The flow of Kepler problem is not complete

$$\dot{x} = v, \quad \dot{v} = -\frac{x}{|x|^3}, \quad x \in \mathbb{R}^2 \setminus \{0\}$$

$x(t)$ solution, I maximal interval

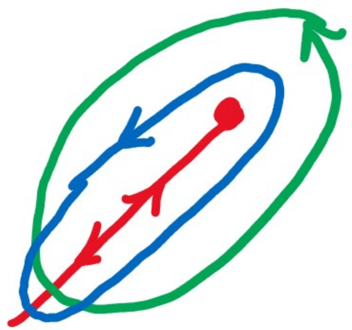
$\vec{c} \neq 0, \quad I = \mathbb{R}$

$\vec{c} = 0, \quad I \neq \mathbb{R}$

ejection $I =]\alpha, \infty[$, collision $I =]-\infty, \omega[$

ejection / collision $I =]\alpha, \omega[$

Ejection/collisions orbits are a limit case of elliptic orbits



a, ω, τ fixed

$e \nearrow 1$

Example: $a = 1, \omega = 0, \tau = 0, e_n \nearrow 1$

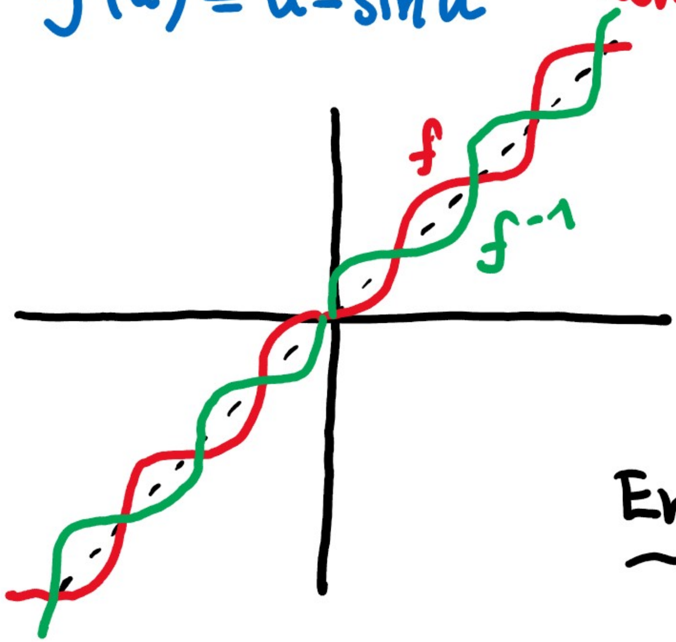
$$x_n(t) = \begin{pmatrix} \cos u_n(t) - e_n \\ \sqrt{1-e_n^2}\, \sin u_n(t) \end{pmatrix}, \quad u_n - e_n \sin u_n = t$$

$$x_n(t) \longrightarrow x_*(t), \quad \text{uniformly in } t \in \mathbb{R}$$

The limit $x_*(t) = \begin{pmatrix} \cos u_*(t) \\ 0 \end{pmatrix}$

$u_* - \sin u_* = t \iff f(u) = t \iff u(t) = f^{-1}(t)$

$f(u) = u - \sin u$ homeomorphism of $\mathbb{R}$, $f'(2n\pi) = 0$



$u_*(t) = f^{-1}(t)$ is not C^1

$x_*(t)$ ejection / collision solution
on $t \in]2\pi n, 2\pi(n+1)[$

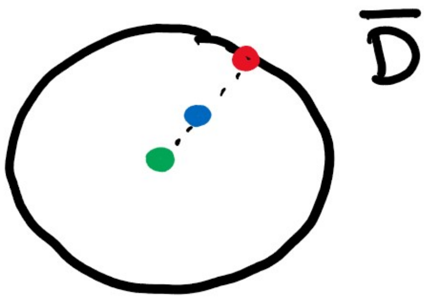
Energy + direction preserved at collisions

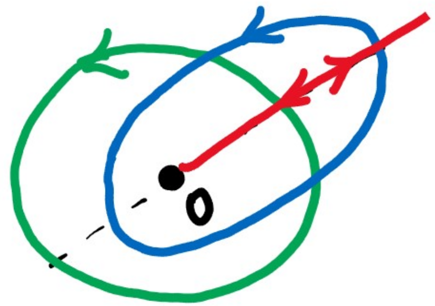
Topology of $\tilde{\mathcal{P}}_T$ (i)

$\tilde{\mathcal{P}}_T = \mathcal{P}_T \cup \{$T-periodic extension of ejection/collision solutions$\}$

$\mathcal{P}^+_{T,n} \approx D \times \mathbb{R}/_{2\pi\mathbb{Z}}$ $\quad (\vec{e}, \bar{\theta})$, $\quad |\vec{e}| < 1$, $\theta = a^{2/3}\tau$

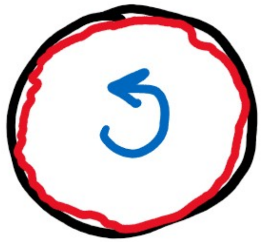
$\tilde{\mathcal{P}}^+_{T,n} \approx \bar{D} \times \mathbb{R}/_{2\pi\mathbb{Z}}$ $\quad |\vec{e}| \leq 1$





Topology of $\tilde{\mathcal{P}}_T$ (ii)

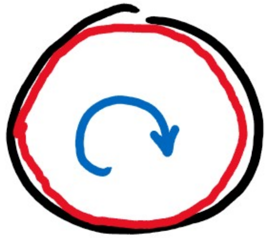
$\tilde{\mathcal{P}}^{+}_{T,n}$:



$\times \mathbb{R}/2\pi\mathbb{Z}$

$\overline{D}$

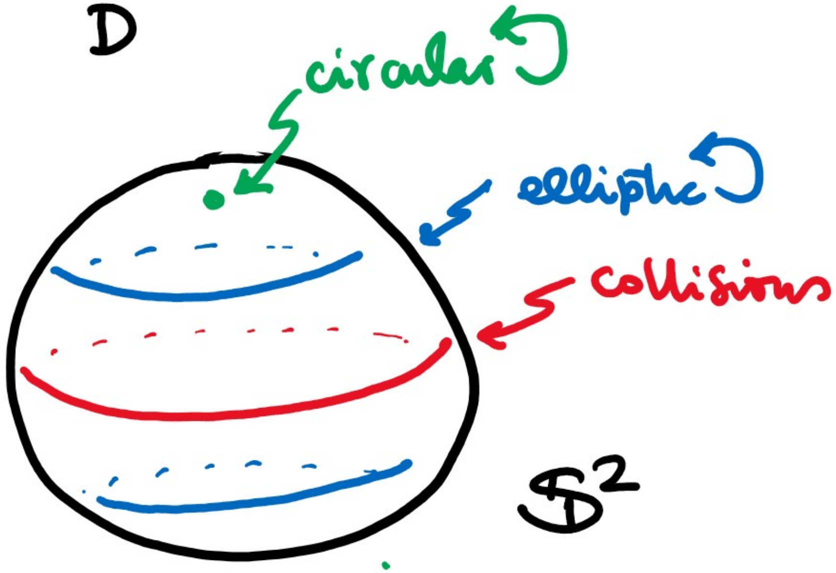
$\tilde{\mathcal{P}}^{-}_{T,n}$:



$\times \mathbb{R}/2\pi\mathbb{Z}$

$\overline{D}$

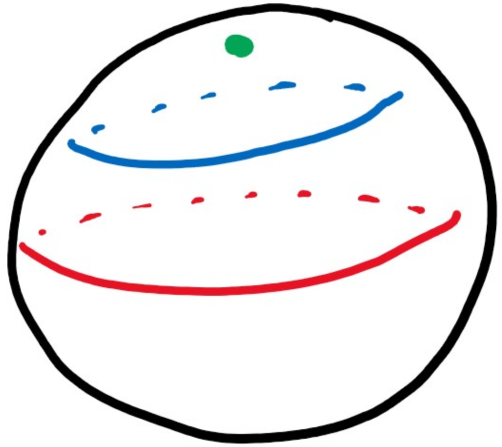
$\tilde{\mathcal{P}}_{T,n}$:

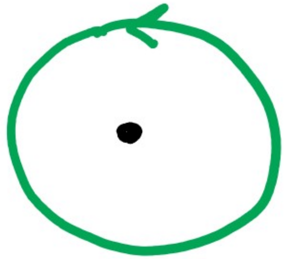


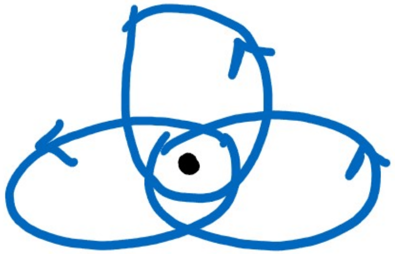
$\times \mathbb{R}/2\pi\mathbb{Z}$

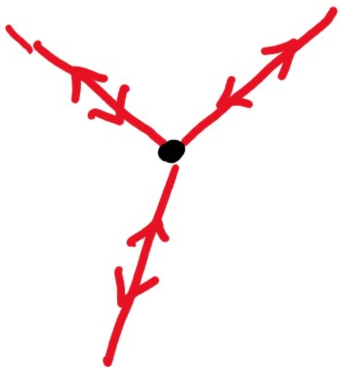
S^2

Topology of $\tilde{\mathcal{P}}_T$ (iii)









$$\tilde{\mathcal{P}}_T = \bigcup_{n=1}^{\infty} \tilde{\mathcal{P}}_{T,n}$$

$$\cancel{\tilde{\mathcal{P}}_{T,n} \approx S^2 \times S^1}$$

The connected components are compact manifolds