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Forecasting total natural-gas consumption in Spain by using the stochastic Gompertz innovation diffusion model

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Abstract

The principal objective of the present study is to examine the possibilities of using a Gompertz-type innovation diffusion process as a stochastic growth model of natural-gas consumption in Spain, and to compare our results with those obtained, on the one hand, by stochastic logistic innovation modelling and, on the other, by using a stochastic lognormal growth model based on a non-innovation diffusion process. Such a comparison is carried out taking into account the macroeconomic characteristics and natural-gas consumption patterns in Spain, both of which reflect the current expansive situation characterizing the Spanish economy. From the technical standpoint a contribution is also made to the theory of the stochastic Gompertz Innovation diffusion process (SGIDP), as applied to the case in question.

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Keywords: Stochastic Gompertz, logistic and lognormal models; Natural-gas consumption; Estimation and prediction

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Nomenclature

$\hat{a}$,$\hat{b}$	maximum likelihood of the drift parameters a and b
$\hat{c}$	estimator of the diffusion coefficient c
DLR	dynamic linear-regression
$\mathrm{d}x(t)$	stochastic Ito's differential of $x(t)$
E	mathematical expectation
$F_{N(0,1)}$	cumulative normal standard-distribution
IEA	International Energy Agency
ktep	thousand metric tons of oil equivalent
$m(t)$	trend function
$m(t\|s)$	conditional trend function
SGIDP	stochastic Gompertz innovation diffusion process
SDE	stochastic differential-equation
v_{lower}	lower limit of the confidence interval
v_{upper}	upper limit of the confidence interval
$w(t)$	Wiener standard process
$x(t)$	total natural-gas consumption
α	conditional confidence interval

1. Introduction

Various deterministic and stochastic models have been applied to describe and forecast the evolution of natural-gas consumption in different situations (residential, industrial or national consumption, in large geographic areas, in stable or developing economies, etc.). For example, Maddala et al. [1] proposed a dynamic linear regression (DLR) model to estimate short-run and long-run elasticities of residential demands for natural gas in the USA for each of 49 states, as a function of the real per capita personal income, the real residential natural-gas price, the real residential electricity price and the heating and cooling degree days. Subsequently, Batalgi et al. [2] discussed problems that might arise in evaluating forecasts produced with the above model, particularly as regards shrinkage estimators. Sarak and Saturau [3] described a deterministic model to forecast natural-gas consumption for residential heating in certain areas of Turkey, based on previous studies performed by Durmayaz et al. [4]. Other studies have examined the total (domestic and industrial) consumption of natural gas and other fuels in large geographic areas. For example, Siemek et al. [5] consolidated earlier studies by Hubbert [6] and by Al Fattah et al. [7], proposing a deterministic model based on the logistic growth curve to describe and forecast natural-gas consumption in Poland, taking into account the macroeconomic context and the economic cycles affecting the country. Stochastic logistic growth models have also been used in relation to the consumptions of various fuels, with special attention to that of electricity. For example, Giovanis and Skiadas [8] described and forecasted total energy consumption in the USA and in Greece, developing estimation methods that were based on statistical inference by continuous sampling for this type of logistic diffusion, and obtained good results. Other models of non-logistic deterministic growth, notably the Gompertz curve, have been widely used to describe phenomena such as the diffusion of technological innovations and the marketing of new products. A representative example is the model proposed by Franses [9] concerning the sale of new cars in the context of cointegrated trivariate systems fitted to market data from the Netherlands.

Darrat [10] discussed statistical problems associated with the Franses model [9], analyzing questions related to cointegration, and suggested the possibility of using other explanatory variables. Skiadas and Giovanis [11] applied the stochastic version of Bass's classical growth-model to the study of electricity consumption in Greece.

This paper, in which we examine the possibilities of using a SGIDP as a stochastic growth model of natural-gas consumption in Spain, is structured as follows: in the next section, we define this process as a solution of Ito's stochastic differential equation (SDE) and then, using Ito's formula, the analytical expression of this process is found, after which the trend and conditional trend functions are determined. In Section 3, the parameter estimators of the proposed process are derived by two methods, firstly, the maximum likelihood based on continuous sampling used to estimate the parameters in the drift coefficient; the second is used to approximate the parameters in the diffusion coefficient. Therefore, a confidence interval of the model is obtained. In the last section, the model is applied to time-series data of natural-gas consumption in Spain and provides sufficiently good results. Our results are compared with those obtained, on the one hand, by stochastic logistic innovation modelling and, on the other, by using a stochastic lognormal growth model based on the non-innovation diffusion process.

2. The stochastic Gompertz innovation diffusion process

2.1. The SGIDP model

The stochastic version of the Gompertz innovation diffusion process, can be defined ([12] and [13]) by the following SDE

$$\mathrm{d}x(t) = (ax(t) - bx(t)\log x(t))\mathrm{d}t + cx(t)\mathrm{d}w(t), \quad x(s) = x_s, \tag{1}$$

$c > 0$, $x_s \in \mathbb{R}_+^*$ and $w(t)$ is a one-dimensional Wiener standard process, with independent increment $w_t - w_s$ normally distributed with mean zero and variance $t - s$, for $t \geqslant s$.

By applying the Ito formula to the transformation $\mathrm{e}^{bt}\log(x(t))$, and if we denote by $\gamma = a - c^2/2$, we obtain the solution of the Eq. (1) (which presented the analytic expression of SGIDP) in the following form

$$x(t) = \exp\left(\log(x_s)\mathrm{e}^{-b(t-s)} + \frac{\gamma}{b}(1 - \mathrm{e}^{-b(t-s)})\right)\exp\left(c\int_s^t \mathrm{e}^{-b(t-\tau)}\mathrm{d}w_\tau\right).$$

2.2. Trend and conditional trend functions of the SGIDP

The conditional trend function of the process is given by

$$m(t \mid s) = \mathbf{E}(x(t) \mid x(s) = x_s)$$
$$= \exp\left(\log(x_s)\mathrm{e}^{-b(t-s)} + \frac{\gamma}{b}\left(1 - \mathrm{e}^{-b(t-s)}\right)\right)\mathbf{E}\exp\left(c\int_s^t \mathrm{e}^{-b(t-\tau)}\mathrm{d}w_\tau\right).$$

The random variable in the last expression, is normally distributed with mean zero and variance $c^2\int_s^t \mathrm{e}^{-2b(t-\tau)}\mathrm{d}\tau$ and so its expectation can be calculated using the (Gardiner [14]) relation

$$\mathbf{E}(\exp(z_t)) = \exp\left(\frac{1}{2}\mathbf{E}(z_t)^2\right),$$

where z_t is a zero-Gaussian random process. Then, we have

$$\mathbf{E}\left(\exp\left(c\int_s^t \mathrm{e}^{-b(t-\tau)}\mathrm{d}w_\tau\right)\right) = \exp\left(\frac{c^2}{2}\int_s^t \mathrm{e}^{-2b(t-\tau)}\mathrm{d}\tau\right).$$

After substitution, we obtain the final form of the conditional trend function of the process

$$m(t \mid s) = \exp\left(\log(x_s)\mathrm{e}^{-b(t-s)} + \frac{\gamma}{b}\left(1 - \mathrm{e}^{-b(t-s)}\right) + \frac{c^2}{4b}\left(1 - \mathrm{e}^{-2b(t-s)}\right)\right). \quad (2)$$

With the initial condition $P(x(0) = x_0) = 1$, the trend function produces

$$m(t) = \exp\left(\log(x_0)\mathrm{e}^{-bt} + \frac{\gamma}{b}\left(1 - \mathrm{e}^{-bt}\right) + \frac{c^2}{4b}\left(1 - \mathrm{e}^{-2bt}\right)\right). \quad (3)$$

These functions are used in the final section to forecast the future values of the model.

3. Inference in the SGIDP

In this section, we examine the SGIDP estimation parameters. Two methods are presented, the first of which is used to estimate the drift parameters a and b by the maximum likelihood principle in continuous sampling, while the second is used to approximate the parameter in the diffusion coefficient c^2 (the white noise).

3.1. Estimation of drift parameters

The two parameters in the drift a and b are to be estimated from an observed sample path $\{x(t); t \in [0, T]\}$: for this, we suppose that we have observed the process in the interval $[0, T]$, then the likelihood estimators of the parameters [15] are given by the following equations:

$$\hat{a} = \frac{\left(\int_0^T (\log(x(t)))^2 \mathrm{d}t\right)\left(\int_0^T \frac{\mathrm{d}x(t)}{x(t)}\right) - \left(\int_0^T \log(x(t))\mathrm{d}t\right)\left(\int_0^T \frac{\log x(t)}{x(t)}\mathrm{d}x(t)\right)}{T\int_0^T \log^2(x(t))\mathrm{d}t - \left(\int_0^T \log(x(t))\mathrm{d}t\right)^2}, \tag{4}$$

$$\hat{b} = \frac{\left(\int_0^T \log(x(t))\mathrm{d}t\right)\left(\int_0^T \frac{\mathrm{d}x(t)}{x(t)}\right) - T\left(\int_0^T \frac{\log x(t)}{x(t)}\mathrm{d}x(t)\right)}{T\int_0^T \log^2(x(t))\mathrm{d}t - \left(\int_0^T \log(x(t))\mathrm{d}t\right)^2}. \tag{5}$$

In practice, as we do not have continuous sampling, we must consider approximations based on the discrete observations of the process at times $t_0 = 0, \dots, t_n = T$ (discrete sampling). By using conditioned likelihood based on the transition density of Gompertz diffusion, very complicated equations are obtained in [16], which, nonetheless, can be solved numerically. An alternative method, used in the present study, is to replace the stochastic integrals in expressions (4) and (5) by Riemann integrals, applying Ito's formula, and then approximating the integrals by the trapezoidal method.

3.2. Estimation of the noise coefficient

In order to estimate the coefficient c, we can extend the procedure proposed in [17] for estimating the coefficient diffusion for a linear SDE with multiplicative noise to the case of a non-linear SDE with multiplicative noise: the method is the same as in [18]; the resulting estimator having the following form:

$$\hat{c} = \frac{1}{T-1}\sum_{t=2}^{T} \frac{\mid x(t) - x(t-1) \mid}{\sqrt{t x(t) x(t-1)}}.$$

3.3. A confidence interval of the (SGIDP)

Let $v(s,t) = X(t) \mid x(s) = x_s$. As we mentioned in the Section (2.2), it is known that the Ito integral $c\int_s^t \mathrm{e}^{-2b(t-\tau)}\mathrm{d}w_\tau$ is Gaussian with mean zero and variance $c^2\int_s^t \mathrm{e}^{-2b(t-\tau)}\mathrm{d}\tau$. Then a random variable z is given by

$$z = \frac{\log(v(s,t)) - \mu(s,t)}{v(s,t)} \sim N(0,1),$$

where

$$\mu(s,t) = \log(x_s)\mathrm{e}^{-b(t-s)} + \frac{\gamma}{b}\left(1 - \mathrm{e}^{-b(t-s)}\right),$$

$$v^2(s,t) = \frac{c^2}{2b}\left(1 - \mathrm{e}^{-2b(t-s)}\right).$$

A α% conditional confidence interval for z is given by $P(-\epsilon \leqslant z \leqslant \epsilon) = \alpha$. From this, we can obtain a confidence interval of $v(s,t)$ with following form: $v_{\text{lower}}(s,t) \leqslant v(s,t) \leqslant v_{\text{upper}}(s,t)$, where,

$$v_{\text{lower}}(s,t) = \exp\{\mu(s,t) - \xi v(s,t)\}, \tag{6}$$

$$v_{\text{upper}}(s,t) = \exp\{\mu(s,t) + \xi v(s,t)\}, \tag{7}$$

with $\xi = F^{-1}_{N(0,1)}((1+\alpha)/2)$ and where $F^{-1}_{N(0,1)}$ is the inverse cumulative normal standard distribution.

By Zehna's theorem, the estimated trend, conditional trend functions and the confidence interval of the process can be obtained from (2), (3), (6) and (7) by replacing the parameters by their estimators.

4. Application to gas consumption in Spain

In Spain, the proportion of natural gas within the total energy consumption increased consistently during the period 1973–2000. In particular, between 1990 and 2000, natural-gas consumption rose from 7.5 to 14.2% of the total energy-consumption in Spain, while the proportions of final energy derived from oil and electricity, in the same period, varied from 67.4% to 64.1% and from 18.1% to 18.8%, respectively. During a similar period, according to International Energy Agency (IEA) data, the consumption of the above sources of energy in OECD countries varied as follows: 18.86–19.58% (gas); 52.2–52.86% (oil); 17.50–19.66% (electricity). With regard to the European Union, the respective figures were: 20.6–23.2% (gas); 46.03–48.1% (oil) and 18.06–19.5% (electricity). There has been a notable increase in the contribution of natural gas to energy consumption in Spain, in comparison with EU and OECD countries.

In the Spanish market, the total consumption of primary energy obtained from natural gas presents structural characteristics similar to those referring to final energy consumption. Other characteristics of the energy market in Spain can be consulted in [19].

The endogenous consumption pattern in Spain, in absolute terms, also presents a clear upward trend. Between 1973 and 2000, the final consumption of energy obtained from natural gas rose from 763 to 12292 ktep (thousand metric tons of oil equivalent), while between 1990 and 2000, from 4531 to 12292 ktep (an increase of 171.3%). With respect to the total consumption of primary energy derived from natural gas, the increase between 1990 and 2000 was even greater, at 204.46%. Finally, the separation, within total demand for gas (final energy), of domestic-commercial use from industrial use (including electricity generation and cogeneration), reveals values of 18% and 82%, respectively (estimated data for 2002).

The energy market in Spain has been characterized in recent decades by very important quantitative and structural changes, especially concerning natural gas as a source of energy. Moreover, this has taken place in a context of an expanding phase of the economic cycle and significant social changes.

The SGIDP is applied to the data of total natural-gas consumption in Spain from 1973 to 2000. These data were provided by the Ministry of Economic of Spain [19] and are included in Table 1.

We use the 25 first data of the above time series in order to estimate the parameters of the process using the methods described in Sections (3.1) and (3.2). By using the Matlab package, the resulting values of the estimators are: $\hat{a} = -0.0108$, $\hat{b} = -0.0144$ and $\hat{c} = 0.0322$. The data from 1998 to 2000 are used to make forecasts of the future values of the process, with the trend and conditional trend functions given by expressions (2) and (3) and the confidence interval (given $\alpha = 95\%$) in the expressions (6) and (7). The results are summarized in Tables 2 and 3.

The performance of the SGIDP for the forecasting period using the trend and conditional trend function is illustrated in Figs. 1 and 2.

Finally, in order to evaluate the results obtained using the SGIDP in studying our data series, we compared it with two alternative models; the first being the stochastic logistic innovation process [11] and the second is the stochastic lognormal model [20]. The results obtained are shown in Fig. 3.

Table 1
Total natural-gas consumption (in ktep) in Spain

Years	1973	1974	1975	1976	1977	1978	1979
Data	763	820	901	1034	1136	1220	1252
Years	1980	1981	1982	1983	1984	1985	1986
Data	1220	1184	1178	1110	1549	1768	2004
Years	1987	1988	1989	1990	1991	1992	1993
Data	2463	3153	4116	4531	4999	5154	5130
Years	1994	1995	1996	1997	1998	1999	2000
Data	5647	6550	7325	8162	9688	10934	12319

Table 2
Predictions from trend function of the process

Years	Real data	Trend function	Confidence interval
1998	9688	9718	(6510–13,966)
1999	10,934	10,981	(7270–15,934)
2000	12,319	12,430	(8133–18,211)

Table 3
Predictions from conditional trend function of the process

Years	Real data	Conditional trend	Confidence interval
1998	9688	9197	(8626–9795)
1999	10934	10943	(10264–11656)
2000	12319	12373	(11604–13178)

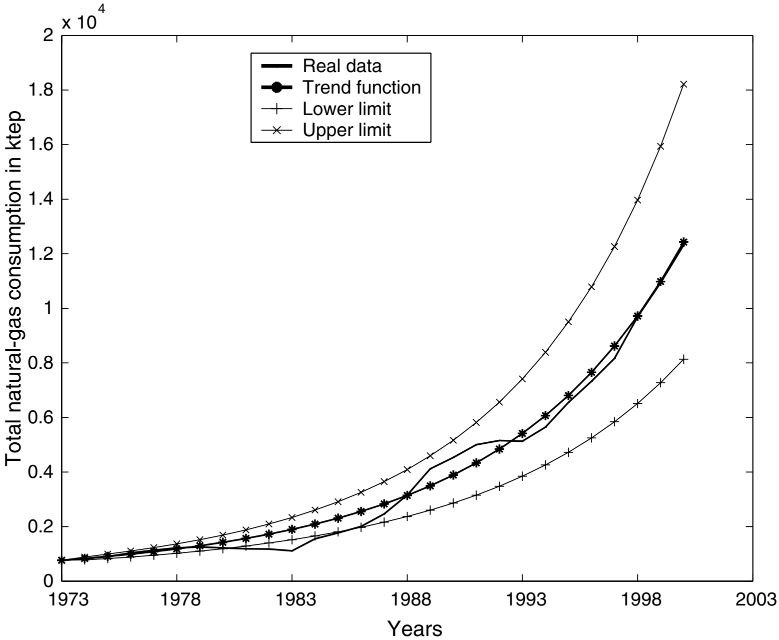

Fig. 1. Real data versus a trend function.

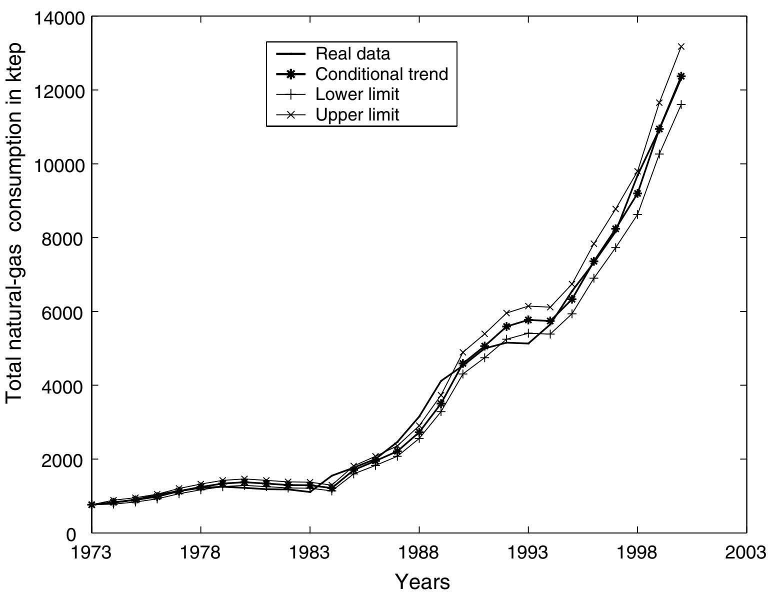

Fig. 2. Real data versus a conditional trend function.

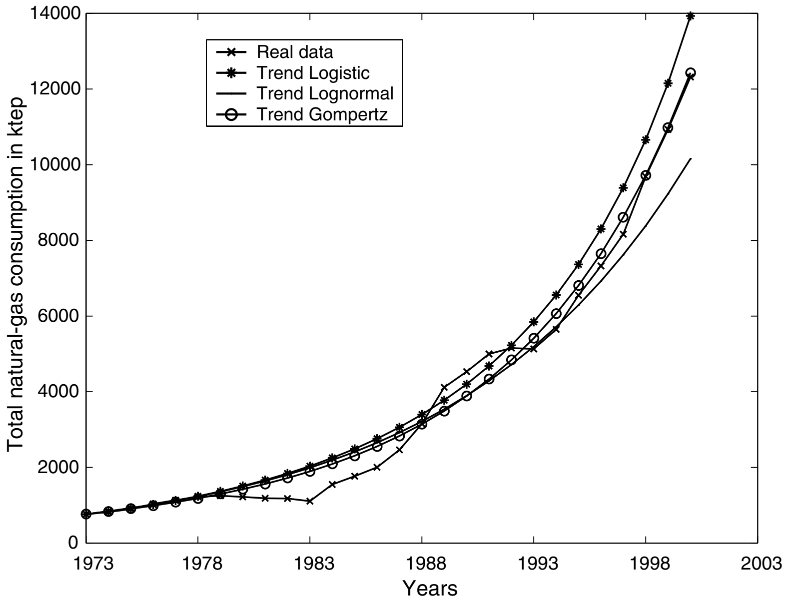

Fig. 3. Real data versus trends of the three processes.

5. Conclusions

- By fitting a Gompertz stochastic model of diffusion and innovation to the data for total consumption of final energy obtained from natural gas in Spain during the period 1973–1997, a good description of the series and good short-medium term forecasts (1998–2000) are obtained.
- The description and forecast using the conditioned trend are considerably better than those based on the trend alone, although they are only optima in the short term (year on year).
- For the period in question, the Gompertz model is found to be more suitable than other stochastic diffusion growth models, namely the logistic (diffusion–innovation) and the lognormal (diffusion–non innovation) models.
- Further studies are required to examine data fitting for versions of the above-mentioned stochastic diffusion processes that incorporate exogenous factors given by non-endogenous variables such as economic and demographic data, using techniques (see, for example [20,21]) that have been successfully applied in other fields. Thus, we could improve the long-run fits and forecasts achieved, by taking into account the influences on gas consumption of significant variables within the socio-economic environment.

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