 - \frac{B(\frac{n_R}{n_E+n_R} - \frac{n_R}{n_R+n_P})}{y_R}]y_P] + B(\frac{n_E}{n_P+n_E} - \frac{n_E}{n_R+n_E})}{kNy - y_P}$ and P is the largest group. #

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