On the surfaces with the same mean curvature in the Euclidean 3-space and the Lorentz-Minkowski 3-space, and the $H_R = H_L$ surface equation.

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Spacelike surfaces in the Lorentz-Minkowski 3-dimensional space $\mathbb{L}^3$ can be endowed with another Riemannian metric, the one induced by the Euclidean space $\mathbb{R}^3$. Those surfaces are locally the graph of a smooth function $u(x, y)$ satisfying $|Du| < 1$. If in addition they have the same mean curvature with respect to both metrics, they are the solutions to a certain partial differential equation, the $H_R = H_L$ surface equation.

It is well known that the only surfaces that are simultaneously minimal in $\mathbb{R}^3$ and maximal in $\mathbb{L}^3$ are open pieces of helicoids and of spacelike planes, [3]. The proof of this result consists in proving that those surfaces are ruled surfaces. And finishes using that the only ruled surfaces which are maximal and minimal at the same time are the plane and the helicoid.

In this talk we consider the general case of spacelike surfaces with the same mean curvature with respect to both metrics. Firstly, we prove the existence of examples with non-zero mean curvature. Afterwards, we show that our surfaces do not have any elliptic points. As an application of this result, jointly with a classical argument on the existence of elliptic points due to Osserman [4], we present several geometric consequences for the surfaces we are considering. Finally, we focus on the $H_R = H_L$ equation. Its character is studied, some uniqueness results for its Dirichlet problem are given, as well as some uniqueness and non-existence results for entire solutions.

References

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