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TACKLING COMPLEXITY IN MATHEMATICS TEACHING DEVELOPMENT: USING THE TEACHING TRIAD AS A TOOL FOR REFLECTION AND ANALYSIS

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ABSTRACT. This paper reports research that attempts to make sense of the complexity of mathematics teaching and its development at secondary school level. The research was conducted in partnership between two teachers and two educator/researchers over one school term in two U.K. schools. A theoretical construct, the teaching triad, was used as an analytical device (by the researchers) and as a reflective agent for teaching development (by the teachers). The focus of analysis was the interactions between teacher and students at whole class and small group level. Both micro- and macro-analyses were undertaken. We present details of the processes involved in examples from the teaching of one teacher as she translated theoretical aims into classroom practice. The use of the triad allowed access to complexity, involving both psychological and sociological elements, and to the position of a sincere teacher with respect to competing forces in the educational system. The potential of the triad for teacher and teaching development is discussed.

KEY WORDS: analysis of teaching, classroom interaction, cognitive and affective sensitivity complexity of mathematics teaching, effective learning, teaching development, teaching triad

INTRODUCTION

This project involved a collaboration between two teachers and two educator/researchers to explore the contribution of a theoretical construct, *the teaching triad* (TT), to the construction and analysis of mathematics teaching and its development in secondary school classrooms. It took place over one school term (a third of the academic year) in which the teachers' lessons were observed by one or both researchers; the teachers were interviewed between lessons, and the team of four met periodically to examine issues arising from the research.

The study was designed to explore the use of the triad as a tool to

- analyse classroom data to provide insights into mathematics teaching, particularly its complexity and issues for teachers;
- encourage teachers' reflection in all aspects of teaching.

The study shows that use of the triad in these ways revealed details of complexity and allowed insights into teaching issues for both researchers and teachers. We were able to go beyond simplistic judgments about lessons to search out cognitive and affective factors in learning and the wider social issues and concerns that impinge on classroom decisions. Such a revealing of factors and issues in a context of teacher/researcher collaboration provided a powerful environment for considering teaching development.

However, generality here is seen not in the particular issues that the research has raised, nor in the power of the triad to reveal particular issues; it is in the methodology of the use of the triad, the analytical process that reveals issues, and the collaborative working of the research participants in making such research possible. We take seriously the words of Cooney, who writes

> But if we are to move beyond collecting interesting stories, theoretical perspectives need to be developed that allow us to see how those stories begin to tell a larger story. That is we should be interested in how local theories about teachers can contribute to a more general theory about teacher education. (Cooney, 1994, p. 627)

We are interested in how our approach, using the teaching triad, can contribute to a more general theory of describing and interpreting teaching practice, and ultimately to indications for teacher education.

THEORETICAL BACKGROUND

The Teaching Triad Research

The *teaching triad* emerged from an ethnographic study of *investigative mathematics teaching* (Jaworski, 1994) of a small number of mathematics teachers. Very briefly, this involved engaging students in open-ended and problem-solving tasks through which curriculum-designated mathematical topics would be approached and students' mathematical thinking and understanding fostered. The study led to identification of general characteristics of investigative teaching and to a theoretical construct, *the teaching triad*, which linked the generalized characteristics to three 'domains' of activity in which teachers had been seen to engage: *management of learning* (ML); *sensitivity to students* (SS) and *mathematical challenge* (MC). This triad attempted to provide a framework to capture essential elements of the complexity of the observed teaching and to generalize these to mathematics teaching more widely.

Briefly, *management of learning* describes the teacher's role in the constitution of the classroom learning environment by the teacher and students. It includes classroom groupings; planning of tasks and activity; setting of norms and so on. *Sensitivity to students* describes the teacher's

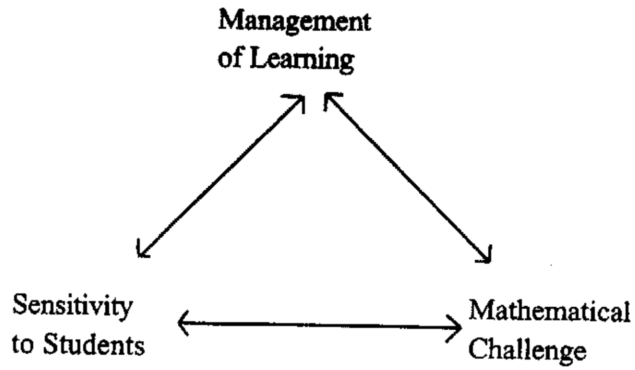

Figure 1. The teaching triad.

knowledge of students and attention to their needs; the ways in which the teacher interacts with individuals and guides group interactions. *Mathematical challenge* describes the challenges offered to students to engender mathematical thinking and activity. This includes tasks set, questions posed and emphasis on metacognitive processing. These domains are closely interlinked and interdependent as our current research shows. One major aim of the research described in this paper was to explore the teaching triad beyond the situations from which it arose and to consider its applicability and uses for teachers and researchers more widely.

The following episode from our research (data extract: 4J/9: the fourth observed lesson of a teacher, Jeanette, with her Year 9 class) illuminates the elements of the triad and provides initial insight into its use.

A lesson opening[1]

Jeanette's class was divided into groups of four, five or six students according to students' choice. She gave each group sheets of squared paper, some card, and a sheet describing a problem. Multilink cubes were available for use. The problem was to design a box to contain 48 cubes, each of side 2 cm, using a minimum amount of card. Each group had to work on this task and provide a group solution. The lesson opened with an interactive whole class session in which the task was clarified, described briefly as follows.

Jeanette invited someone to come to the board and draw a 2 cm cube. Michael accepted the invitation and drew a 2 cm square. Some students said that it was a square (i.e. not a cube). Jeanette acknowledged it as "a cube facing the front". Another student came forward and drew a cube. Jeanette asked him "Can you write your dimensions as Michael has done?" The teacher then clarified the problem: "You need to make a box to fit 48 of those cubes. The squared paper is to try out some designs. It is a group project. Everyone needs to be busy." She paused, waiting for silence in the room. "The company will give a prize to the one who has used the least card. We want to find the best shape and size of box".

ML In our analysis, in the category of *management of learning*, we recognized the teacher's organization of the class in groups; her planning and organization of the investigative task; her management of the whole class situation in which she introduced the task; and her responses to the students who participated. All of these are part of the teacher's organization of the learning environment, her developing of social skills within the classroom setting, and her creation of opportunity to engage in mathematics. In the

SS category of *sensitivity to students*, we placed the teacher's mode of introduction of the task, and her particular responses to students. She wanted all students to be able to make a start on the task; hence she offered a familiar practical context, resources, and clarification of the problem. She involved students in her introduction and was careful to value, and encourage others to value, their contributions. As well as valuing Michael's contribution, her words "a cube facing the front" acknowledged another perspective of

MC what students saw as a square. Thus she opened up *mathematical challenge* (MC), initially manifested in the problem itself, and enhanced through contextual emphasis on optimization in the box.

The teaching triad research, which is the basis of this paper, was research into mathematics teaching in naturalistic settings – secondary school mathematics classrooms – aiming to study the full complexities of teaching and reveal the tensions, issues and dilemmas that teachers face in constructing effective learning situations for students. The research did not try to promote any particular forms of teaching or approaches to teaching. It did not seek overtly to educate teachers or develop teaching. Its starting point was the teaching of the two teachers who were participants in the research. A study of their real lessons, in real circumstances was a source of opportunity to look at the potential of the triad to inform teaching, and to gain insights into teaching. We shall show in this paper how insights gained were related to developments in a teacher's thinking about teaching and potentially the development of teaching itself. This suggests the triad as a potential tool for teaching and teacher development.

Complexity

Three guiding assumptions in this research were:

- that the complexity of teaching-learning situations is such that any attempt to provide simple descriptions is 'rapidly shown to be hollow' (Bauersfeld, 1988); false

- that the essence of teaching/learning processes lies in classroom interactions, the analysis of which provides the key to understanding the complexity acknowledged above;
- that teachers work from a position of sincerity and professionalism in seeking for students' mathematical development.

The first two of these follow from the work of Bauersfeld and colleagues in Germany (e.g. Bauersfeld, 1988) and from our own professional experience before the research began. It is through a close scrutiny and analysis of interactions that we gain insights into teaching complexity and start to appreciate the myriad issues that lie beneath teaching decisions (Cooney, 1988; Cobb et al., 1990; Jaworski, 1994).

Without the third assumption, it is hard for effective collaboration between teachers and researchers to begin. Inevitably, in looking critically at teaching, one comes up against the apparent success or lack of success of a teacher's approach in relation to students' learning. It is important to get beyond such surface judgments. A strength of this project was that we were able to address the roots of success, or, importantly, perceived lack of success, with the teachers through a common endeavour, a mutual respect and an appreciation of the problematic areas with which we were dealing jointly.

Values and Beliefs

Both teachers and researchers came to this project with strong views about the nature of teaching that should lead to effective learning of mathematics for students in classrooms. Inevitably, such views were based with varying degrees of explicitness on theoretical perspectives that we can trace in the literature.

At the root of the observed teaching were teachers' aims for students' understanding of mathematics. This involves developing what various scholars have called *relational* understanding (e.g., Skemp, 1976) or *conceptual* understanding (e.g., Brown, 1979). Teachers wanted students to go beyond ritualized knowledge (Edwards & Mercer, 1987) to appreciate reasons for procedures and rules, to relate apparently different areas of mathematics to each other, and to apply mathematical learning to solving problems in mathematics and the wider world.

The words we use here are those of researchers and theoreticians. Teachers did not express their beliefs and intentions in such terms. Rather they spoke about the classroom processes that they were trying to foster, the kinds of activities that would encourage thinking, and ways in which they would encourage students to work together. Thus, dialogue was important: students were encouraged to articulate their processes and perceptions; collaborative activity was encouraged: sharing and contrasting ideas, and reflecting critically; activities requiring exploration and inquiry were introduced with the negotiation of outcomes. Teachers believed that such classroom approaches and the resulting learning environment would be conducive to the deep levels of mathematical understanding they sought in students. In such beliefs they were in accordance with much of the mathematics education community currently in seeking to foster both individual and social forms of learning and leading to a conceptual understanding of mathematics (e.g., Jaworski, 1994; Ball, 1996; Lampert, 2001; Wood et al., 2001).

A Methodological Approach to Analyzing and Developing Teaching

The research here is related to other studies that explore mathematics teaching in naturalistic settings, to describe and analyse in detail the nature of teaching that seeks students' conceptual understandings of mathematics. Such studies focus on students' learning of mathematics, on the sociological construction of the classroom environment, and on the roles of language and interactions in fostering mathematical learning (see for example, Lampert, 1985; Williams & Baxter, 1993; Jaworski, 1994; Fennema et al. 1996; Wood et al., 2001). We see in such studies a significant shift over the last decade towards a knowledge base related to a reflective approach to developing teaching. This knowledge base documents styles or approaches to teaching as well as dilemmas experienced by teachers in interpreting these styles or approaches. The teaching triad research adds to this knowledge base. In addition, it develops a methodological approach to a study of teaching based on an analysis of classroom interactions. An assumption is that the teaching will result in classroom interactions, the discourse of which can be analyzed by studying episodes in the dialogue between students and teacher (micro-analysis). This analysis can be reviewed critically alongside social factors in the wider educational system in which teaching takes place (macro-analysis).

A focus on such discourse and resulting dialogue fits with most of the projects referenced above. Whether the discourse is one of students articulating their strategies for solving problems (Fennema et al., 1996), one of students challenging their peers' arguments for a particular solution (Wood, 2001), or one of discussing the results of a classroom investigation or experiment (Jaworski, 1994) it is possible to use the dialogue as the basis for studying teaching and its outcomes, and learning about the issues and dilemmas for teachers. Most research reports from these various research projects use dialogue to *illustrate* issues in teaching. However, the rigorous use of dialogue actually to *analyse* teaching is less common. Such

analysis has been a hallmark of the work of a number of researchers in Germany (e.g. Voigt, 1985; Steinbring, 1998). Steinbring's analyses have involved an epistemological perspective that scrutinizes the mathematics on which teacher and students are focusing and explores ways in which the mathematical concepts are addressed. What is similar in our work is the line by line micro-analysis which forms the basis of more detailed conceptualizations of the teaching situation.

These conceptualizations enable us to go beyond simplistic judgments on the success, or otherwise, of teaching to the tensions and dilemmas that affect teaching decisions at both micro and macro levels. The fine details of a brief action or interaction can be related to the wider social concerns in which these actions and interactions reside. In earlier writing about the teaching triad we identified the concept of 'harmony' (between the elements of mathematical challenge and sensitivity to students) as a key factor in explaining the apparent success of a teaching/learning episode (Jaworski & Potari, 1998). In our writing here, we explore particularly a situation where there is a lack of harmony, where teaching would be judged critically to fail in its objectives. Micro- and macro-analyses, using the triad as a tool, enable us to seek out the finer details of the complex influences on the teaching situation and its relation to teachers' associated beliefs. In these analyses we found ourselves drawing on constructs that have been used and debated widely in considerations of teaching and its development; those of *scaffolding* (e.g. Bruner, 1985; Williams & Baxter, 1996) and *funneling* (Bauersfeld, 1988). Williams and Baxter talk about *analytical* scaffolding and *social* scaffolding; the former relating to students cognitive development and the latter to their social development. These terms seemed to relate to concepts of cognitive and affective sensitivity. Goldin (in press) discusses the concept of 'affect' and its relation to beliefs and value systems, including how the cognitive and affective representational systems of individuals interact with socially or culturally shared systems. Analysis enabled us to look critically at such constructs, and to see what factors in the micro and macro domains contributed to thinking and behaviour that could be described in those terms.

The teachers in this study used the teaching triad as a tool for their planning of lessons and reflections on students' learning. This tool enabled teachers to be more explicit about their beliefs and intentions for teaching. In addition it allowed reflections on teaching to address occasions where beliefs and actions appeared not to coincide, where teaching did not seem to achieve its declared aims. Here we see an interface between the uses of the triad as a tool for *thinking about teaching* and for *analyzing teaching*. Beliefs, lying in the domain of theory, have to be interpreted into classroom actions through the thinking and professional expertise of the teacher (Cooney et al., 1998). Analyses exposed factors in such interpretation, revealing issues and tensions and encouraging alternative actions and interpretations through an enhanced knowledge base. This means that the teaching triad was acting as a tool for *teaching development* through this research and the activity of the teachers. Such activity and research fits into a **developing paradigm of collaboration** between teachers and researchers in the development of mathematics teaching (Britt et al., 1993; Krainer, 1993; Zack et al., 1997). Partnerships between teachers and researchers are seen as fruitful ways forward in the development of mathematics teaching more widely (Jaworski, 2001).

METHODOLOGY

The Participants

The research was conducted by four participants, two mathematics teachers and two university educator/researchers. It was a partnership in the sense that teachers and researchers agreed to work together for mutual interest, exploration and benefit, although towards differing goals. Researchers sought the nature of the practices observed, issues arising and theoretical conceptualizations of the teaching, using and exploring the *TT* as an analytical device. Teachers considered how the *TT* contributed to their thinking and planning for teaching and evaluation of lessons in order to improve teaching. They designed and taught lessons to a number of classes of students; they discussed with the researchers their specific teaching objectives for each lesson before teaching and their reflections on practices and outcomes after each lesson. They read and commented on initial drafts of the analyses that the researchers produced. In this section we provide a methodological account of our work with both teachers. However, to communicate key issues in the practice of any teacher requires considerable detail and hence space in the paper, so we present detailed analyses of the teaching of only one of the teachers (Jeanette). Where different issues arose from Sam's teaching, these will be addressed in a further paper.

The teachers work in secondary comprehensive[2] schools (for ages 11–18 years) both of which are partners with the university in an initial teacher education (ITE) programme in which the two teachers had each worked as a mentor to student teachers. Jeanette's school is a boys' school, designated as a 'Technology College', for which extra funding is provided bringing with it extra responsibilities for technology use within the school. Sam's is a mixed school. Both teachers are experienced mathematics teachers, well regarded by their peers and with positions of responsibility in their schools. Jeanette is Head of Year in her school where she is responsible for students' well-being within her year, their academic progress, behaviour, and self esteem. She liaises with parents to enable students to fulfil their potential during their time in school. Sam is the head of the mathematics department in his school, responsible for teaching and assessment of the mathematics curriculum. Both had worked in an earlier project, the Mathematics Teachers Enquiry (MTE) Project, to study teachers' classroom research into their own teaching (Jaworski, 1998). There they came to know about the teaching triad and had expressed an interest in using it explore their teaching. The language of the triad was a frequent part of their vocabulary in talking about teaching.

Both teachers' chief aims for teaching involve the mathematical understanding of their students and they have a 'reform agenda' as the basis for developing their teaching such as is recommended in the UK Cockcroft Report or the US Professional Standards for Teaching Mathematics (Department of Education and Science, 1982; National Council of Teachers of Mathematics, 1991). In addition, in England, teachers are required by law to follow the National Curriculum which is tested throughout primary and secondary schooling, satisfy the institutional requirements in their schools, and consider the current demands of society for the education of children. Fulfilling their academic aims and satisfying this multitude of requirements leads to issues and tensions some of which this research reveals.

The Data Collection

Data were collected

(1) through participant observation of lessons by the researchers (involving the researcher in sitting, observing, making notes, and occasionally talking with students) where field notes, audio recordings, and occasionally video recordings, were produced;
(2) from interviews in the form of conversations between researchers and teachers before and after each lesson, audio recorded;
(3) from team meetings between all participants for a discussion of research and teaching, audio recorded.

All recordings were transcribed.

We studied classes in years seven to twelve (ages 11–17 years). Classes were 'sets' in which students were grouped according to teachers' perceptions of their mathematical achievement – *lower* (L), *middle*, (M) or *higher*

(H). For Jeanette, for example, sets in years nine (M) and ten (H) were observed. Teaching varied according to the different characteristics of each class.

The conversations before and after each lesson were informal and concerned issues about planning and teaching that were initiated either by the researchers or by the teachers. Three team meetings (each lasting about three hours) were held during the period of the research and here more general issues about teaching and learning were discussed. Conversations and meetings enabled a development of mutual understanding within the team, nurturing trust and respect; offered further data to broaden the global characterization and contribute to macro-analyses, and acted in a validatory role in which inferences and interpretations could be checked through a discussion of issues.

Data Analysis

Our analyses have focused particularly on classes in which the teachers had explicit goals for changing their way of working with the class. In the case of Jeanette, on whom we now focus particularly, it was her Year 9 class in which she had explicit goals for changing her way of working with this class. She had taught this class in the previous year and had an extremely good relationship with the students. However, she was dissatisfied with the degree of mathematical challenge she was offering: "Am I offering them enough challenge? . . . Am I pushing them enough?"

Analysis of data was in two phases: Firstly on-going analysis during data collection involved an inferential, descriptive and interpretative process. Inferences about the teaching were made during classroom observations and during the reading of notes and transcriptions. For each lesson a summary was written describing briefly the various parts of a lesson (starting activity, group work, etc.) and including interpretations relating to the inferences made. These were checked with teachers during conversations before or after subsequent lessons. From these summaries patterns were abstracted which described the teaching with a particular class, and sometimes, but not necessarily, across classes. These patterns form what we have called a global characterization of the observed teaching – a description in broad detail, attending to styles of teaching and interaction without addressing the finer detail of students' learning and understanding. Each characterization was related to the teacher's expressed beliefs and intentions.

The second phase of analysis related to the teaching triad and was done by the researchers at *micro* and *macro* levels. In micro-analysis the transcribed text from each lesson was read and divided into sections

according to the pattern of activity in the lesson. Each section was then split into smaller parts according to changes in teachers' interactions. For example, in a section where the teacher was working with one particular group of students, a smaller part might involve her interaction with just one of the students in the group. The discourse of this subsection was analyzed line by line looking in fine detail at the language and meanings of teacher and students in relation to the elements of the triad. At the end of each subsection inferences were made about relationships between the elements. During this process the researchers tested their inferences and developed their conceptualization of the *TT*, both with reference to the empirical data and also in their own discussions of details of the analysis. Certain sections were analyzed separately by each researcher leading to a comparison of outcomes and a validation of inferences and their relation to the *TT*.

Following micro-analysis, a macro-analysis was done to attempt to explain why a certain teaching behaviour occurred. This behaviour was linked to relevant parts of the wider set of data including conversations and team meetings, and also to lesson summaries from Phase 1 analyses. The macro-analyses allowed linkages to a wider sociological frame such as constraints of curriculum and assessment, time pressure, parental expectations, familiar patterns of classroom interaction, the particular aptitudes, experiences and expectations of students in any class and elements of teachers' views, beliefs and intentions.

Micro- and macro-analyses are complementary, and the analytic process involved a cycling between the two. For example, when wider issues emerged in the macro-analysis, a cycling back to further micro-analysis was done to see whether finer attention to detail might reveal particularities or consequences of the wider issues; when details of the interrelationship of *sensitivity to students* and *mathematical challenge* emerged from micro-analysis, attention to the wider issues often served to root such interrelationships in the realities of social concern. Moreover, the global analysis of the first phase contributed to linkages in the data. This process was successful in gaining deeper insights into the complexity of the teaching and explaining teaching tensions and issues as the following sections will show.

AN ANALYSIS OF JEANETTE'S TEACHING

In this section we offer detailed examples of our analytical process to show how the triad enabled key teaching issues to be revealed taking us beyond superficial judgments on teaching. First we provide a global characterization of Jeanette's teaching from our first phase of analysis. This characterization reveals general aspects of the teacher's behaviour, deriving from observations in her classroom, and uses the teacher's own words to connect behaviour with beliefs and intentions for teaching. We then go into the second phase of analysis to consider two separate cases. Case 1 refers to a situation where the teacher's goals, to be sensitive to students needs *and* to challenge them mathematically, seem to be achieved: i.e., the teaching seems *effective* in terms of students' mathematical development. The analytical process helps us to look critically at the meaning of the term 'effective' and to link it to the concept of *harmony* of the three elements of the *TT*. In Case 2 in contrast, the teacher's actions seem either to neglect students' thinking or not to challenge it further, so that the outcome seems *ineffective* in terms of students' mathematical development. In this case it is tempting to say that there is a gap between what the teacher aims to achieve and what happens in her actual teaching. However, micro- and macro-analyses reveal a complexity of factors contributing to the apparent discrepancy. As a result, we are able to appreciate better the issues that the teacher faces in making decisions about approaches to teaching. The first case was reported in Jaworski and Potari (1998) and therefore will be dealt with here more briefly than the second case. Following a discussion of the two cases, we look more closely at the teacher's own thinking and development with reference to her use of the triad.

Phase 1 Analysis: A Global Characterization of Jeanette's Teaching

Jeanette's teaching is characterized by an emphasis on individual and group work while whole class discussion takes place mainly for sharing ideas from the group work, for reviewing a test or homework, for introducing and clarifying a task and, relatively rarely, for introducing a new concept. She mostly coordinates whole class discussion, rather than leading it. The group work is mainly on a task of investigative and practical nature while the individual work is based on the textbook or on a structured worksheet which describes a new topic and provides a number of problems for the students to consider related to this specific topic. Jeanette's role is mainly that of a *facilitator* of learning (c.f., Scott-Nelson, 2001), and includes a form of *scaffolding* (Bruner, 1985) which she describes as 'pushing' or 'pulling' her students.

> To push them I will ask an open ended question. What happens if we do this or have you thought about that, or why have you done this here? To pull them I will point to something, sometimes not even say anything, come out with a question and I will know the child well enough … I will actually sit with them and, depending on their ability perhaps, will read a whole question together, or will go through. I will say right lets draw this diagram, you draw it, what do we know, what information have we got? And it's much more guided: what do we need to find here, have we all the information we need, how are we going to set about doing this? Just the usual way through a problem. I will leave it there then, I will start them on a next question and sometimes maybe sit there for five questions, gradually drawing back, so that I am doing less and less of the question with them. (Data extract: TM2: team meeting 2)

In her interactions with the students she typically encourages reflection and negotiation by asking them to explain and justify what they have done, by praising their attempts and encouraging continuation and extension of their work. She provides help by encouraging peer cooperation, by building links between current and previous work, by simplifying the challenge, and by providing emotional support parallel to the cognitive. In some cases her attempts to help a student result in closed questions leading the student towards an answer, or she provides an answer. However, she conceives of teaching as *process* oriented rather than as a *product* oriented activity and aims for the emergence of students' ideas and strategies, and for building their autonomy, self confidence and understanding as indicated in the following data extracts (Data extract: 1DJ9: discussion *after* Jeanette's first observed lesson in Year 9):

> In the whole class discussion, I would start with the right one [response from the groups], or the better one, and get the boys to discuss, to explain how they found that. Because those who had not got it right, it's better for them to realize by themselves, instead of saying OK let's take that one. What is wrong with it?
>
> I do always emphasize particularly with them that just because I am showing one method of doing things … there are other ways of doing things; certainly other ways of seeing how things come together … I think that they are quite happy with the idea that they can do things in different ways.

Phase 2 Analysis: The Two Cases

We focus, here, on a series of three lessons (of 70, 35, 70 minutes respectively) focusing on mathematical concepts of volume and surface area. One researcher observed the first and last lessons and discussed the middle one with Jeanette. Case 1 is taken from the first lesson and Case 2 from the third. We have chosen the lessons to be representative of those with the particular class and of the analytical process as a whole. The issues emerging should be seen as indicative of the power of the analytical process to reveal issues rather than representative of the issues it reveals. It is impossible to address all possible issues even for one lesson, as the emerging complexity shows.

Effective teaching: Interacting with a group of students (Case 1)

The teacher wanted students to understand the nature of the least surface area for a cuboid of given volume. We have described earlier her opening activity focusing on the construction of a box to fit 48 cubes each of side 2 cm. Groups of students worked on this task. We focus first on an interaction between Jeanette and two boys, Tom and Stewart. The boys had two different organizations of the 48 cubes: Tom with 48 cubes in a line; Stewart with a $2\times4\times6$ ($4\times8\times12$ cm) cuboid. They had each drawn nets of their solids to enable them to calculate surface area (respectively 776 cm^2 and 352 cm^2) for a volume of 384 cm^3. The teacher looked, with the boys, at these two cuboids and through her questioning and their responses we gain evidence of Stewart's appreciation of the key mathematical concept (T: teacher, S: Stewart).

Transcription 1. Dialogue from Volume and Area 1 (Data extract: 4J9).

1. T: Now I want you to think why Stewart's is less
2. S: Cause mine's higher and wider and
3. Tom: It's easier to fit in the trolley. [He refers to a supermarket trolley]
4. T: It is easier to fit in the trolley, yes.
5. S: Because mine's got more height and width than Tom's.
6. T: Right, so it will be (many voices) consequently its been made
7. S: Shorter
8. T: Shorter, (many voices in the background)
9. S: More compact
10. T: Right. Compact. Good. Right now, is Stewart's model the most compact model you can come up with, or is there anything better? Well. I don't know. Let's look at James and John's to see if they've done better.
11. [Some interruption here from other students to whom the teacher responds]
12. T: Stewart, you have done really well so far, OK?
13. S: Yes.
14. T: But you need to make sure you are listening in to the others' designs as well.

A line by line analysis of this dialogue is given in Jaworski and Potari (1998). We therefore omit it here. We exemplify our line by line process of analysis in Case 2.

In this dialogue the students seem actively involved in a process of construction of mathematical concepts; also in negotiation in which explanation, justification, and elaboration are essential features (c.f., Wood et al., 1993). The teacher's questions lead to Stewart's articulation of the word 'compact', his own word, that reflects an image that seems to fit well with the concept of minimum area. It is an example of what Jeanette described as 'pushing' her students and is characterized in line 1 by the teacher offering a mathematical challenge that fits with the current stage of students' thinking (their construction of the two differing models). The teacher's actions might be seen to involve sensitivity to students in both

cognitive and affective domains: what Williams and Baxter (1996) have called 'analytic scaffolding' – scaffolding directed at cognitive development (e.g., lines 1, 6 & 10) and social scaffolding directed at affective needs, seen in her later encouragement of Stewart (lines 12–14).

This dialogue can be seen as a special case of a pattern the teacher described as follows:

> In an investigative lesson, I provide the stimulus for the initial problem and then give them some time to explore. And so I now see a bit on a hillside, a bit rocky, fairly open, maybe a bit bleak. Some of them will stay very close to me, not physically, but close to me metaphorically, not close to the problem or their friends. Others will start perhaps to go round the problem, trying things, maybe coming back, making sure they are doing alright and then go off again. One or two will skedaddle down the path and find something very interesting or get nowhere and come back again. And so my role will be making, I will make sure that the ones who want to stay with me are walking with me round, maybe round or round about, encouraging them to go off. Sometimes I feel I push them and they won't go. And perhaps I will leave them and come back and they haven't gone anywhere and then I will have to take them on with me a little bit further and then try and push them off again and they will go. ... When they've gone off on their own track and, and they don't think they've got anywhere, and they ask for help to come back that's fine, it's when they've gone off on their own track and I'm not sure whether they are getting anywhere or not. ... When do you give them a rope and pull them back? (Data extract: TM2)

For some of the students, the *mathematical challenge* can be seen mainly in the initial task as they take responsibility for guiding their own learning with very little input from the teacher. For other students the *mathematical challenge* is more tailored to their needs by the teacher offering hints or asking questions to start them articulating the situation, or by extending the task to new possibilities for exploration. The *management of learning* works in two planes in this type of teaching: in the first plane it is the actual interaction with the individual student; in a second plane it is the coordination of different actions and decisions enabling the teacher to meet the different needs of all the students in the class.[3] We see in this session a situation where the teacher values and at the same time challenges students' mathematical thinking by supporting them both emotionally (praising, encouraging) and cognitively (questioning, seeking clearer conceptual articulation). We would claim, in this case, that the three elements of the teaching triad are in *harmony*. The teacher through her *management of learning* creates an environment where the *sensitivity to students* works in both affective and cognitive domains to make the *mathematical challenge* appropriate to students' needs and thinking. In Case 1 we see agreement between theory, the teacher's perceptions about teaching and learning, and practice, the implementation of these beliefs in practice.

	Dimensions	Volume	Surface area
Cubes (cm)	1) 4x4x3 (8x8x6)	48 (384 cm³)	320 cm²
	2) 48x1x1 (96x2x2)	48 (384 cm³)	776 cm² (*discussed in the following dialogue*)
	3) 6x2x4 (12x4x8)	48 (384 cm³)	352 cm² (*added subsequently*)

Figure 2. Summarizing students' findings on the board.

Complexity in teaching: Reviewing the homework, forming generalizations (Case 2)

This episode took place in the third lesson on the packing problem. The students had completed the group work in this task and had reported some of their findings to the whole class. The teacher had asked them, for homework, to explain the task and how their group set about it and to put down some results about the characteristics of the boxes they constructed.

As part of this lesson the teacher led a plenary session to write onto the board some of the results from the students' work. She drew a table, as shown in Figure 2, and invited students to give her a set of dimensions. Two sets were given by students and written into the table by the teacher. In each case Jeanette asked how the box would look and the value of its volume. The third entry was added during subsequent discussion.

The following dialogue is from the whole class discussion between Jeanette and the students in their attempts to calculate the surface area of the second box entered in the table (T: teacher, P: Peter, S: another student).

Transcription 2. Dialogue from Volume and Area 3 (Data extract: 6J9).

1. T: Right. Now, how am I going to find out the surface area?
2. S: This is where I got stuck.
3. T: Right. This is where people got stuck. Peter (the only one who puts his hand up) you obviously got an answer. Could you tell us how you started that problem? What did you do to work out the surface area?
4. P: I drew a net.
5. T: Right. You actually drew a net. Can you describe for me roughly how it looks like?
6. P: Is one . . . (he hesitates)
7. T: (draws on the board a strip of 2 × 96) How many are like that? (Figure 3a)
8. P: 4
9. T: Right. 4 like that. (She draws again. Figure 3b)

3a 3b 3c

Figure 3. Building the net for the $48 \times 1 \times 1$ cuboid.

10. P: Then one on this side and another one on the other
11. T: (draws the two faces of 2 × 2) (Figure 3c) You need to write some dimensions. This time you need to write some dimensions in cms. You need to work out the surface area in cms. Peter what is this length here? (length l in Figure 3c)
12. P: 96
13. T: 96, good. What about all the way down there (she shows the total width of the four faces (8 cms, d in Fig. 3c)
14. P: . . . (cannot respond)
15. T: No, ok. not to worry. What about this bit there (the width, w in Fig.3c)
16. P: 2
17. T: 2 cms, up here
18. P: that's 2
19. T: that's 2 as well. Down here is also 2 cms. What about these little end bits here (the faces 2 x 2)?
20. P: 2 by 2
21. T: The total in this case is (on the board she wrote: total = 192 + 192 + 192 + 192 + 4 + 4 =).
22. T: Peter, what is the answer? Has everybody found the answer for this one?

According to the teacher's metaphors, this situation might be described as "**standing on the platform, offering them a hand and pulling them up**". Here the teacher wants her students to have specific outcomes (solutions to the questions about finding surface area) and uses questioning to try to achieve these outcomes.

We offer the following outline of our line by line micro-analysis, using the triad. *SS/A* and *SS/C* indicate sensitivity to students in the affective and cognitive domains respectively. Comments in brackets, with query, indicate interpretations regarding teacher intentions that are checked in macro-analysis.

- 1–3. The teacher indicates a *MC* which the students had been asked to consider in their homework. A student admits that he had difficulty and the teacher responds at an affective level by stating that this was difficult for most students (SS/A). She asks Peter to give an answer as he is the only one offering to contribute (ML – time pressure, resulting in not asking other students?). She asks Peter to describe his process of working (possibly to explore his thinking, SS/C/A, and to encourage the sharing of his ideas with the other students SS/A/C, through this ML).

- 4–10. Here, Peter says that he drew a net and the teacher asks him to describe how he did it (SS/A/C). She does not encourage Peter to show his working, for example by asking him to draw his net on the board. (*ML* – time pressure?) Instead, she manages the situation herself, using Peter's contributions to demonstrate the process to the other students. *MC* seems to be lacking here. It seems that in this part of the dialogue there is mostly *management of learning* directed towards the teacher's mathematical objectives.
- 11–20. The teacher poses a new challenge of finding the dimensions of the net (*MC*) and gives reasons why this is important. She reduces the challenge by offering small steps in the form of short closed questions. In line 14, Peter cannot respond to one of these questions, so (at 15) the teacher offers Peter emotional support and splits the question further so that in the end Peter gives a successful answer. The interaction here is similar to the previous one (4–10), lacking challenge, but showing sensitivity to Peter at an affective level. Here, *A* appears to dominate over *C* which may be due largely to the time factor.

Two issues seem to emerge from the above analysis: (i) the nature of the *mathematical challenge* and how it changes in relation to the other two elements of the triad, (ii) the dual nature of *sensitivity to students* and its relation to *mathematical challenge*. The *mathematical challenge* starts with a problem which seems to require both conceptual and factual considerations of finding the surface area of 3-D solids. From the way that the teacher manages the discussion, it seems that conceptual aspects are mainly implicit while the factual ones are revealed in small distinct steps. As a result, *mathematical challenge* seems to disappear, *management of learning* dominates and does not provide access to students' thinking, largely because *sensitivity to students* operates mainly in the affective domain. This is characterized by the teacher's appreciation of students' difficulties and her reinforcement of classroom norms of respect and inclusion shown in her support and encouragement for Peter. However, the episode seems to lack balance between the cognitive and affective dimensions of *sensitivity to students*. The missing cognitive dimension reflects a corresponding lack of challenge.

Micro-analysis reveals a three stage 'elicitation pattern' similar to that described by Voigt (1985, p. 80) which Bauersfeld (1988, p. 36) has called a '**funnel pattern**'. The teacher posed an open task to offer different possibilities for the students to explore; she then guided them towards a solution, and in the end produced a solution herself. Micro-analysis is (necessarily) limited to the dialogue in the episode to help us explain and interpret the interactions. However, at a macro-level, other issues/concerns become evident. The teacher had found from the students' homework that they had difficulties with some parts of the task, so she wanted to make sure that all the students could complete the task and develop some computational skills on the topic of the area and volume of 3-D solids. At the same time she did not want to spend a lot of time in this phase of the lesson, as she wanted the students to move to her next activity – a text book exercise for practice and consolidation of techniques in finding areas and volumes. This led her to reduce the challenge and guide the students towards her content oriented objectives. Time pressure, to achieve curriculum goals in the available time, was evident. Her shift to the focus on skills needed to answer questions in the textbook resulted from this time pressure. We could only conjecture, in micro-analysis, that time was a factor, but our conversations with the teacher, both in school after the lesson and in a later team meeting, confirmed this tension.

Before moving the class to working on the exercise, the teacher finally wound up the investigative activity by asking about the third column in the table. Tom and Stewart contributed their understandings of compactness, giving a good explanation of how surface area can be minimized. Here there seemed to be a possibility to re-enter a conceptual level of interaction, enabling the whole class to understand the meaning of compactness through the presentation of Tom and Stewart. However, following their explanation, the teacher ended the whole class session. Why did she choose not to build on these potentially fruitful contributions to work towards an abstracted conclusion?

To address this question, we need to look again to a macro-analysis. In discussion with Jeanette about her decision not to extend the challenge we see that she perceived some students still to have difficulties and wanted to be sensitive to the wider needs:

> I felt that there were only a few that had actually got that idea and I think if I'd carried it forward and said well what would be a more perfect number to use, that would have lost a lot of them. That's why I left that; but I think it's something to come back to when we come to look at something like this again; looking for a more perfect solution. (Data extract: D5J/9)

She wanted the cubes activity to help students to address these concepts in a more abstract context – the exercise in the textbook – which was part of her planning to enable students to achieve curriculum objectives.

Moreover, it was Friday afternoon and many students were excitable and inattentive.

The funneling pattern discussed above is not one with which Jeanette feels comfortable. Analysis shows two major factors motivating her approach: her serious concern for students' self-esteem, resulting often in sensitivity that is effective in the *affective* but not always in the *cognitive* domain, and a need to address her curriculum objectives under time and social pressures. As Head of Year in her school, students' wellbeing and the school's responsibilities to students with respect to parental and societal expectations are central to her views on education (we quote from her own words later). The curriculum and time pressures are not just inconvenient impositions due to political forces, but also play a major role in interpreting responsibility towards students' wider educational fulfillment and success. These factors acting together can result in a reduction in challenge, and in leaving students with ill-formed concepts, and a dependency on instrumental skills. Jeanette's recognition of a need for greater challenge is elaborated here in the particular detail of interactions: practical wisdom adds to theoretical ideas and creates opportunity for the future to recognize patterns and develop alternatives, either in pre-planning or in the classroom, as the next section shows.

The Teaching Triad as a Tool for Reflection and Development of Jeanette's Thinking about Teaching

Jeanette used the teaching triad for planning her lessons and for reflecting on and analyzing her teaching actions. For the lessons above, Jeanette drew the diagram in Figure 4 and tried to fit her planning to this diagram.

She conceived a dynamic interrelationship of the elements. First, she started to consider the kind of *mathematical challenge* she could offer to the students. Then she moved to *management of learning* where she considered the resources she needed to offer, the classroom organization and the encouragement of communication in the class. *Sensitivity to students* was seen in providing students with the chance to explore, discuss, and argue but also to encourage the building of confidence in their own ideas. She also saw *sensitivity to students* as the basis for making *mathematical challenge* meaningful to students. The following extract indicates how the teacher conceived the relationship between her planning and the *TT*.

When I'd written my lesson plan, I went on to reflect on the triad and it was the mathematical challenge that came out first. It comes out in my mind as well as to the top of the triangle. But with them [this particular group of students] . . . I don't know, maybe it's because it's a practical lesson it's sort of management of learning which is the next aspect of the triad that I actually started to focus on. And they were more as prompts for me to have the resources on hand, to work as a group and then to go around and see how well they share their ideas. And then, that was interesting, it was then that I considered sensitivity and so . . . I obviously, I wanted them to have a chance to explore and discuss and argue their ideas. But also, within that, I would encourage them to build their confidence to their own ideas, so what I am looking for then is how . . . if they've got a chance to explore and discuss their ideas and to explore some things and put their own point of view forwards how will that then extend the mathematical challenge. That's what I am looking for in Year 9. (Data extract: D4J9)

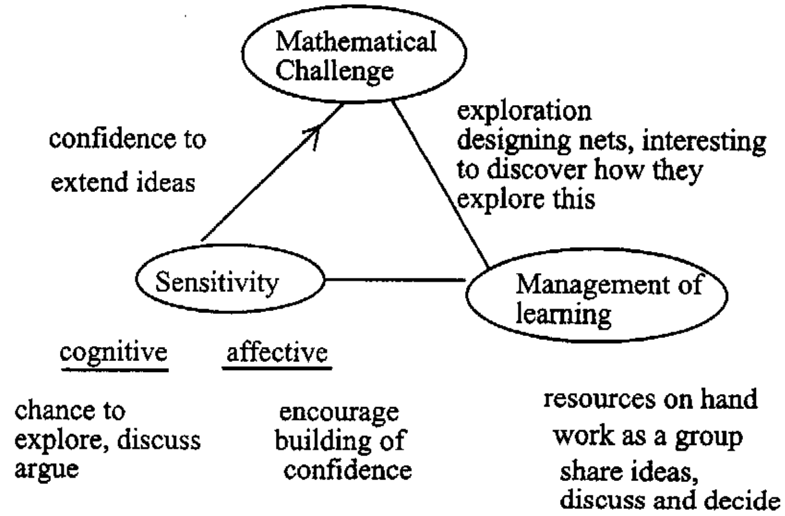

Figure 4. Jeanette's fitting of her planning to the *TT*.

Later, at a meta-level, she reflected, "Now I am focusing much more on it [the triad] and I am just starting, I am just interpreting my actions." This shows Jeanette's own overt recognition of her analytical activity in planning and reflecting on her lessons. She is surprised that in her planning the *mathematical challenge* has a priority even in this group where she thought initially that she did not offer enough challenge in her teaching. She has also started to see sensitivity and challenge ('bits' of the triad, as she calls them) very much related to affective and cognitive interactions – notions we introduced to her – and this helps her to go more deeply into analysis of her teaching actions.

With Year 9, I can't explain why but I feel I'm more in tune with their affective and cognitive and I wonder if that's part of the movement between the challenge and the sensitivity bits. (Data extract: D4J9)

Jeanette feels that she should work next on *management of learning*. She has started questioning how this element is related to the management of the resources and to the flow of the teaching in moving from one topic to the next, in order to make sure that she has organized an appropriate environment in which students can work. In one discussion after her teaching, she recognized differences between planning and teaching which shows her recognition of interactions between the three elements of the triad.

When I plan something, I think I know how I am going to challenge them mathematically with what I'm presenting. And I can make sure by managing resources and time that it's going to work, that they're at the right stage that they can move on. And I know which ones are going to need extra help. Now, then when you're actually teaching and you're actually in the lesson and you're thinking on your feet the whole time, things will change, you have to be prepared to change. Therefore then, within the lesson, maybe my perception of the mathematical challenge perhaps changes slightly which has a knock-on-effect to management of learning and sensitivity towards students ... you've planned it carefully but now you've got to change so thinking quickly on your feet, the sensitivity bit – will they be able to carry on with this for homework? How will they feel if they get stuck? Therefore, how much [homework] do I actually set them? Or should I just say OK, no homework tonight? (Data extract: 4DJ9)

Through this description from Jeanette, we can see the dynamic nature of her use of the triad in the way that the three elements interact in the complexity of planning and interpreting her teaching. We also see how she has started to elaborate the concept of *management of learning* and how this is related to the process of teacher's decision-making in classroom action.

Towards the end of the project in the discussion at a group meeting, Jeanette showed a recognition of factors, other than those coming from the classroom culture, which influence her teaching. Thus, she developed further her conceptions of *management of learning*. In the following extracts, Jeanette sees a difficulty in operating according to her teaching theories as a number of different factors act in her decision making process. Here, the *management of learning* is influenced by external pressures: the system of Examination (GCSE: General Certificate of Secondary Education[4]), the government's educational policy, parents' and students' attitudes, the school's policy and its role as a technology college, and the teacher's different roles in the school:

The caring side, the side of me that is sensitive to students, makes me think I still have to build up their confidence. They find it difficult, so they stop. I have to work on their confidence, but the management of learning side – for those who are in five months time sitting their GCSEs, and there is so much riding on technology, on improving our standards. I've got a C group which is the grade Cs who are being targeted to improve and ... all that pressure piles in. And whilst one side of me says yes, keep building up their confidence, give them easy questions, the other side is – I'm getting so frustrated because I know that it is just years and years of apathy built up. And I'm positive, and the head of the year side of me who meets a lot of parents and sees parents with these kids, I am convinced ... that parents are not allowing their kids to take risks they do far too much for them.

By the time they get to year 11 ... they must be very confused because they have a diet of a lot of teachers who keep them very much pinned down, do this, do that, and then at home they are not thinking for themselves either because their parents are doing everything for them, or they will give in to them very easily. And there is a number of parents who come to me and say 'what can I do, he doesn't want to come to school?' What can I do to try to teach mathematics in the middle of all that? I'm actually finding it quite stressful, much more so than in any other school [I have worked] and I think it's because there are so many, so many conflicts going on there. (Data extract: TM3)

Here we see Jeanette looking critically at her practice, questioning, acknowledging issues, recognizing tensions. It is clear that decisions, for her, go considerably beyond immediate issues of sensitivity and challenge, cognition or affect in particular mathematics lessons. However, it is use and consideration of the triad that have revealed the necessity to see the finer details of mathematical learning within the complexity of these wider issues. We have not been able to discuss here the finer details of development of Sam's teaching due to space considerations; however, these details were also considerable, and particularities which take us beyond the issues addressed here will be discussed in another paper. It would have been valuable to explore further the effects of Jeanette's and Sam's enhanced awareness on their subsequent classroom practice but the period of this research limited what was possible.[5]

PERCEPTIONS OF THE TEACHING TRIAD EMERGING FROM THIS RESEARCH

In striving to teach effectively (as teachers) or to talk sensibly about effective teaching (as teachers and educators) there are various principles, theories and beliefs to which we aspire. These are laid out at length in documents like the NCTM standards and the Cockcroft Report. Ultimately, however, these principles have to be translated into action for the classroom. With whatever sincerity a teacher tries to follow such principles, there is a complex set of psychological and sociological factors that have to be considered in constructing, managing and achieving classroom teaching and learning. The triad as an analytical tool helps to recognize, interpret, and understand processes of translation from principles to outcomes. The triad as a developmental tool helps to make such translation more effective.

The Triad as an Analytical Tool

Analyses here have shown that while *mathematical challenge* can be seen to operate largely in the cognitive domain, *sensitivity to students* seems to work in both the cognitive (C) and the affective (A) domains for the mathematical learner. Cognitive sensitivity concerns the appreciation and recognition of students' thinking which can then be developed further by appropriate challenge. Affective sensitivity includes fostering students' personal beliefs in and valuing of their ability to do mathematics and think mathematically; it concerns students' wellbeing and positive attitude within the classroom setting, and does not always connect obviously with a cognitive dimension. It is possible to foster a good classroom atmosphere in which students are happy, confident and well-motivated but do not achieve the quality of mathematical thinking of which they are capable. Jeanette suspected that this was the case for her Year 9 class: she had worked hard on students' self esteem, but felt that she had neglected *mathematical challenge*. In trying to gain some access to what might be considered 'effective' teaching, we recognized that a balance between sensitivity in the cognitive domain and mathematical challenge seemed crucial. When such a balance seemed to be achieved we referred to it as a *harmony* between challenge and sensitivity. We asked about the extent to which harmony depended also on affective sensitivity.

For example, we have seen that Jeanette's teaching of her Year 9 class included three main considerations: i her concern for students' self esteem, ii an aim to offer a higher degree of mathematical challenge, and iii a need to deal with factors of time, curriculum, examination requirements, parental expectations and so on. Self esteem and challenge are associated with *principles* of teaching: for example, principles that self esteem is necessary for cognitive development; and that cognitive development is unlikely to happen without suitable challenge. Factors of time, curriculum, examinations, and parental issues are all a part of situational complexity. Analysis linked elements of the triad to elements of teaching principles and situational complexity. Here, Jeanette's concern for self esteem translated into classroom actions that we described as sensitivity to students of an affective nature. In line by line analysis, we identified particular words and phrases as showing affective sensitivity – praising students, guiding their actions, listening to their explanations. Challenge was seen in tasks set by Jeanette and questions she offered. However, challenge needs to be judged carefully to fit with students' current levels of cognition. Also, even if there is some degree of cognitive fit, challenge may not be taken up if the affective dimension is not also appropriate. Micro-analysis examines relationships between these dimensions mainly at a psychological level,

identifies actions and elements of dialogue that contribute to harmony or disharmony, and leads to interpretations of motivating factors in a teacher's decision-making.

The struggle of a teacher to enable students to understand or to gain access to a concept was seen to involve pedagogic processes of *scaffolding* (Bruner, 1985) or *funneling* (Bauersfeld, 1988, p. 36). Such processes are linked to the teacher's management of the learning situation (ML). We analyzed these processes further in terms of the *TT*. The following sequence of stages in an interaction can be seen to develop into a funneling process.

1) The teacher offers a challenge to which the student cannot respond.
2) The teacher scaffolds in some way, reducing the challenge in order to engage the student in the ideas. This indicates sensitivity in the cognitive domain.
3) Further scaffolding is offered, increasing sensitivity in either the cognitive or the affective domains, reducing challenge further to enable the student to engage.
4) (3) is repeated until either the student engages or the teacher tells the student what is required.

If at any stage in the above process, the student takes up the challenge, the process stops and the student engages with the challenge. Thus at stages (2) or (3) the scaffolding might be seen to have been successful, with sensitivity and challenge meeting harmoniously for the student to engage actively with the mathematics. However, if the student is not able to take up the challenge at any of the earlier stages, the funneling process results in the teacher giving the answer, or in the challenge becoming negligible for the student or both teacher and student.

As educators we use words like scaffolding and funneling to refer to pedagogic concepts in general terms. Scaffolding is often regarded positively as a helpful process (e.g. Bruner, 1985; Williams & Baxter, 1996). However, it might lead, also, to encouragement of a student's dependency on the teacher – a 'crutch' (Jaworski, 1990). Funneling, on the other hand is offered to describe an ineffective teaching approach (Bauersfeld, 1988). When we take the trouble to analyse closely particular interactions in which we perceive scaffolding or funneling, we recognize that the terms themselves are insufficient to carry the complexity for the teacher of the interactions with the student. Voigt (1994) writes critically of teachers continuing traditional patterns of interaction in the classroom perhaps contradicting the teachers' beliefs or intentions, "teachers never realise that traditional patterns of interaction are still alive in their classrooms nor that they contradict the teacher's intentions" (p. 288). We believe that our analyses show that this position may be more complex than it seems at first sight. In the case of Jeanette, that was certainly revealed as we studied interactions at micro and macro levels. The triadic analysis goes some way to making evident this complexity and enabling us to address it, particularly when harmony seems lacking.

In the lack of harmony between challenge and sensitivity, we can see where alternative words or actions by the teacher might have allowed the students to engage more effectively with the mathematical ideas. For example, in Case 2, we might ask *why* the teacher did not elicit more carefully the thinking of students in building up the table of results. Her funneling activity seemed contrary to her expressed aims for work with students, and different from her approach in small group work. Macro-analysis provided explanations for the perceived disharmony. Tensions for the teacher in navigating psychological and sociological factors led to ineffective outcomes in students' achievement.

It was in such considerations that we started to recognize the need to link a triadic analysis to the wider sociological concerns impinging on the classroom and the teachers. For example, we see in Case 2 that Jeanette's decisions motivating interaction were related to wider sociological concerns as much as to cognitive factors. In a pragmatic immediacy, where decisions cannot wait on extensive reflection, the teacher recognized that further work needed to be done where students' mathematical conceptualization was concerned. There was not enough time, mood or cognitive readiness to deal with her planned mathematical objectives at that time. So she decided to return to these at a later date.

The Triad as a Tool for Teaching Development

The teachers' reasons for participating in the research was that it would give them the opportunity to explore how the triad might be of use in enhancing their thinking, planning and classroom activity. We saw that Jeanette used the triad in planning her lessons and thinking about outcomes of lessons. Unsurprisingly, the language of the triad permeated our discussion about teaching and making sense of that teaching, particularly in discussions between teacher and researcher about the lesson that was to be taught, or had just been observed. Such discussions took place in the immediacy of classroom decisions such as those discussed above, and before any detailed analysis of the data was possible. In the timescale of this research, it was not possible to research the impact of analyses on the thinking of the teachers in the longer term. Much of the more detailed analysis took place after data collection ended. Another phase of research would have been necessary to test out the impact of analyses on teachers' thinking

and future teaching. This was a limitation of the research described here.

However, during the timescale of this research, the triad served as a stimulus for talking in depth about issues in teaching. The lengthy team meetings provided evidence of the teachers' thoughtful attention to issues that had been raised as a result of classroom activity and subsequent discussion, and provided considerable evidence of the wider social concerns that motivated or constrained teaching. Such raising of issues fed back into teachers' classroom activity.

Thus, we see that there was impact from use of the triad for the teachers' development of teaching during the timescale of the project. However, change in teaching is a lengthy process as we see, extensively, from the literature in mathematics education and beyond (e.g. Fennema & Scott Nelson, 1997; Fullan & Hargeaves, 1997). We need to ask what potential there is for teaching development that builds on what this project has achieved.

CONCLUDING THOUGHTS

The triad was a tool which provided access to complexity and helped us to perceive dualistic aspects of teaching (e.g. funneling/scaffolding, conceptual/procedural) in a more integrated way in terms of both pupils' cognitive and affective development (SS) but also in terms of teachers' obstacles and decisions (ML). Its use led us to consider wider social concerns alongside cognitively focused factors (such as harmony), and to realize the impossibility of separation of the cognitive and the social. It acted as a tool for our own learning and development by bringing these factors and relationships to a clear focus in both theoretical and practical domains, enabling a dynamic linking of theory and practice.

These elements provide the basis of a theory of teaching by making possible explicit links between teaching principles, classroom interaction and teachers' thinking about cognitive and social factors. We can see possibilities for a methodological applicability that goes beyond substantive outcomes. By this we mean that the practices we developed in analyzing teaching are available for wider use, with some confidence that the outcomes will be fruitful in raising issues central to practice, enabling teachers' serious engagement with these issues, and ultimately creating possibilities for modifying practice. For example, we ourselves are using the triad for analyzing teaching in other places and at other levels, such as undergraduate level.[6] The triad might also be used to address apparent inconsistencies that have been reported in the literature between teachers' beliefs and their observed teaching (Raymond, 1997; Cooney, 1985; Voigt 1994; Skott, 2001). Also, we see the triad entering the discourse in our teacher education programmes and becoming a tool with which beginning teachers can start to analyse their own practices. Both teachers in our project are mentors in one of these programmes, and use the language of the triad with their student teachers. In all of these ways we see the triad contributing to the developing thinking and practices of teachers and teacher educators.

Finally, it needs to be said that the triad is no more than a tool. The critical dimension of mutual exploration developed between teachers and educators through the use of the triad is in our view one key to successful teaching development.

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NOTES

1 Some of the following discussion appeared in Jaworski and Potari (1998).
2 'Comprehensive' implies a full range of achievement of students within the school as measured according to standardised achievement levels and testing.
3 These two planes have important connections with Vygotsky's two planes: those of the inter- and intra-psychological in learning and development (Vygotsky, 1978, p. 57). Such analysis waits for another paper.
4 The GCSE is the national examination across all subjects at 16+.
5 This research was conducted during the sabbatical leave of the first author.
6 Jaworski, B. (in review), *Sensitivity and Challenge in University Mathematics Teaching*.

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