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Electricity consumption in Morocco: Stochastic Gompertz diffusion analysis with exogenous factors

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Abstract

This paper proposes a means of using stochastic diffusion processes to model the total consumption of electrical power (including distribution and transport losses) in Morocco, as recorded by the official data for total sales published by Office Nationale de l'Électricité (ONE), the Moroccan electricity authority. Two models of univariate stochastic diffusion were used: the time-homogeneous Gompertz Diffusion Process (HGDP) and the time-non-homogeneous Gompertz Diffusion Process (NHGDP). The methodology proposed is based on the analysis of the trend function; this requires the analyst to obtain fits and forecasts for the consumption of electrical power by means of the estimated trend function (conditioned and non-conditioned). This latter function is obtained from the mean value of the process and the maximum likelihood estimators (MLE) of the parameters of the model. This estimation and the subsequent statistical inference are based on the discretised observation of the variable "electricity consumption in Morocco", using annual data for the period 1980–2001. The fit and forecast are improved by using macroeconomic exogenous factors such as the gross domestic product per inhabitant (GDP/inhab), the final domestic consumption (FDC) and the gross fixed capital formation (GFCF). The results obtained show that NHGDP, (with the above three exogenous factors) provides an adequate fit and medium-term forecast of electricity consumption in Morocco.

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Keywords: Time-homogeneous and non-homogeneous Gompertz diffusion; Exogenous factors; Discrete sampling, Fits and forecasts; Electricity consumption

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Nomenclature

$\hat{a}$	estimator of the parameter a
A'	transpose of A
AR	autoregressive model
ARIMA	autoregressive Integrated Moving Average model
DH	Dirham: Moroccan currency
ECTF	estimated conditional trend function
ETF	estimated trend function
FDC	final domestic consumption in 10^6 Dirhams
GDP/inhab	gross domestic product per inhabitant in Dirhams
GFCF	gross fixed capital formation in 10^6 Dirhams
HGDP	homogenous Gompertz diffusion process
I_n	identity matrix of order n
$\mathbb{L}$	likelihood function
MLE	maximum likelihood estimator
$\mathcal{N}_1$	univariate normal distribution
NHGDP	non-homogenous Gompertz diffusion process
ONE	state electricity authority in Morocco
p.d.f	probability density function
SDE	stochastic differential equation
$x(t)$	random electricity consumption at time t
Λ_1	univariate lognormal distribution

1. Introduction

In recent decades, the total consumption of electricity in Morocco has been characterized by a sustained, significant increase. According to official figures [1] during the period 1992–2002 the annual rates of variation ranged from 2.69% (1992/93) to 6.9% (1997/98), while consumption in 2002, at 14085 Mkwh, was 61.4% higher than in 1992. During the earlier period 1982–1992, consumption increased even more rapidly, with annual rates of variation ranging from 3.72% (1983/1984) to 8.51% (1986/1987); total consumption in 1992 was 89.5% higher than in 1982, when it was 4605.5 Mkwh. This pattern of electricity consumption over the two periods mentioned, with a slight deceleration in the 1990s, is of great interest with respect to the modelling procedures discussed in this paper. As concerns total consumption of primary sources of energy in Morocco, it is noteworthy that in 2002 this was 52.47% higher than in 1992. In the latter year, the consumption of electrical energy represented 32.9% of that of total primary energy, while in 2002, the corresponding figure was 34.89%. Thus, the proportion of electrical energy within total primary energy consumption rose significantly during the above period. The electrical sector, hence, is an essential and increasingly significant component in the energy market in Morocco. Other variables related to the total consumption of electrical energy, such as "installed capacity" or "number of consumers (domestic and industrial)" also presented high, sustained annual rates of growth throughout the period 1982–2002, in line with the above-described increase in total consumption of electricity. Turning to the generation of electricity in Morocco, to date this has mainly been based on thermal generation, followed by hydroelectric power generation and other, minor, sources such as wind power. In 2002, according to official data [1], 92.1% of electricity was generated in thermal power stations, 6% in hydroelectric plants, with the rest being derived from wind power and other sources. In 2002, the production of thermally-generated electricity was 13067.7 Mkwh, which was 75.8% higher than the corresponding figure for 1992 (7431.8 Mkwh). Between 1992 and 2002, hydroelectric generation varied greatly, depending on the climatologic circumstances affecting Morocco; production ranged, thus, from a minimum of 443.1 Mkwh in 1993 to a maximum of 2062 Mkwh in 1997. Between 1982 and 1992, a similar pattern of hydroelectric power generation was observed. With respect to the fuel used in thermal generation, it is still mainly coal; in 2002, for example, 88% of production was derived from coal, 11.8% from fuel-oil and 0.2% from gasoil. This heavy dependence on coal must be taken into account in planning future environmental policy in Morocco. Finally, let us consider the fact that in 1996, 128.3 Mkwh of electrical energy was imported; by 2002, this figure had risen to 1392.1 Mkwh, with a peak value of 2363.2 Mkwh being recorded in the year 2000.

The above-described trend of rising electricity consumption forms part of a context of socio-economic development in Morocco that is clearly expansionary. In general, significantly rising interannual rates of growth have been recorded for all the macroeconomic indicators, such as available gross national income, gross fixed capital formation and, in particular, the gross domestic product. Thus, the GDP at current prices for the period 1992–2002 presented interannual growth rates ranging from 0.54% to 14.5% (except in 1995 and 1997, with rates of −1.51% and −0.3%). In the year 2002, the value of the GDP was 397,781.9 million DH (Dirham: Moroccan currency), which was 63.14% higher than the corresponding value for 1992. A similar pattern is observed for GDP at 1980 constant prices; for example, between 2000 and 2002, interannual growth rates were 6.29% and 3.2%, respectively.

The same remarks are valid for the GDP of energy (in the services sector of GDP), which, at current prices, increased during the period 2000–2002 at interannual rates of 6.8% and 0.54%. At constant 1980 prices, these represent, respectively, growth rates of 7.84% and 3%. Population increase in Morocco presents high, sustained annual growth rates; thus the population in 2002 was 15.98% higher than in 1992. We conclude, therefore, that modelling the evolution of energy consumption in general and of total electrical energy consumption in particular, and obtaining short and medium-term forecasts, is highly useful to obtain a better understanding of the past evolution of the economy in Morocco and to forecast its future development, and also to evaluate the repercussions of this consumption on the international energy market. Hence, the aim of this study was to model the global consumption of electricity and to derive short and medium-term consumption forecasts, as the first phase of a broader analysis of the market for energy in Morocco.

This study examines the fitting and forecasting of the evolution and the short and medium term behaviour of total domestic and industrial consumption of electricity in Morocco. Two growth models, based on HGDP and NHGDP, respectively, are analysed. In each case, the results obtained are compared and the most suitable model is selected. The following comments describe the precedents for the methodology applied, to highlight the particular contribution of this study. In recent decades, deterministic diffusion models (S-shaped curves) have been developed and successfully fitted to growth phenomena in many scientific fields, and in particular to the study of the diffusion of technical innovations or of new mass-market products. For example, models of deterministic growth such as the logistic, Bass, Richard and Gompertz types, among others, are based on growth hypotheses that, in principle, are valid for modelling the evolution of certain real phenomena. In particular, the Gompertz model has been successfully used to describe the growth of animal and cell populations and to study manufacturers' stock levels (e.g. [2,3]). These types of models have been extended to their respective stochastic versions, formulated from the stochastic differential equations that describe the respective hypotheses in random terms. Particularly well known is the Gompertz stochastic process used for theoretical modelling, described for example by Ricciardi [4], Ferrante et al. [5] and by Gutiérrez et al. [6], in the latter case it being applied to studying the consumption of natural gas in Spain. In these studies, the HGDP was fitted by statistical inference based on continuous sampling of the phenomenon under study, using the general theory applicable to the question (see, for example, [7]). Furthermore, HSLDPs have been extended to non-homogeneous versions, based on non-homogeneous diffusions whose trend function varies according to given time-dependent exogenous factors. For example, in [8], a non-homogeneous Gompertz stochastic process is used to study the prices of new housing in Spain.

The consumption of electrical energy has been the object of many studies, from various standpoints and in which a large variety of geographic areas and socio-economic situations are considered. Numerous studies have addressed the evolution and forecasting of the consumption of electrical energy, for specific periods or in relation to particular economic or climatologic variables, using statistical techniques such as regression analysis and econometric modelling (including time series models such as AR and ARIMA). Recent examples include the studies by Saab et al. [9], for Lebanon; Ranjan et al. [10] in Delhi; Shiu and Lam [11], who studied the relation between electricity consumption and GDP in China; Egeliglu et al. [12], in Northern Cyprus; Jaber et al. [13], in Jordan; and Gustafsson and Probert [14], in the case of the Swedish carpentry industry. Another approach is the use of one of the above-mentioned growth models to describe the evolution of electricity consumption, in global terms (annual observed data) for certain geographic or economic areas. Such a study can be made using either deterministic models or stochastic growth models. Recent studies of this type include those by Brodger and Tay [15,16], who used a logistic model applied to data for New Zealand, and by Zaid and Brodger [17], who compared logistic and Harvey models with respect to the domestic, non-domestic and total consumption of electricity. All the latter models were deterministic. In the stochastic case, Giovanis and Skiadas [18] used a stochastic logistic process to examine electricity consumption in Greece and in the USA, while Skiadas and Giovani [19] used a Bass stochastic process to study electricity consumption in Greece. Taking the above context into account, the present paper describes a study of the total consumption of electricity in Morocco, using HGDP and NHGDP stochastic growth models, as an alternative to the logistic and Bass models and other types described above. The fits are based, technically, on discrete samples recorded (annual data for electricity consumption), which makes it necessary to apply a different statistical inference methodology from that used to fit the models in earlier studies (see, for example, [6]).

Finally, it is interesting to note that in the above-mentioned applications, and in general, stochastic growth models are clearly well suited to describe the evolution of phenomena in diverse fields and to predict future trends, allowing the use of statistical inference in fitting and forecasting, from observed data.

2. Homogenous Gompertz diffusion process (HGDP)

This is a time-homogeneous diffusion process $\{x(t);\, t_0 \leqslant t \leqslant T\}$ for which the infinitesimal moments are time-independent and given by [5,6]

$$A_1(t,x) = ax - \beta x \log(x) \tag{1}$$

$$A_2(t,x) = \sigma^2 x^2 \tag{2}$$

where a, β and σ are time-independent parameters, the parameter σ is the diffusion coefficient, the parameter a is the intrinsic growth rate and β is a growth deceleration factor.

The trend function (with the initial condition $P(x(t_1) = x_1)$) of the process is

$$\mathbb{E}(x(t)) = \exp\left\{\log(x_1)\mathrm{e}^{-\beta(t-t_1)} + \frac{a - \sigma^2/2}{\beta}(1 - \mathrm{e}^{-\beta(t-t_1)}) + \frac{\sigma^2}{4\beta}(1 - \mathrm{e}^{-2\beta(t-t_1)})\right\} \tag{3}$$

and the conditional trend function is

$$\mathbb{E}(x(t)|x_s) = \exp\left\{\log(x_s)\mathrm{e}^{-\beta(t-s)} + \frac{a - \sigma^2/2}{\beta}(1 - \mathrm{e}^{-\beta(t-s)}) + \frac{\sigma^2}{4\beta}(1 - \mathrm{e}^{-2\beta(t-s)})\right\} \tag{4}$$

For the inference in this process, see for example [6,20] for a continuous case and [21,22] for a discrete case. In the present study, in order to use the estimators of the parameters of this model in the application, they are obtained as a particular case of those of the non-homogeneous model, which is considered in the following section.

3. Non-homogenous Gompertz diffusion process (NHGDP)

3.1. The NHGDP model

This is a time non-homogeneous diffusion process $\{x(t);\, t_0 \leqslant t \leqslant T\}$ solution of the following stochastic differential equation (SDE)

$$\mathrm{d}x(t) = [h(t)x(t) - \beta x(t)\log(x(t))]\mathrm{d}t + \sigma x(t)\mathrm{d}w_t, \quad x(s) = x_s \tag{5}$$

Here, the intrinsic growth rate is affected by exogenous factors and has the form $h(t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i g_i(t)$, where $g_i(t)$ (exogenous variable) are time-continuous functions in $[t_0, T]$, β, σ and α_i, for $i = 0, \ldots, q$ are time-independent parameters (to be estimated), and $w(t)$ is a standard Wiener process.

3.2. Transition density and trends functions

By applying the Ito formula at the time-dependent transformation $y_t = \mathrm{e}^{\beta t}\log[x(t)]$, we have

$$\mathrm{d}y_t = (h(t) - \sigma^2/2)\mathrm{e}^{\beta t}\mathrm{d}t + \sigma \mathrm{e}^{\beta t}\mathrm{d}w_t, \quad y_s = \mathrm{e}^{\beta s}\log(x_s) \tag{6}$$

The solution to this is

$$y_t = y_s + \int_s^t (h(\tau) - \sigma^2/2)\mathrm{e}^{\beta\tau}\mathrm{d}\tau + \sigma\int_s^t \mathrm{e}^{\beta\tau}\mathrm{d}w_\tau \tag{7}$$

from which we obtain the solution to the original SDE:

$$x(t) = \exp\left\{ e^{-\beta(t-s)} \log(x_s) + \int_s^t (h(\tau) - \sigma^2/2) e^{-\beta(t-\tau)} d\tau + \sigma \int_s^t e^{-\beta(t-\tau)} dw_\tau \right\} \tag{8}$$

Making use of the fact that the random variable $\int_s^t e^{-\beta(t-\tau)} dw_\tau \sim \mathcal{N}(0, \int_s^t e^{-2\beta(t-\tau)} d\tau)$, we can deduce the distribution of the random variable $x(t)|x(s) = x_s$, which is exactly the univariate lognormal distribution $\Lambda_1(\mu(s,t,x_s),\sigma^2 v^2(s,t))$, where

$$\mu(s,t,x_s) = e^{-\beta(t-s)} \log(x_s) - \frac{\sigma^2}{2\beta}(1 - e^{-\beta(t-s)}) + \int_s^t h(\tau) e^{-\beta(t-\tau)} d\tau \tag{9}$$

$$v^2(s,t) = \frac{1}{2\beta}(1 - e^{-2\beta(t-s)}) \tag{10}$$

Thus the p.d.f. of the process is

$$f(y,t \mid x,s) = [2\pi\sigma^2 v^2(s,t)]^{-\frac{1}{2}} y^{-1} \exp\left\{ -\frac{[\log(y) - \mu(s,t,x_s)]^2}{2\sigma^2 v^2(s,t)} \right\} \tag{11}$$

After applying the properties of the above distribution, and considering the initial condition, $P[X(t_1) = x_1] = 1$, the trend function of the NHGDP process is given by

$$\mathbb{E}(x(t)) = \exp\left\{ \log(x_1) e^{-\beta(t-t_1)} + \frac{\alpha_0 - \sigma^2/2}{\beta}(1 - e^{-\beta(t-t_1)}) + \frac{\sigma^2}{4\beta}(1 - e^{-2\beta(t-t_1)}) \right\} \times \exp\left(\sum_{i=1}^q \alpha_i \int_{t_1}^t g_i(\tau) e^{-\beta(t-\tau)} d\tau \right) \tag{12}$$

The conditional trend function in this case is given by

$$\mathbb{E}(x(t)|x_s) = \exp\left\{ \log(x_s) e^{-\beta(t-s)} + \frac{\alpha_0 - \sigma^2/2}{\beta}(1 - e^{-\beta(t-s)}) + \frac{\sigma^2}{4\beta}(1 - e^{-2\beta(t-s)}) \right\} \times \exp\left(\sum_{i=1}^q \alpha_i \int_s^t g_i(\tau) e^{-\beta(t-\tau)} d\tau \right) \tag{13}$$

3.3. Inference on the model

In the present study, with discrete sampling, we estimate the parameters β, σ and α_i for $i = 0,\ldots,q$ of the model by applying MLE methodology and assuming equally-spaced time intervals ($t_j - t_{j-1} = h$), with $h = 1$. Let us consider a discrete sampling of the process $x_1,x_2,\ldots,x_n$ for times $t_1 < t_2 < \cdots < t_n$ and assume an initial distribution $P[X(t_1) = x_1] = 1$. Then the associated likelihood function can be obtained from Eq. (11) by the following expression

$$\mathbb{L}(x_1,\ldots,x_n,\mathbf{a},\sigma^2) = \prod_{j=2}^n f(x_j,t_j \mid x_{j-1},t_{j-1}) \tag{14}$$

In order to work with a known likelihood function and to calculate the estimators in the simplest possible way, the discrete sampling is transformed as follows: $\mathbf{u}_{i,\beta} = v_\beta^{-1}(\gamma_\beta, \int_{t_{i-1}}^{t_i} g_1(\tau) e^{-\beta(t_i-\tau)} d\tau, \ldots, \int_{t_{i-1}}^{t_i} g_1(\tau) e^{-\beta(t_i-\tau)} d\tau)'$ and $v_1 = x_1$, $v_{i,\beta} = v_\beta^{-1}(\log(x_i) - e^{-\beta} \log(x_{i-1}))$, for $i = 2, \ldots ,n$ with thus, the likelihood function can be written as

$$\mathbb{L}_{v_2,\ldots,v_n}(\mathbf{a},\beta,\sigma^2)=[2\pi\sigma^2 v_\beta^2]^{-(n-1)/2}\exp\left(-\frac{1}{2\sigma^2}(\mathbf{v}_\beta-\mathbf{U}'_\beta\mathbf{a})'(\mathbf{v}_\beta-\mathbf{U}'_\beta\mathbf{a})\right) \tag{15}$$

where $\mathbf{a}=(\alpha_0-\frac{\sigma^2}{2},\alpha_1,\ldots,\alpha_q)'$, $\mathbf{v}_\beta=(v_{2,\beta},\ldots,v_{n,\beta})'$, $v_\beta=v(t_{i-1},t_i)$ and $\gamma_\beta=(1-e^{-\beta})/\beta$, $\mathbf{U}_\beta$ is the $(q+1)\times(n-1)$ matrix, whose rank is assumed to be $q+1$, given by $\mathbf{U}_\beta=(\mathbf{u}_{2,\beta},\ldots,\mathbf{u}_{n,\beta})$

By differentiating the log-likelihood function with respect to $\mathbf{a}$ and σ^2, we obtain the following equations:

$$\mathbf{U}_\beta\mathbf{v}_\beta=\mathbf{U}_\beta\mathbf{U}'_\beta\mathbf{a} \tag{16}$$

$$(n-1)\sigma^2=(\mathbf{v}_\beta-\mathbf{U}'_\beta\mathbf{a})'(\mathbf{v}_\beta-\mathbf{U}'_\beta\mathbf{a}) \tag{17}$$

The third likelihood equation is obtained from the derivative with respect to β, making use of Eqs. (16) and (17), thus

$$\left(v_\beta^{-1}\mathrm{e}^{-\beta}\mathbf{l}'_x-\mathbf{a}'\frac{\partial\mathbf{U}_\beta}{\partial\beta}\right)(\mathbf{v}_\beta-\mathbf{U}'_\beta\mathbf{a})=0 \tag{18}$$

where $\mathbf{l}_x=(\log(x_1),\ldots,\log(x_{n-1}))'$ and $\frac{\partial\mathbf{U}_\beta}{\partial\beta}$ is the matrix whose elements are the derivatives, with respect to β, of the components of $\mathbf{U}_\beta$.

After some algebraic rearrangement, the maximum likelihood estimators of $\mathbf{a}$ and σ^2 yield

$$\widehat{\mathbf{a}}=(\mathbf{U}_{\hat\beta}\mathbf{U}'_{\hat\beta})^{-1}\mathbf{U}_{\hat\beta}\mathbf{v}_{\hat\beta} \tag{19}$$

$$(n-1)\widehat{\sigma}^2=\mathbf{v}'_{\hat\beta}\mathbf{H}_{\mathbf{U},\hat\beta}\mathbf{v}_{\hat\beta} \tag{20}$$

where the matrix $\mathbf{H}_{\mathbf{U},\beta}$ is the symmetric and idempotent matrix given by

$$\mathbf{H}_{\mathbf{U},\beta}=\mathbf{I}_{n-1}-\mathbf{U}'_\beta(\mathbf{U}_\beta\mathbf{U}'\beta)^{-1}\mathbf{U}_\beta.$$

Taking into account the Eqs. (19) and (20), this latter expression can be expressed as

$$\left(v_\beta^{-1}\mathrm{e}^{-\beta}\mathbf{l}'_x-\mathbf{v}'_\beta\mathbf{U}'_\beta(\mathbf{U}_\beta\mathbf{U}'_\beta)^{-1}\frac{\partial\mathbf{U}_\beta}{\partial\beta}\right)\mathbf{H}_{\mathbf{U},\beta},\mathbf{v}_\beta=0 \tag{21}$$

It should be noted that in the latter equation it is not possible to explicitly calculate the estimator of β because the components of the matrix $\mathbf{U}_\beta$ depend on the integrals $\int_{t_{i-1}}^{t_i} g_j(\tau)\mathrm{e}^{-\beta(t_i-\tau)}\mathrm{d}\tau$, and the calculations require the expressions of the functions $g_j(t)$ in the intervals $[t_{i-1},t_i]$, which in practice are only known from the observed, discrete data of these exogenous variables. The $g_j(t)$ must be obtained, and so one objective of the present study is to obtain them using polygonal functions, as described in the following section.

Remarks

- As described below, we obtain the estimators of the β, a and σ parameters for HGDP as a particular case of NHGDP, by means of the expressions we have established Eqs. (19)–(21). Taking into account the non-existence of exogenous variables in the HGDP model, that is, that $g_i(t)=0$ for $i=1,\ldots q$, the matrix $\mathbf{U}_\beta$ is reduced to the vector row $\mathbf{U}_\beta=\gamma_\beta v_\beta^{-1}\mathbf{U}$ with $\mathbf{U}=(1,\ldots,1)$. Then Eq. (21) in this case can be written as: $\mathrm{e}^{-\beta}\mathbf{l}'_x\mathbf{H}_\mathbf{U}\mathbf{v}_\beta=0$, where $\mathbf{H}_\mathbf{U}=\mathbf{I}_{n-1}-\mathbf{U}'(\mathbf{U}\mathbf{U}')^{-1}\mathbf{U}$, Then, making use of the fact that $\mathbf{v}_\beta=v_\beta^{-1}(\mathbf{l}_x-\mathrm{e}^{-\beta}\mathbf{J}_x)$, with $\mathbf{J}_x=(\log(x_2),\ldots,\log(x_n))'$, the estimator of β is expressed in the following explicit form:

$$\hat{\beta} = \log\left(\frac{\mathbf{J}_x'\mathbf{H}_\mathbf{U}\mathbf{J}_x}{\mathbf{J}_x'\mathbf{H}_\mathbf{U}\mathbf{I}_x}\right) \tag{22}$$

The estimators of the parameters a and σ^2 are obtained from Eqs. (19) and (20), and in the present case are given by

$$\hat{a} = \gamma_{\hat{\beta}}^{-1} v_{\hat{\beta}} (\mathbf{U}\mathbf{U}')^{-1}\mathbf{U}\mathbf{v}_{\hat{\beta}} + \frac{\hat{\sigma}^2}{2} \tag{23}$$

$$\hat{\sigma}^2 = \frac{1}{n-1}\mathbf{v}_{\hat{\beta}}'\mathbf{H}_\mathbf{U}\mathbf{v}_{\hat{\beta}} \tag{24}$$

These estimators are the ones obtained by Nafidi [21] and Gutiérrez et al. [22], and are used for fitting and forecasting in the case of HGDP in the following section.

- By Zehna's theorem, the estimated trend function ETF and the estimated conditional trend function ECTF of the HGDP and of the NHGDP can be obtained from Eqs. (3), (4), (12) and (13) by replacing the parameters by their estimators.

4. Application

Let us take the endogenous variable, $x(t)$, to be total electricity consumption (expressed in 10^9 kWh), including transport and distribution losses, in Morocco. The following exogenous variables are considered: GDP per inhabitant, GDP/inhab; final domestic consumption, FDC; gross fixed capital formation GFCF (expressed in 10^6 DH at current prices). All these data, shown in Table 1, are annual and observed for the period 1980 to 2001 (Source: Haut Commissariat au Plan. Direction de la Statistique, Maroc. Statistiques: Comptes et agregats de la nation. www.statstic-hcp.ma[23]).

Normal practice is not to take the real value of the exogenous variables in each observation time, but rather the increment relative to the previous instant. We denote these values by y_{ij}; $i = 2,\ldots,n$; $j = 1,2,3$ with $y_{1j} = 0$, $j = 1,2,3$.

Let us assume these variables $g_j(t)$ to be polygonal functions in time in the intervals $[t_{i-1},t_i]$ and that they have the following form:

$$g_j(t) = y_{i-1,j} + (y_{i,j} - y_{i-1,j})(t - t_{i-1}), \quad t_{i-1} \leqslant t \leqslant t_i, \;\; i = 2,\ldots,n, \;\; j = 1,2,3 \tag{25}$$

Then

$$\int_{t_{i-1}}^{t_i} g_j(\tau)\mathrm{e}^{-\beta(t_i-\tau)}\mathrm{d}\tau = \gamma_\beta\left(y_{i-1,j} + (y_{i,j} - y_{i-1,j})\frac{\beta - 1 + \mathrm{e}^{-\beta}}{\beta(1-\mathrm{e}^{-\beta})}\right), \quad i = 2,\ldots,n, \;\; j = 1,2,3 \tag{26}$$

and so the columns of the matrix $\mathbf{U}_\beta$ can be expressed as follows

$$\mathbf{u}_{i,\beta} = v_\beta^{-1}\gamma_\beta(1, z_{i1}(\beta), z_{i2}(\beta), z_{i3}(\beta))', \quad i = 2,\ldots,n \tag{27}$$

where

$$z_{i,j}(\beta) = y_{i-1,j} + (y_{i,j} - y_{i-1,j})\frac{\beta - 1 + \mathrm{e}^{-\beta}}{\beta(1-\mathrm{e}^{-\beta})}, \quad i = 2,\ldots,n, \;\; j = 1,2,3 \tag{28}$$

The data for 2000 and 2001 are used to forecast and compare the values observed with those obtained with the proposed models.

Table 1
Observed values

Year	Real data in 10^9 kWh	GDP/inhab in DH	FDC in 10^6 DH	GFCF in 10^6 DH
1980	4.0938	3833	64517.6	16478.4
1981	4.3236	3984	70392.3	20511.8
1982	4.6055	4550	80578.4	25376.9
1983	4.9552	4759	85929.6	24232.9
1984	5.1400	5272	98644.1	25953.8
1985	5.5067	5943	112794.9	29927.5
1986	5.9023	6948	133635.5	32991.3
1987	6.2830	6888	135765.0	31632.0
1988	6.6744	7818	152107.1	37234.7
1989	7.1332	8182	162551.0	44174.1
1990	7.6543	8804	179191.9	51059.2
1991	8.1015	9832	205658.9	53863.2
1992	8.7264	9708	212111.3	54364.0
1993	8.9614	9742	216523.5	56718.9
1994	9.4954	10713	245283.4	57899.8
1995	9.8971	10676	251475.4	60386.6
1996	10.2283	11894	281254.4	61940.9
1997	10.7945	11657	275217.4	65786.0
1998	11.5400	12385	296257.2	75739.3
1999	12.2460	12239	295240.7	81895.9
2000	12.8380	12335	310657.7	85422.1
2001	13.4520	13126	333793.1	85374.9

FDC: final domestic consumption; GFCF: gross fixed capital formation; GDP/inhab: gross domestic product per inhabitant.

In the homogeneous case HGDP, the estimated parameters are obtained using expressions Eqs. (22)–(24), and their values are $\hat{a} = 0.08068$; $\hat{\beta} = 0.01160$ and $\hat{\sigma}^2 = 0.00017$.

In the non-homogeneous case NHGDP, using the polygonal form of the above exogenous variable in Eq. (25), and the columns of the matrix $\mathbf{U}_\beta$ in Eq. (27), a study was carried out in which first one, then two and finally three exogenous factors were introduced into the model. The best fit and forecast were obtained in the latter case , and the estimator of the β parameter was obtained by numerically solving Eq. (21) $\hat{\beta} = -0.0014$. From this value, we calculate the estimators $\hat{\mathbf{a}} = (0.0426;\ 0.3256;\ -0.1945;\ 0.0872)'$ and $\hat{\sigma}^2 = 0.000108$.

The coefficients (0.3256; −0.1945; 0.0872) provide us information about the degree and the sign of the influence for each exogenous factor.

Table 2 shows the values predicted, using the two versions of the proposed model, from the ETF and the ECTF for 2000 and 2001 for some combinations of exogenous factors (GDP/inhab; GDP/inhab and FDC; GDP/inhab and GFCF; GDP/inhab, FDC and GFCF).

The following methodology was applied in this respect: The process is adjusted by applying the observed data of the endogenous variable, $x(t)$ = total electricity consumption, and those of the three exogenous factors (GDP/inhab, FDC and GFCF), for the years 1980–1999. For the years 2000 and 2001 the corresponding observed data was also available. Having adjusted the process and, in particular, with the adjusted expressions of the trends, ETF and ECTF (see Section 3, Remarks), the values for the endogenous variable are calculated for the years 2000 and 2001, using the above-cited adjusted expressions, but now adding one

Table 2
Forecasts using the proposed models

		HGDP		NHGDP							
				GDP/inhab[a]		GDP/inhab FDC[b]		GDP/inhab GFCF[c]		GDP/inhab FDC GFCF	
Year	Data	ETF[d]	ECTF[e]	ETF	ECTF	ETF	ECTF	ETF	ECTF	ETF	ECTF
2000	12.838	12.905	12.891	12.834	12.818	12.797	12.782	12.864	12.848	12.838	12.824
2001	13.452	13.576	13.507	13.520	13.524	13.427	13.470	13.516	13.489	13.461	13.461

FDC: final domestic consumption; GFCF: gross fixed capital formation; GDP/inhab: gross domestic product per inhabitant; ETF: estimated trend function and ECTF: estimated conditional trend function.
[a] In Dirham.
[b] In 10^6 Dirham.
[c] In 10^6 Dirham.
[d] In 10^9kWh.
[e] In 10^9kWh.

or two summands (one for the year 2000, and two for the year 2001) in the part of these estimated trends:

$$\exp\left(\sum_{i=1}^{q}\alpha_i\int_{t_1}^{t}g_i(\tau)\mathrm{e}^{-\beta(t-\tau)}\mathrm{d}\tau\right) \tag{29}$$

This part of the estimated trend is the part that dependent exclusively of the exogenous factors.

These two summands are then dealt with using the polygonal methodology described in Section 4 of this paper.

It is possible to extend the projection of the values of the endogenous variable through the year 2010, using a reasonable scenario about the evolution of these factors. To achieve this, a hypothesis must be formulated, such a hypothesis could be as follows: During the period 2002–2010, the exogenous factors considered will increase steadily, with "annual relative increases" of 0.03 (GDP/inhab), 0.05 (FDC) and 0.47 (GFCF). This hypothesis seems a reasonable one for the following reasons: the "mean annual relative increase"

Table 3
Forecasts 2000–2010 using the three exogenous factors

Year	ETF in 10^9 kWh	ECTF in 10^9 kWh
2000	12.838	12.824
2001	13.461	13.461
2002	14.1686	14.1684
2003	14.9018	14.9016
2004	15.6740	15.6738
2005	16.4875	16.4872
2006	17.3444	17.3441
2007	18.2472	18.2469
2008	19.1984	19.1981
2009	20.2007	20.2003
2010	21.2569	21.2564

ETF: estimated trend function; ECTF: estimated conditional trend function.

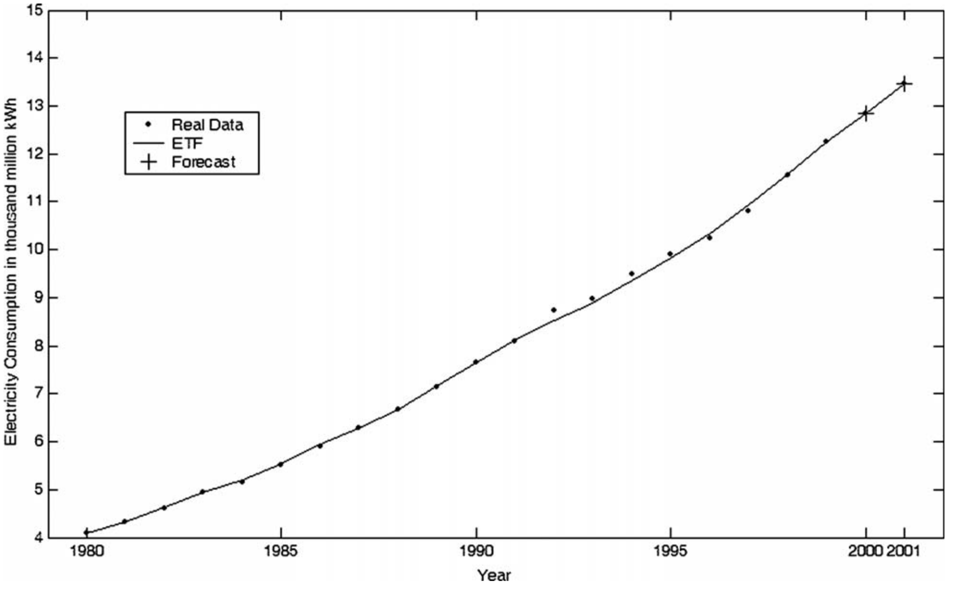

Fig. 1. Fits (1980–1999) and forecasts (2000 and 2001) using the ETF of NHGDP with the exogenous factors GDP/inhab, FDC and GFCF.

has been calculated for the last 10 years for which data are available (1992–2001) for each of the three exogenous factors in question, and these values are, respectively 0.03039, 0.05059 and 0.04792. In fact, the above-mentioned hypothesis is a reasonable scenario,

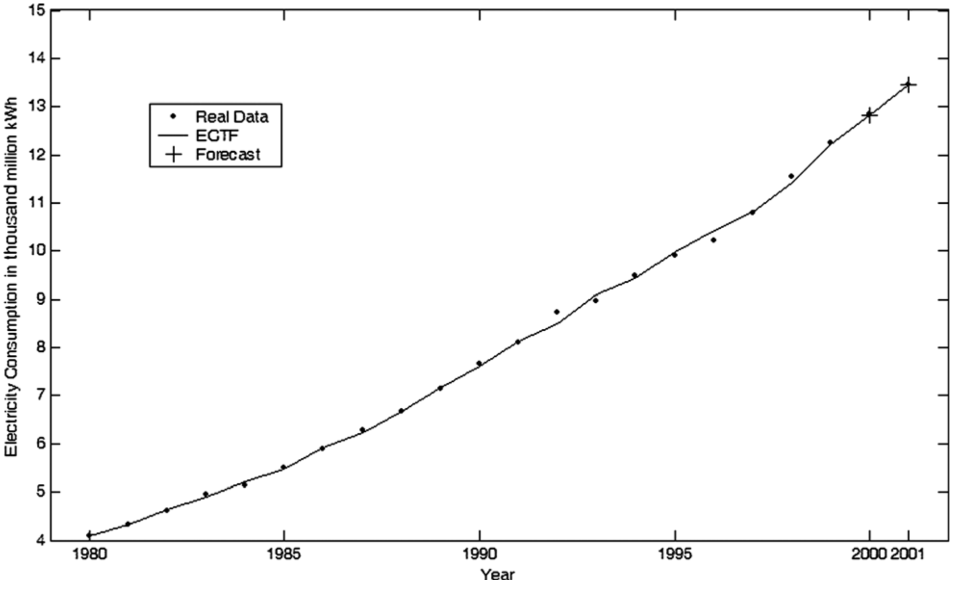

Fig. 2. Fits (1980–1999) and forecasts (2000 and 2001) using the ECTF of NHGDP with the exogenous factors GDP/inhab, FDC and GFCF.

meaning that during each year in the period 2002–2010, the mean annual increase presented by the exogenous variables is a constant one, with approximate values of 0.03, 0.05 and 0.047, respectively. By applying this hypothesis and the methodology described in the above paragraph, the estimated values of the endogenous variable through the year 2010 have been calculated (Table 3).

Finally, Figs. 1 and 2 show the fits and the forecasts, of the model NHGDP with the exogenous factors GDP/inhab, FDC and GFCF, respectively, from the ETF (Fig. 1) and the ECTF (Fig. 2).

5. Conclusions

An original methodology is presented, based on stochastic non-homogeneous Gompertz diffusion processes, which makes it possible to incorporate exogenous factors that may influence the behaviour of the endogenous variable being studied.

This methodology was applied to the case of electricity consumption in Morocco, and three exogenous factors, from among a wider set, were identified as significantly improving the description of the series analysed (1980–1999) and the medium-term forecasts (2000–2001). Table 2 shows the forecasts derived from the homogeneous model (without exogenous factors) and those obtained from the model with exogenous factors.

The methodology enables us to simultaneously model, to a high degree of accuracy, the Gompertz trend of endogenous variable (electricity consumption) and the influence of exogenous variables (GPD/inhab, FDC and GFCF) on such a trend.

Longer-term forecasts can be made with the methodology described, within a specific macroeconomic scenario defined by the hypothesis of the growth of the exogenous variables in question, such as those of the interannual growth forecasts of GDP/inhab and of domestic consumption (Table 3).

This methodology can be applied to other energy sources and other geopolitical areas, investigating the exogenous factors that are most appropriate for each case.

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