Problemas de Steklov mixtos en dimensión 2

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Resumen

En 1902 fue considerado el problema de Steklov [2] sobre una superficie Riemanniana compacta y con frontera Lipschitz. Desde entonces ha sido ampliamente estudiado. En particular, Weinstock [3] comenzó el estudio de cotas superiores para el primer valor propio en 1954 y sus resultados fueron extendidos al resto de valores propios por Hersch, Payne, and Schiffer [1] en 1975.

La charla se centrará en mostrar como el estudio de cotas superiores para los problemas de Steklov mixtos proporciona resultados que mejoran las cotas superiores obtenidas por Hersch, Payne, and Schiffer.

Este es un trabajo conjunto con Emily B. Dryden, Carolyn S. Gordon, Asma Hassannezhad, Allie Ray y Elizabeth Stanhope.

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El segundo teorema de Minkowski para variedades Riemannianas.

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Resumen

En esta charla, presentaremos generalizaciones del segundo teorema de Minkowski a variedades Riemannianas. Por ejemplo, explicaremos por qué los grafos ponderados, los toros Riemannianos de cualquier dimensión o las superficies Riemannianas con volumen normalizado admiten siempre una base de homología inducida por geodésicas cerradas cuyo producto de longitud está acotado superiormente por alguna constante que depende únicamente de su topología.

Trabajo conjunto con S. Karam y H. Parlier.

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A new approach to helicoidal surfaces in the 3-sphere

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Resumen

We deal with helicoidal surfaces in the unit 3-sphere $\mathbb{S}^3$, i.e. surfaces invariant under the action of the helicoidal 1-parameter group of isometries given by the composition of a translation and a rotation in $\mathbb{S}^3$. This class includes the rotational ones. We propose a new approach to their study ([**?**]), introducing the notion of the *spherical angular momentum* of the profile curve of the helicoidal surface. It will play a key role since it determines the geometry of the helicoidal surface joint to its pitch. Inspired by [**?**], we show that we can prescribe the mean curvature of a spherical helicoidal surface in terms of a function depending on the distance to its axis, obtaining a one parameter family of helicoidal surfaces with that prescribed mean curvature. Moreover, we reduce any type of Weingarten condition (i.e. any functional relation between the principal curvatures) on a rotational surface to a first order differential equation on the momentum of its generatrix curve. As a first application, we identify (searching in [**?**] and [**?**]) the minimal surfaces in $\mathbb{S}^3$ that play the role of classical catenoids and helicoids and describe all the minimal spherical helicoidal surfaces connecting them, proving that they are their *associated* surfaces (in the sense of [**?**]).

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Extending submanifolds from Euclidean spaces to symmetric spaces of noncompact type

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Resumen

In this talk I will present a simple method that allows to enlarge submanifolds from certain totally geodesic, flat submanifolds of symmetric spaces of noncompact type to the whole ambient symmetric space, in such a way that the codimension, the mean curvature, and other geometric properties are preserved. As an application, I will explain how one can construct the first examples of inhomogeneous isoparametric hypersurfaces in each symmetric space of noncompact type and rank at least three. This is based on a joint work with Víctor Sanmartín-López [**?**].

Referencias

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Weingarten surfaces and Bernstein type problems

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Resumen

A surface in Euclidean 3-space is an elliptic Weingarten surface if its mean curvature H and Gaussian curvature K are related by a smooth, elliptic equation $W(H,K) = 0$. A well known open problem, proposed for instance by Rosenberg and Sa Earp in 1994, is to solve the Bernstein problem for this class of surfaces, that is: are planes the only entire elliptic Weingarten graphs? Up to now, it is only known that the answer is positive if the Weingarten equation is uniformly elliptic; this follows from a deep theorem by L. Simon on entire graphs with quasiconformal Gauss map.

In this talk we present two theorems. In the first one, we extend the solution to the Bernstein problem in the uniformly elliptic case to multigraphs, proving that planes are the only complete uniformly elliptic Weingarten surfaces whose Gauss map image lies in an open hemisphere. In the second one, we will solve in the affirmative the Bernstein problem for Weingarten graphs for a large class of non-uniformly elliptic Weingarten equations.

Upper bounds for the first eigenvalue of geodesic and extrinsic balls

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Resumen

In this talk I will explain how to obtain upper bounds for the first eigenvalue for a geodesic ball in a Riemannian manifold via an orthogonal symmetritation of the metric tensor and, moreover I will explain how to obtain upper bounds for the fundamental tone when the manifold admits an isometric, minimal, and proper immersion into the Euclidean space in terms of the growth of the volume of its extrinsic balls. This is a joint work with Erik Sarrión Pedralva [**?**] and independently, a joint work with Pacelli Bessa and Panagiotis Polymerakis [**?**].

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Variations for submanifolds in graded manifolds

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Resumen

The aim of this talk is to present the deformability properties of submanifolds immersed in graded manifolds that are a generalization of Carnot manifolds. We consider an area functional defined on submanifolds of fixed degree immersed into a graded manifold equipped with a Riemannian metric. Since the expression of this area depends on the degree, not all variations are admissible. It turns out that the associated variational vector fields must satisfy a system of partial differential equations of first order on the submanifold. Moreover, given a vector field solution of this system, we provide a sufficient condition that guarantees the possibility of deforming the original submanifold by variations preserving its degree. In the one-dimensional case, the integrability of compact supported vector fields depends on the surjection of the holonomy map at the endpoints. As in the case of singular curves in sub-Riemannian geometry, there are examples of isolated surfaces that cannot be deformed in any direction. This talk is based on my joint work with G. Citti and M. Ritoré.

Espacios con curvatura positiva.

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Resumen

El clásico Teorema de Gauss-Bonnet implica que la única superficie compacta orientable que admite una métrica de curvatura Gaussiana positiva es la esfera. En esta charla discutiremos posibles generalizaciones de dicho resultado a dimensiones superiores. Más concretamente, en la primera parte de la presentación repasaremos algunos resultados conocidos sobre variedades que admiten una métrica Riemanniana de curvatura seccional, de Ricci o escalar positiva. En la segunda parte complementaremos estos resultados considerando condiciones de curvatura intermedias.

Sasakian structures on tangent sphere bundles

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Resumen

There is an extensive bibliography on the Riemannian geometry of the tangent sphere bundle T_rM of any radius $r > 0$, equipped with the induced metric $\widetilde{g}^S$ from the Sasaki metric g^S on the tangent bundle TM. See G. Calvaruso [?] for surveys of the unit tangent sphere bundle T_1M and O. Kowalski and M. Sekizawa [?] of T_rM, for an arbitrary radius $r > 0$.

With respect to the *standard almost complex structure* J, the Sasaki metric on TM is almost Kähler. Then T_rM in (TM, J, g^S) inherits an almost contact metric structure $(\varphi, \xi, \eta, \widetilde{g}^S)$, where ξ is the Hopf vector field $\xi = -JN$, N being its outward normal unit vector field. Moreover, the pair $(\frac{1}{2}\eta, \frac{1}{4}\widetilde{g}^S)$ on T_1M is K-contact if and only if (M, g) has constant sectional curvature 1. In this case, T_1M is Sasakian.

Sasakian manifolds can be considered in many respects as the class analogous to that of the Kähler manifolds for the odd dimensional case.

In this lecture, some of the results given a recent paper [?] about the existence of Sasakian structures on T_rM, where (M, g) has no constant sectional curvature, are exposed. As far as the author knows, there are no examples satisfying these conditions in the literature. More precisely, we show that the tangent sphere bundle $T_r(G/K)$, for any $r > 0$, of any compact rank-one symmetric space G/K, admits a unique Sasakian structure whose characteristic vector field is the Hopf vector field. Such a structure can be expressed as an induced structure from an almost Hermitian structure on the punctured tangent bundle $T(G/K) \setminus \{\text{zero section}\}$.

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El problema isoperimétrico en la 3-esfera sub-riemanniana

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Resumen

En la 3-esfera unidad $\mathbb{S}^3 \subset \mathbb{R}^4$ consideramos la estructura sub-riemanniana obtenida al restringir la métrica de curvatura constante 1 a la distribución ortogonal a las fibras de la fibración de Hopf. Estudiaremos el problema que surge al minimizar el área horizontal entre todas las superficies compactas y embebidas de clase C^2 que encierran un volumen dado. Analizaremos los puntos críticos del problema, destacando que las soluciones son esferas o toros con CMC (curvatura media sub-riemanniana constante). Construiremos una familia de esferas de revolución y justificaremos que estas esferas son candidatas naturales para resolver el problema. Finalmente, emplearemos herramientas de segunda variación para probar que todos los toros CMC son inestables, lo que nos permitirá concluir que las soluciones son congruentes a las esferas descritas.

Referencias

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Índice de subvariedades mínimas y compactas de las esferas de Berger

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Resumen

Se estudia el índice de las subvariedades mínimas y compactas de las esferas de Berger. A diferencia de la esfera estándar, en este caso aparecen ejemplos estables, los cuales son caracterizados bajo ciertas condiciones.

También, en el caso de superficies, se clasifican las de índice uno.