Maximal surfaces, Born-Infeld solitons and Ramanujan's identities

Abstract: In the first part of these lectures we make an observation that the maximal surface equation and Born-Infeld equation (which arises in physics in the context of nonlinear electrodynamics) are related by a wick rotation. Using this observation we present a method to construct a one parameter family of complex Born-Infeld solitons (solutions of Born-Infeld equation) from a given one parameter family of maximal surfaces. We shall also show that a Born-Infeld soliton can be realised either as a spacelike minimal graph or timelike minimal graph over a timelike plane or a combination of both away from singular points. In the next part we discuss a different formulation for describing maximal surfaces in Lorentz-Minkowski space, $\mathbb{L}^3$, using the identification of $\mathbb{L}^3$ with $\mathbb{C} \times \mathbb{R}$. This description of maximal surfaces will help us to give a different proof of the singular Björling problem for the case of closed real analytic null curve. As an application, we show the existence of maximal surface which contains a given spacelike closed curve and has a special singularity. Finally in the last part of these lectures we show the connection of maximal surfaces to analytic number theory through certain Ramanujan's identities.