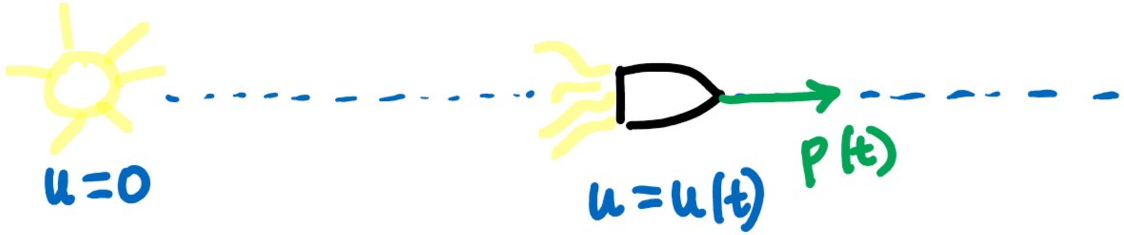
$d = 1$

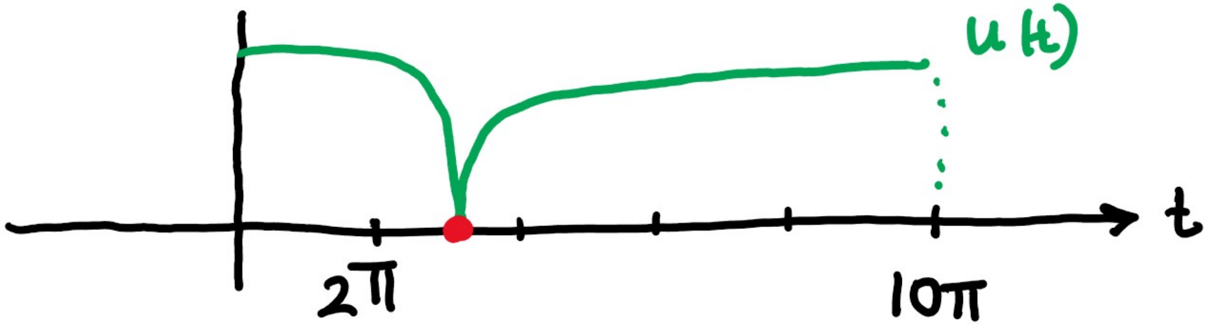
$$\ddot{u} = -\frac{1}{u^2} + p(t), \quad u \geq 0$$



rocket flight

$p : \mathbb{R} \to \mathbb{R}$ is C^1 and 2π-periodic

$\forall N \geq 1$ $\exists$ two generalized $2\pi N$-periodic solutions having exactly one collision on $[0, 2\pi N[$



$N = 5$

Cooperative systems

$$\dot{u} = v, \quad \dot{v} = -\frac{1}{u^2} + p(t), \qquad (u,v) \in]0,\infty[\times \mathbb{R}$$

Kamke condition

$$\dot{u} = f(t,u,v), \quad \dot{v} = g(t,u,v)$$

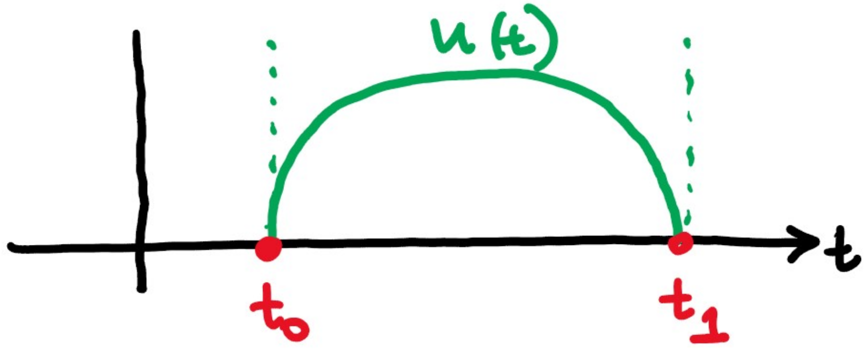
$$\frac{\partial f}{\partial v} \geq 0, \quad \frac{\partial g}{\partial u} \geq 0$$

Monotone flow

$$u(t_0) \leq U(t_0), \; \dot{u}(t_0) \leq \dot{U}(t_0) \Rightarrow u(t) \leq U(t), \; \dot{u}(t) \leq \dot{U}(t)$$

$t \geq t_0$ before collisions

The Successor map



$$S : (t_0, E_0) \longmapsto (t_1, E_1)$$

$$E_0 = \lim_{t \to t_0^+} \left[\frac{1}{2} \dot{u}(t)^2 - \frac{1}{u(t)} \right], \quad E_1 = \lim_{t \to t_1^-} \left[\frac{1}{2} \dot{u}(t)^2 - \frac{1}{u(t)} \right]$$

$p(t)$ 2π-periodic $\Longrightarrow$ $S(t_0 + 2\pi, E_0) = S(t_0, E_0) + (2\pi, 0)$

t_0 angular variable

$$S : \mathcal{D} \subset \left(\mathbb{R}/2\pi\mathbb{Z}\right) \times \mathbb{R} \to \left(\mathbb{R}/2\pi\mathbb{Z}\right) \times \mathbb{R}$$

$$\mathcal{D} = \{ (t_0, E_0) : t_1 < \infty \}$$

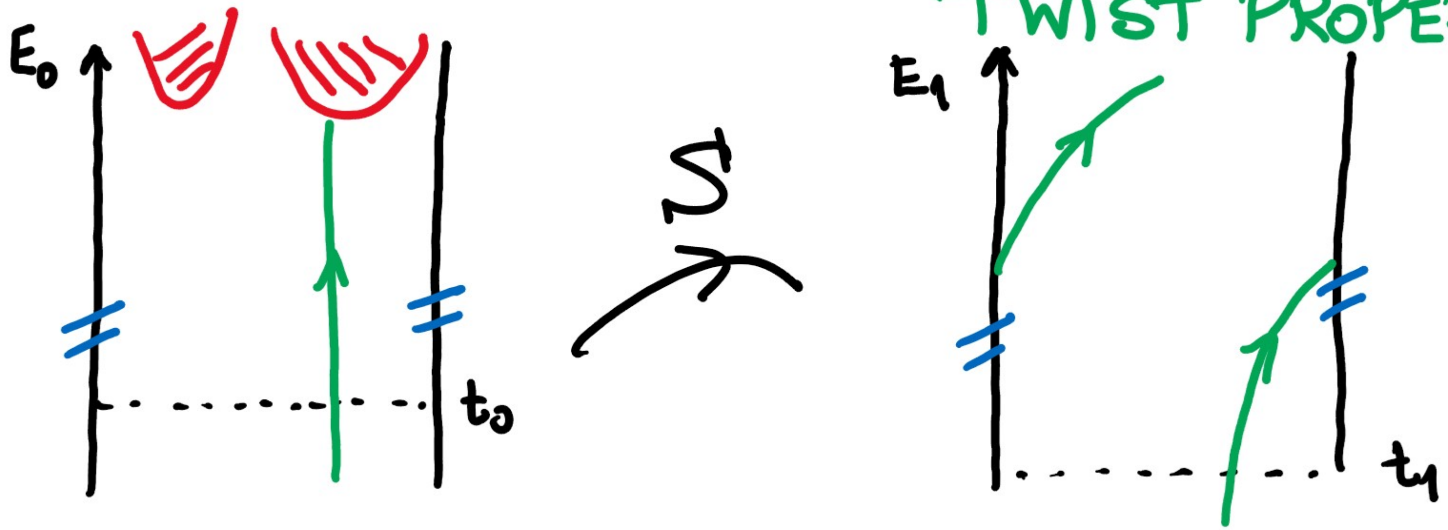
Two properties of S

i) $\mathcal{D} = \{ (t_0, E_0) : E_0 < \Psi(t_0) \}$, $\Psi : \mathbb{R}/_{2\pi\mathbb{Z}} \to \mathbb{R} \cup \{+\infty\}$

lower semi-continuous

ii) $\forall t_0 \in \mathbb{R}$, $E_0 \longmapsto t_1(t_0, E_0)$ is strictly increasing

TWIST PROPERTY

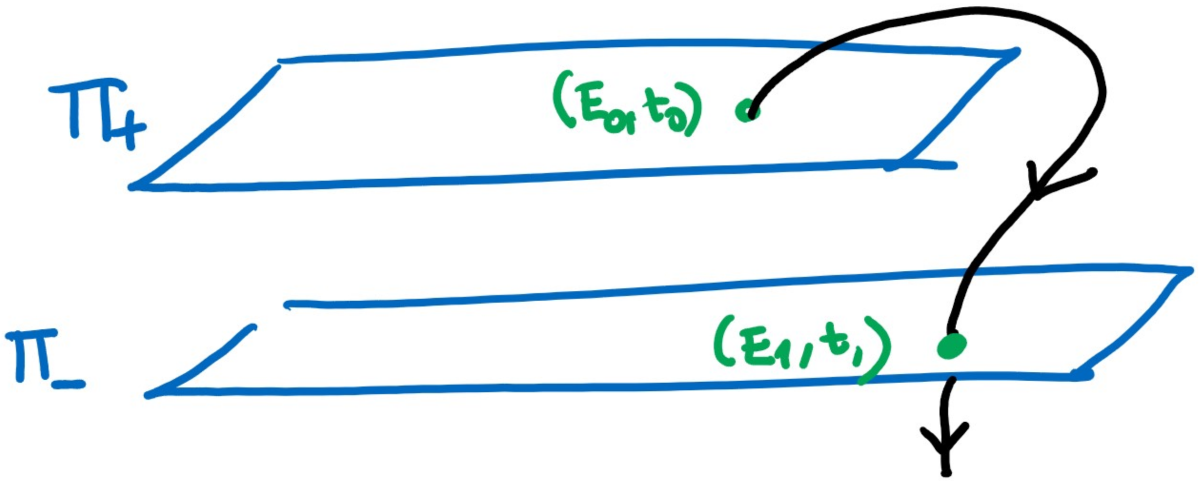


S is C^1

Levi-Civita regularization, $u \in \mathbb{C} \rightsquigarrow (z, w, E, \bar{t})$

$\{z = \bar{z},\ w = \bar{w}\}$ invariant set $(d = 1)$

$$\Pi_{\pm} = \{ z = 0,\ w = \pm 2\sqrt{2} \}$$

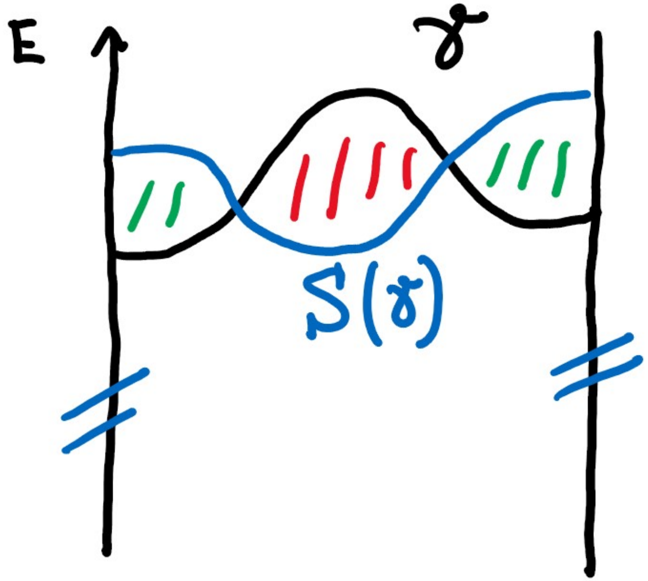


$\Pi_{\pm}$ transversal sections

$\Downarrow$

S smooth if $p(t)$ smooth

S is exact symplectic



γ Jordan curve $\gamma \not\sim 0$

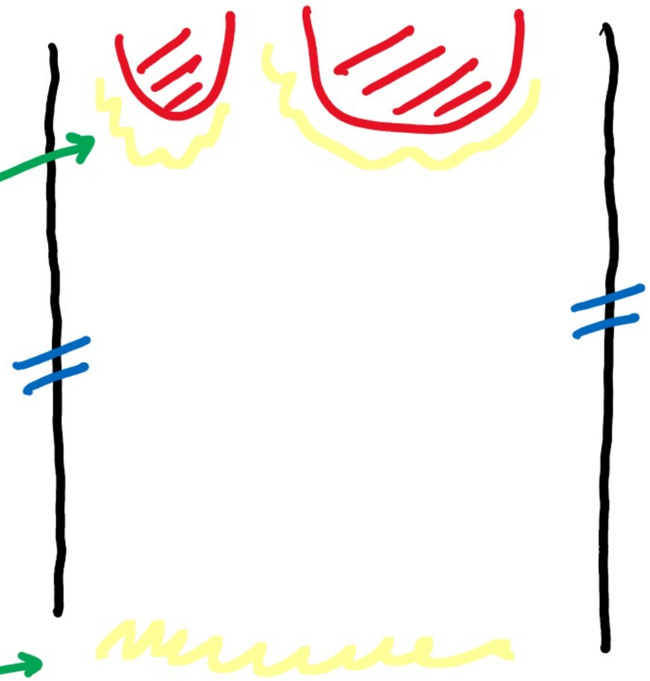
Red area = Green area

Proof: LC is Hamiltonian $\Rightarrow$ Regularized flow is symplectic

+ Stokes th

Application of Poincaré-Birkhoff Theorem

large twist



small twist

$t_1(t_0, E_0) - t_0 \longrightarrow +\infty$

if $(t_0, E_0) \longrightarrow \partial\mathcal{D}$

$t_1(t_0, E_0) - t_0 \longrightarrow 0$

if $E_0 \searrow -\infty$

$t_1 = t_0 + 2N\pi \;,\; E_1 = E_0$ has at least 2 solutions