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ANNE M. RAYMOND and MARYLIN LEINENBACH

COLLABORATIVE ACTION RESEARCH ON THE LEARNING AND TEACHING OF ALGEBRA: A STORY OF ONE MATHEMATICS TEACHER'S DEVELOPMENT

ABSTRACT. This paper presents the story of a mathematics teacher's transformation that resulted from her engagement in collaborative action research. The collaborative research, conducted by a university mathematics educator and an eighth-grade mathematics teacher (Marylin), centered on investigating the outcomes of implementing a 'Hands-On Equations' approach to teaching algebra. This algebraic study, which is briefly reported herein, serves as a vehicle for examining the questions, reflections, and changes brought forth by Marylin throughout the collaborative process. Thus, the project of investigating the teaching and learning of algebra can be viewed as the 'sub-study' within the broader study of the effects of collaborative action research on the mathematics teacher involved in the investigation. Within this paper, issues related to the broader study of collaborative action research in the mathematics classroom, that are connected to Marylin's transformation, are discussed. These issues include the characterization of and goals for collaborative action research as well as the challenges for collaborative action research in mathematics classrooms.

1. Introduction: Marylin's dilemma

Marylin had a dilemma. Because her middle school (grades 6–8) had elected to implement an 'algebra for all' program, she suddenly found herself, after teaching middle school mathematics in the United States mid-west for 22 years, faced with teaching algebra to a wide range of students (ages 11–13), many of whom would not have normally been encouraged to pursue algebra in middle school. She was not confident that she would be able to efficiently teach algebra to all of the students through a traditional textbook approach. Fortunately, she was afforded the opportunity to attend a workshop on teaching algebra through a manipulative-driven algebra program.

As Marylin began to teach the innovative program (Hands-On Equations[1]) during the tenth week of school, she started to worry about whether or not the program was effective. Specifically, she was concerned that some students would become dependent on the manipulatives and might not fare well when faced with traditional teaching methods in future high school

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mathematics courses. She was also concerned about the depth to which students would learn algebra and how they would perform on the end of the year standardized algebra test required to be taken by all algebra students, particularly since her colleagues at the middle school and high school (with students ages 14–18) taught strictly via algebra textbooks and were skeptical of her hands-on teaching practices.

Marylin called Anne, a university researcher, and introduced herself and her dilemma, and asked if Anne might be interested in talking with her about a possible collaborative action research project. The two met to discuss the potential for research. After some discussion, it was clear that the questions of most interest to Marylin were (a) How does the use of these mathematics manipulatives in an algebra class affect students' confidence and interest in solving algebraic problems? (b) How does the use of these mathematics manipulatives in an algebra class affect students' ability to correctly solve algebraic equations and (c) Will the students' retention of algebraic skills learned via manipulatives last beyond the eighth-grade experience? The question of primary interest to Anne was, 'How will engagement in collaborative action research affect Marylin as a mathematics teacher?'

Both Marylin and Anne agreed to document their explorations into these questions as they delved into their first collaborative action research project. Thus, the project of investigating the teaching and learning of algebra serves as the focus of a 'sub-study' within the broader context of studying of the effects of collaborative action research on the mathematics teacher involved in the investigation.

2. THE BROADER STUDY: EXAMINING EFFECTS OF COLLABORATIVE ACTION RESEARCH ON THE MATHEMATICS TEACHER

2.1. *What is collaborative action research?*

Collaborative action research is becoming an increasingly recognized field of educational research (Cardelle-Elawar, 1993; Clift, Veal, Johnson and Holland, 1988; Miller and Pine, 1990). It provides a medium for teachers to systematically look at the problems they face in their classrooms in an effort to find practical solutions. Although the definition of action research is still in flux, leaders in the field offer some characterizations. Miller and Pine (1990), for example, define action research as 'an ongoing process of systematic study in which teachers examine their own teaching and students' learning through descriptive reporting, purposeful conversation, collegial sharing, and critical reflection for the purpose of improving classroom practice (p. 57)'. Clift, Veal, Johnson and Holland (1988) suggest that collaborative action research between university and school faculty is characterized by its focus on practical problems of individual teachers or schools and its emphasis on professional development and support for collaboration between teachers and university staff.

TABLE I

Goals of collaborative action research

- To bridge the gap between universities and schools
- To provide opportunities for teacher enhancement
- To stimulate classroom reform
- To enhance the professional status of teaching
- To improve teaching and learning
- To generate theory and knowledge

2.1.1. *Goals of Collaborative Action Research*

To further characterize the nature of collaborative action research, six goals (see Table I) of collaborative action research have been identified (Raymond, 1996). A primary goal suggested by many (Calhoun and Glickman, 1993; Cardelle-Elawar, 1993; Miller and Pine, 1990) is *to bridge the gap and strengthen the relationship between universities and schools.* Collaboration between university and school personnel fosters communication and respect between participants. In addition, collaborative action research centers around issues that hold interest to both university and school partners. Thus, the existence of common interests and common goals helps to bridge the often-present gap between researcher and teacher.

Collaborative research between university researchers and classroom teachers *presents opportunities for a more 'action-oriented' approach to teacher enhancement* (Clift et al., 1988). As teachers are encouraged to reflect upon and systematically examine aspects of their classrooms, they are likely to make changes based on observations that lead to improvements in their classrooms. Collaborative action research partnerships create rich opportunities for the professional development of teachers (Rafferty, 1995). Further, theorists claim (Cardelle-Elawar, 1993) and studies show (e.g. Raymond and Hamersley, 1995) that teacher-inspired *action research has the potential to result in immediate classroom reform* because the results are more context specific and meaningful to the teacher. When teachers are involved in investigations of their concerns within their own classrooms, they often make discoveries related to their teaching practice that stimulate change.

There are broader goals of action research. According to Miller and Pine (1990), 'This 'action research' role for teachers can *enhance the professional status of teaching ...*' (p. 57). Teachers involved in classroom research that leads to improved teaching set an example for excellence in reflective practice'. In addition, results of action research projects are often shared and reported by classroom teachers within the educational community through conferences, workshops, and publications. This type of professional activity presents teachers in a new light. They are viewed as reflective, capable educators who understand what it means to look at one's practice critically and affect change when necessary.

At the heart of the teacher research movement is the assumption that *teachers will improve classroom practices and enrich their understanding of teaching* by participating in action research projects. Unlike traditional educational research, the purpose of action research is to transform practice. However, while the improvement of teaching and learning is a key goal of the teacher research movement, there is no literature which supports the claim that teachers involved in this type of activity have, indeed, utilized the knowledge they obtain and ultimately improved their practice on a *long-term* basis. 'No longitudinal studies exist which investigate the actual use of new skill or the permanence of change in self-perception or behavior which results from participation in an action research project' (Smulyan, 1987, p. 67). Also, Rogers, Noblit, and Ferrell (1990) contend that while teacher research may in fact, improve some teachers' practice, it does not have that effect on all.

Finally, Miller and Pine (1990) suggest that *collaborative action research may be a valid means of generating theory and knowledge*. There is much debate about whether or not action research should be valued beyond its teacher enhancement and reform opportunities. Noffke (1994) contends that one of the contemporary challenges of action research is to address the question of whether findings from action research can truly contribute to the body of research on education and educational reform or whether action research must be considered a singular form of research with methodologies unique to the field.

2.2. *Issues raised in the broader study*

Collaborative action research projects set in the mathematics classroom have not been reported in large numbers. However, there are reports of inservice programs aimed at guiding teachers to question their mathematics teaching (e.g., Jaworski, 1991) and to bring a level of investigation

into their teaching (e.g., Jaworski, 1987). These types of research efforts are vital because central to collaborative action research are the issues of questioning, reflection, and change. When questions are raised, teachers need a means of systemic investigation so that they can sufficiently answer their questions. Consequently, through such investigation, comes reflection and change.

In the opening of this paper we were introduced to Marylin's questions. In order for the reader to make sense of Marylin's reflections and the changes in Marylin's outlook on mathematics teaching that resulted from the collaborative action research process, we first provide some details about the collaborative study itself so you may better understand the vehicle through (the context in) which the transformations occurred.

3. THE SUB-STUDY: EXAMINING A HANDS-ON APPROACH TO ALGEBRA

3.1. *Questioning the learning and teaching of algebra*

Much has been studied about students' abilities to perform algebraic tasks, students' abilities to understand algebraic concepts, and ways by which students' algebraic understanding can be improved (Bell, 1976; Booth, 1984; Kieran, 1981; Lee and Wheeler, 1989; Noss, 1986). There is some debate as to what has made algebra the apparent 'gateway' to mathematics. In a summary on research about the learning and teaching of school algebra, Kieran (1992, p. 390) poses these questions, 'What makes the comprehension of school algebra a difficult task for the majority? Is the content of algebra the source of the problem? Or is it the way it is taught that causes students not to be able to make sense of the subject?'

Sfard (1991) suggests that abstract mathematical notions can be conceived in two ways: structurally (as objects) and operationally (as processes). She claims that the shift from a process conception to an object conception does not happen quickly or easily and yet we expect students to make this shift to understand algebraic representations in their first algebra experience. Markovits, Eylon and Bruckheimer (1986) and Vinner (1983) suggest that student difficulties with algebraic notions can be partially explained by students' lack of clarity on the concept of function and the many definitions of functions they encounter. Others contend that attention to developing the notion of variable (Noss, 1986) is a crucial element of furthering the algebra learning process.

There is a scarcity of research on both models of teaching algebra and on the beliefs and attitudes of algebra teachers (Wagner and Kieran, 1989).

More research, however, has been done in the development of innovative algebra curricula (e.g., Rachlin, 1987), and it is this type of innovative material that teachers like Marylin are drawn to when seeking a change in their teaching of algebra.

There has been much effort to bring a more concrete understanding of algebra to middle and high school students through a variety of manipulative tools including computer simulations (e.g., McArthur, 1989). Studies of manipulative use in algebra classrooms have had mixed results. McClung's (1998) study comparing two high school classes, one learning algebra strictly through the use of lecture, homework, and in-class worksheets and the other through lecture, homework, and the manipulatives 'Algeblocks', finds that the students who did not use manipulatives outperformed students taught with manipulatives. In contrast, Hinzman (1997) finds that middle school student performance in pre-algebra is actually enhanced by the use of manipulative materials and that attitudes toward mathematics are significantly more positive when learning with manipulatives.

In the study reported herein, Marylin holds the belief that the means through which students are exposed to algebra makes all the difference in their comprehension. Consequently, Marylin's adoption of the 'Hands-On Equations' algebra materials seemed a natural choice for an algebra teacher who was struggling to revise the middle school algebra experience. However, relying on an innovative curriculum to 'improve things' without careful consideration of the impact of that curriculum left Marylin with doubts. Thus, her efforts to engage in collaborative action research in order to investigate the value of the curricular change was an attempt to not only assess the curriculum itself, but to evaluate her beliefs about the teaching of algebra.

3.2. *The hands-on equations program*

'Hands-On Equations' is a program designed to introduce algebraic concepts to students grades 3 and up (ages 8 and up). It operates with manipulatives that include blue pawns and white pawns, representing x and $-x$, respectively, and red number cubes and green number cubes, representing positive and negative integers, respectively. Students use these materials to represent algebraic equations (on a mat picturing a balanced scale) and then make 'legal moves', such as removing the same number value from both sides of the equation or removing the same number of pawns from both sides, to arrive at a value for the pawn or x. Students are then asked to rebuild the equation with manipulatives and check their answers. Ultimately, students are taught a means of representing the actions with the manipulatives pictorially, making the move toward a more abstract representation of algebraic solutions.

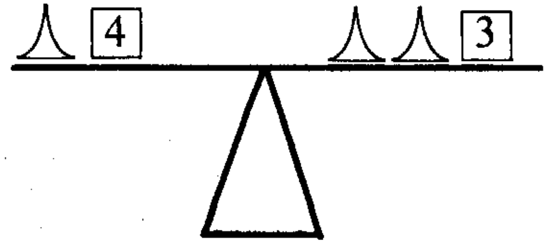

Figure 1. Pictorial representation of $x + 4 = 2x + 3$.

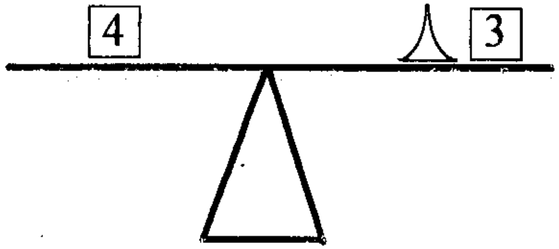

Figure 2. Pictorial representation after the first legal move was performed.

For example, students may be given the problem '$x + 4 = 2x + 3$' to solve. They begin by building the equation with pawns and cubes, placing one blue pawn and a red number cube showing 4 on the left side of their balanced-scale mat and placing 2 blue pawns and a red number cube showing 3 on the right side of their balanced-scale mat. Figure 1 illustrates a pictorial representation of how students would initially build the equation.

Students then perform the first 'legal move' of removing a pawn from both sides of the equation. The result is shown pictorially in Figure 2.

Next, students exchange the red cube numbered with a '4' for two red cubes, one numbered '3' and one numbered '1'. See Figure 3. Finally, the students remove the number 3 cube from both sides of the balanced scale and determine that the pawn is 1, recording $x = 1$.

Students next rebuild the original equation in Figure 1 and then check that a value of $x = 1$ makes sense. They check the work by writing:

$$1 + 4 = 1 + 1 + 3$$

$$5 = 5.$$

The twenty-six lessons progress from simple to challenging, adding in situations involving the distributive property, negative integers, and neg-

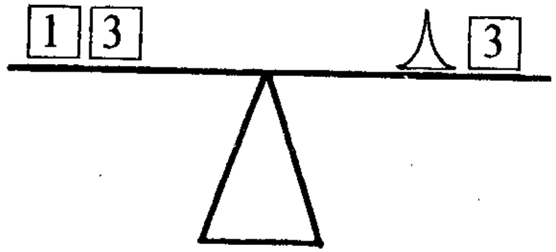

Figure 3. Pictorial representation after number cubes exchanged.

ative pawns. Each new lesson also includes practice problems from prior lessons.

3.3. *A collaborative study in the middle school mathematics classroom*

The roles of the collaborators evolved during the study. At the beginning, Marylin was viewed more as the questioner, the teacher, and the data collector. Anne's role was that of providing background literature, helping to frame data collection, and heading up data analysis. As the study matured, the roles blurred to a point where both were seeking literature on action research, both were questioning the data collection process, and both were equally involved in analyzing data.

This collaborative study had two phases, the first taking place during the school year in which Marylin first taught using the manipulatives. For the first nine weeks of the school year, Marylin taught in a non-manipulative style using the adopted textbook. Following this nine-week period, she implemented the 26-lesson manipulative program. In short, the materials in this program introduce students to a manipulative approach to solving algebraic equations, and guide them through an intermediate pictorial approach, culminating in engaging students in activities that relate the manipulative to the more formal 'high school' algebra. The reader needs to be aware that students were allowed, and encouraged, to use manipulatives during quizzes and tests given during the manipulative program. The tests and quizzes were designed in a format that paralleled the manipulative instruction.

The subjects of the study included five classes of eighth-grade students, approximately 120 students, at a lower class, inner city middle school in the midwest. Each class received its algebra lesson at the same time every day, and for this reason the classes will be referred to by their class period in the daily timetable (1^{st}, 3^{rd}, 6^{th} and 7^{th}). Data collection methods include an end-of-year survey (see Appendix), weekly student reflections, teacher observations and teacher reflections, student work samples and test scores, and a whole-class interview (conducted solely by the university partner). These interviews focused on students' confidence and interest in learning algebra when working with mathematics manipulatives versus working with a textbook.

Data about students' ability to solve algebraic equations were initially gathered through student work samples and student tests scores during both the 'manipulative phase' and the 'book' phase. Some students were also videotaped while demonstrating algebraic solutions during class time. Transcriptions of these videotapes serve as a verifying source of data on students' abilities in algebra. Additional data on students' ability to solve algebraic problems was gathered from a mandatory standardized algebra test given to all eighth-grade students in the middle school at the end of the school year.

The second phase of the study took place the next academic year, during which time the collaborative team continued its investigation of these same students who had moved on to high school algebra. The team investigated the students' high school experience to examine the 'durability' of the results of the manipulative experiences in phase one. In March of the second year of the study, surveys were mailed to approximately 90 students who could be located. Only 19 completed surveys were returned. Of those who completed the survey, eight students were willing to participate in a one-on-one interview during the subsequent summer to talk about the past two years.

3.4. *What did we learn the first year?*

We first compared overall 'class grade averages' from the textbook phase to those earned during the manipulative phase. Note that the class grade average is the average of the grades from all students in the class, including grades earned on homework, quiz, and test scores during this grading period. Table II shows these initial results. In each case, overall class averages were higher during the manipulative phase than the textbook page.

In general, individual students scores were higher during the manipulative phase than during the textbook phase. Some of the differences were quite significant. For example, 23% of the students went from below 'C' scores to scores of 70% or higher, and 42% of the students earned an average of 'A' work on their algebraic work with manipulatives whereas only 14% earned 'A's' during the book phase. On the other hand, 12.5% of students did not have higher scores during the manipulative phase. Of these students, 33% had below 'C' scores during both phases and 60% had

TABLE II

A comparison of class averages during periods of textbook instruction and manipulative instruction

Class period	Class average	
	Textbook	Manipulatives
1st	65.00%	82.38%
2nd	70.47%	81.28%
3rd	75.07%	85.29%
6th	81.40%	87.82%
7th	72.16%	82.10%

a percentage difference between manipulative and book phase scores of less than 5%.

It is difficult to conclude what these numbers tell us. Certainly for Marylin, the results are meaningful to her practice in that the percentages provided some indication that students could solve algebraic problems well with the aid of manipulatives. Thus, it was clear that many students were better able to demonstrate their abilities through the manipulatives and were able to show understanding of algebraic concepts via the manipulatives. On the other hand, these percentages also cause some concern that perhaps the students may have 'needed' the manipulatives to show what they know. Also, since work with the book came before and after the work with manipulatives, it is unclear to what extent the manipulative experience influenced later textbook performance. Thus we were compelled to break down student scores further (see Table III).

Clearly, in every case the class average during textbook instruction decreased after the manipulative instruction period. As before, individual achievement varied. For example, 77% of the students showed a decrease in individual average on textbook work after the manipulative phase. Individual results varied from class to class. All of the students in the first period class showed a decrease, while only 48% of the students in the seventh period decreased. Of all students who showed a decrease, 47% of those students' scores decreased by more than 10% and 21% earned a score of 'D' or less in both textbook periods.

Marylin was quite disturbed by these results. A primary concern was that the manipulatives had weakened the students' abilities to work algebraic problems without manipulatives. She also considered the possibility that the students had not retained the learning they had achieved during the

TABLE III

A comparison of class averages during periods of manipulative work and textbook work before and after manipulative instruction

Class period	Class average		
	Textbook before	Manipulatives	Textbook after
1st	78.36%	82.38%	57.13%
2nd	77.82%	81.28%	68.93%
3rd	76.07%	85.29%	75.40%
6th	87.24%	87.82%	77.65%
7th	74.96%	82.10%	70.33%

manipulative phase. Another possibility was that students may not have been making the connections between the concrete learning and the more abstract learning. Also, it could be that the material in the later part of the textbook was more difficult than earlier material, resulting in lower scores. And yet another likely possibility was that students did not enjoy the work from the textbook as much work with manipulatives and thus, did not put as much effort into their work. Any or all of these conclusions could be valid. However, the real dilemma for Marylin became how did the students explain the reasons for the decrease and how could she change her practice to deal with these unanticipated findings? These questions formed the foundation of questions to ask the students during phase two.

3.4.1. *Standardized test performance*

At the same time Marylin was agonizing over the surprising class averages, an additional piece of data provided a positive twist. On the standardized algebra test given to all eighth-grade students at the middle school, Marylin's students performed satisfactorily, far exceeding the expectations of the administration and colleagues. Approximately 80% of Marylin's students correctly responded to approximately 60% of the test questions. Because test questions were worded and had to be solved in a traditional algebraic fashion, Marylin believed she had successfully helped students bridge the gap between concrete and the more abstract algebra, even though the comparative textbook data left some measure of doubt as to the degree to which students fully made the connections.

3.4.2. *Phase One Survey Responses*

On the first-year survey, students were asked a variety of questions regarding their interest in learning algebra. Only 68 of the 120 students returned completed surveys. One of the questions that these students were asked was, 'How did you feel when you learned that all eighth-grade students would have to take algebra?' Students provided answers such as:

I thought it was going to be really hard ... I didn't want to take algebra. I'd rather take basic math .... I felt a little scared .... I felt that doing algebra in the eighth grade would be fun ... I felt like 'oh no' I'm going to fail this class ... .

However, when asked how they felt about algebra after finishing the manipulative lessons, they expressed:

I felt that it was a neat experience and that it wasn't so hard after all .... Relieved, algebra was a breeze ... I really liked it because it was the most fun .... I kind of liked it because it got easier as the lessons went on .... I felt that I had learned more by the manipulative .... Very comfortable about algebra .... Good, the manipulatives were fun and very helpful ... I proved I could when I thought I couldn't.

When asked which approach to learning algebra they liked better, using the textbook or working with manipulatives, 91% of the students preferred the manipulatives. Explanations they provided included:

There was no homework ... using your hands ... it was easier and funner ... I always scored high ... they were easier to understand ... it wasn't boring ... it was new ... we didn't read much ... everyone got involved ... you could actually see what you were doing ... .

Those who preferred the textbook explained:

manipulatives were too messy ... with the textbook you didn't have to worry about putting things up ... it was more organized

Fifty-seven percent of the students expressed that they learned more algebra when working with the manipulatives. They suggest:

They made me want to learn more ... we could learn faster ... because you see how you get the answer ... when you actually touch the problem it's easier ... .

Those who said they learned more when using the textbook reasoned:

The teacher explained more to us ... the book will always have more learning ... it was more in depth ... book explained more ... doesn't take as long ... more detailed ... the textbook you could read the pages to understand ... .

3.5. *The second year*

Most of the students from Marylin's eighth-grade algebra class went on to the neighboring high school. Relationships between the middle school and high school were not good. The high school teachers did not support 'Hands-On' learning and often referred to this type of teaching at the middle school level as 'playing'. Thus, the atmosphere Marylin's students entered in their freshmen mathematics classes was very different from that in Marylin's classroom. The existence of this disparity between the middle and high schools was of major concern to Marylin, and was actually one of the reasons she initially questioned whether her efforts with manipulatives would be worthwhile in the long run.

TABLE IV

Comparison of high school counselor recommendations for mathematics placement and actual course placement in the freshmen year

Recommendation	Actual	Number of students
Pre-Algebra	Pre-algebra	7
Algebra I	Algebra I	3
Algebra I	Pre-algebra	4
Algebra I	Basic Skills	2
Algebra II	Pre-algebra	1
Algebra II	Algebra I	2

As a bit of background regarding the students' freshmen experience, Table IV shows the mathematics placement of the 19 students who completed the follow-up survey. Most of the students ultimately began their high school mathematics experience below the Algebra I level and none of the students in the survey ended up going directly into Algebra II. With the limited amount of data it is difficult to ascertain how wide-spread these events were. However, from this small population it is clear that the counselors did not initially believe that most of these students could be ready for Algebra II, and it is also clear that most of these students were unable to succeed in their initial placements.

When asked how they were doing in mathematics in their freshmen year, students provided these remarks:

Now that I'm in pre-algebra, I'm doing much better. I like it because I learned most of the stuff last year ... I'm getting an A. I kind of like it but it's not challenging enough ... OK, B's and C's I like it because it's mostly what we did last year ... Not good. They don't take time out to teach really. They just give directions and tell you to do it .... My teacher is weird ... Horrible. He doesn't explain it very well and he goes fast ... .

When asked if there was anything they learned from their eighth-grade work with manipulatives that has helped them in their current math class, they responded:

Yes, the adding and subtracting of negative numbers because we do some of that stuff now ... yes, it helped me understand algebra more ... How to solve proportions ... Most of it made me understand it more ... Yes, they helped a whole lot ... yes the equation work like adding and subtracting from both sides ... .

In the second-year interviews, students were asked to recall specific instances in which the use of manipulatives helped them in high school. Angie said the following:

When we would do equations in algebra in high school, I would sit there and think about getting the pawn (the variable) by itself on one side. I would picture the balance scale in my mind. I would always remember that if I had a '3' I would need to put the opposite color cube on the scale to get zero (additive inverse) ... We had fun!

Melanie could not provide specific examples of how the use of manipulatives in eighth grade helped her in high school. She said, 'I just know that it did'. She added, 'building the equation was what was important in understanding'.

With this type of inconclusive feedback, Marylin was not convinced that there had been a lasting effect, or any transfers or connections made between the eighth- and ninth-grade experiences. However, she is always encouraged by the 'one' student for whom a difference was made. One student that stood out in that second year follow-up was Matt. Upon Matt's visit back to the middle school, he was able to clearly recall his work with manipulatives and to draw a pictorial representation of an algebraic solution to '$2x + x + 4 = x + 6$' following the 'Hands-On' approach. (See Figure 4.)

Matt did the same with '$2(x + 3) = x + 8$'. (See left-hand side of the equation in Figure 5.) He said that he was earning an A in high school pre-algebra. He reported that they had just finished studying the distributive property and exclaimed, 'It's easy!'.

Marylin's reflections on her visit with Matt include the following entry that recounts a response from Matt that really impressed Marylin. She asked Matt if the manipulatives had helped him solve problems. He replied that 'learning algebra was like learning to ride a bicycle. You could learn by a picture or written instructions, but you would learn best by getting on the bicycle, pedaling it, experimenting with what works or what doesn't. Its the same with math'.

When Marylin asked each of the 19 students whether they wished they had experienced a more traditional eighth-grade algebra class, 12 of the 19 respondents said, 'No'. One of the seven students who answered 'yes' to this question explained, 'yes, because the math taught in other junior high schools when they are taking algebra I in 8th grade is the same as what I

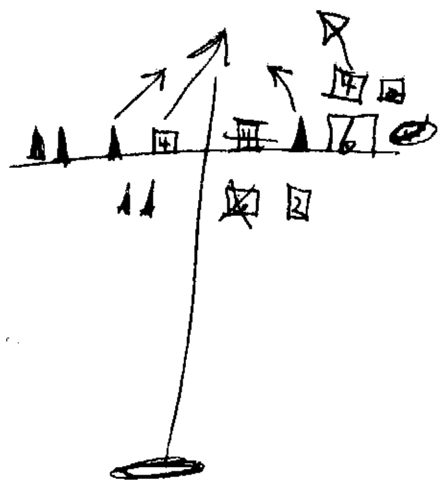

Figure 4. Matt's pictorial representation of $2x + x + 4 = x + 6$.

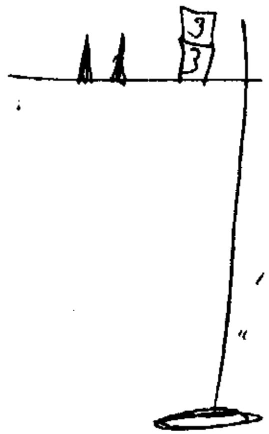

Figure 5. Matt's pictorial representation of $2(x + 3) = x + 8$.

am taking now in 9th', implying that she would have had an easier time in high school if she was repeating learning from eighth grade.

Overall, the findings from the follow-up year were not incredibly positive or enlightening in terms of answering the question of the long-term effects of learning algebra via manipulatives. However, the experience of being able to ask students questions about their learning provided Marylin with much to reflect on regarding her beliefs about learning and her style of teaching. Hence, although the research findings may leave many questions about the initial foci of the action research project, the 'results' in terms of the professional development experience for Marylin were fruitful.

4. Discussion of Issues Related to the Broader Study within the Context of the Sub-Study

4.1. *Marylin's reflections*

Marylin and Anne engaged in countless conversations about teaching and learning mathematics over the two-year period. Each time a new piece of data emerged, they talked about what the data meant. To Marylin, it felt like a roller-coaster ride with continuous ups and downs. At times the data would indicate that things were not going well, and, yet, the next day a student would have an individual 'shining moment' or learning breakthrough, and Marylin would again be convinced that this approach to learning algebra was just what her students needed.

Having weighed all the evidence at the end of the two year period, when asked whether or not she believed her teaching efforts had been worthwhile, Marylin responded 'yes' and 'no'. Yes, because she knows in her heart and from her personal observations of students that her students were learning algebra and were gaining confidence in their mathematical abilities. However, the answer is no when she considers the realities of where her students are headed after they leave her classroom. She admits being frustrated at having her students go from feeling confident about their mathematics to feeling less than adequate in the eyes of their secondary teachers. She reflects that if she is merely 'setting her students up' for a future 'let down' then perhaps she is doing them a disservice by teaching in a hands-on, nurturing fashion.

Although Marylin accepts that there is no way to control what happens in 'other teachers' classroom', she laments that when some teachers work to teach in the spirit of the 'NCTM Standards' (National Council of Teachers of Mathematics, 1989) and others do not, we send conflicting messages to students about what it means to learn and to do mathematics. Thus, we continue the cycle of producing students who are unsure of and uncomfortable with mathematics.

Marylin sums up her reflections on the experience in the following entry:

We [mathematics teachers] are all aware that nationwide pressure to improve students' mathematics education is stronger than ever. Often students are 'lost' to mathematics when they are exposed to the more difficult abstract concepts. Thus, many teachers opt not to teach pre-algebra concepts and are content with general mathematics coursework. Because of this experience with Anne, I am more aware of differing ways in which students process information on their path to understanding math. I now believe that mathematics, particularly algebra, is the gateway for students' futures regarding opportunities for higher education and for broader career choices. Dr Borenson's 'Hands-On Equations' provide an improved opportunity for ALL students to learn, understand and apply math meaningfully in their lives. I want ALL my students to discover math for themselves, and 'Hands-On Equations' does this. It guides them to a higher and more sophisticated levels of mathematical thinking and understanding.

Because of this collaborative experience, and her increased exposure to the *NCTM Standards for School Mathematics*, Marylin created a thoughtful, concrete set of learning goals for her mathematics classes. She expressed these in her reflective writing:

1. I want my students to value mathematics. Students' experiences in school must bring them to believe that mathematics has value for them so they will have the incentive to continue studying math as long as they are in school. Too often, students view traditional math as irrelevant, dull, and routine. This is partly due to students' limited instruction which has place the major emphasis on strictly computation. I want to raise ALL my students' expectations and give them a chance to experience math in a way that allows them to understand and appreciate a higher level of math, that being algebra.

2. I want my students to reason mathematically. The learning of mathematics must engage the students both physically and intellectually. My students must be active learners to discover the mathematical relationships needed to solve equations in a formal algebra class.

3. I want my students to communicate mathematics. To do this, they must work in small groups. Small group cooperative learning gives students an opportunity to talk about ideas and to listen to their peers. In the case of my class, students learn from each other, drawing on their individual strengths. There are no better ways to learn mathematics than by working in groups, teaching each other, and sometimes even arguing about strategies or solutions to a problem.

4. I want my students to gain strength in problem solve and to do this I need to show them a wide variety of mathematical methods to solve problems. With Dr Borenson's system, I show them concrete, a pictorial system, and a written notational system.

5. I want my students to develop confidence in themselves. The ability to succeed in pre-algebra in 7th grade and algebra in 8th grade will depend on their attitudes towards mathematics from their 6th grade [and so forth] experiences. In the math class setting, I want to create an environment that would foster a life-long learning process.

4.2. *Resulting decisions*

It's unclear whether or not this collaborative action research project has a happy ending. Marylin confirmed her own belief that students learn best when they can 'touch' and 'manipulate' mathematics. She's convinced that active involvement and constant reflection on the part of students is vital in coming to understand mathematics. However, Marylin became increasingly uncomfortable with the secondary teachers' perceptions of her type of teaching and their treatment of students in reaction to it. Thus, Marylin decided that a comfortable resolution to her dilemma would be to switch her primary teaching responsibilities at the middle school from eighth grade to sixth grade.

As a sixth-grade teacher, Marylin contends that she can teach in the style she believes best serves students without feeling like she is setting students up for immediate disappointment at the high school level. She hopes to instill in students a love for learning mathematics. She spends a number of weeks in the school year helping students develop understanding of algebraic concepts using her 'Hands-On' materials. Thus, she believes she may be able to lay an algebraic foundation for students early in their middle school experience which they can draw upon as they engage in more traditional algebra experiences in eighth grade and beyond. Marylin is extremely happy with this decision and thoroughly enjoys working with her sixth-grade students.

4.3. *Marylin's professional growth*

This collaborative project not only prompted Marylin to confirm her beliefs about mathematics teaching and to make a major career decision, but it also afforded her a number of other professional growth opportunities. Marylin participated in the writing of two research grant proposals which were both funded through Anne's university's professional development foundation. In addition, Marylin has made presentations and co-presentations at a number of local and national conferences regarding collaborative action research and findings from her own study. Further, Marylin has immensely improved her writing abilities (an area in which she felt particularly weak) in co-authoring three papers about her action research experiences.

Another way in which Marylin has expanded her professional experiences has been through presentations made to pre- and in-service teachers as an invited speaker in undergraduate and graduate classes. In her reflections on these experiences, Marylin states that when she demonstrates the 'Hands-On Equations' approach to teaching algebraic concepts, she hears the reaction, time and again, from teachers and preservice teachers, 'I wish I would have had this when I was studying algebra. Then, maybe, it would have all made sense'.

Marylin also took part in an in-service professional development program in her school corporation, sharing her experiences with her peers. As a result of this involvement, Marylin was asked to be a teacher-in-residence at the local university where she taught a section of an introductory teacher education class which emphasizes middle schools. She later taught an upper-division class entitled 'Teaching and Learning in the Middle School'. She believes that her collaborative experience with Anne gave her the confidence to share with preservice teachers her knowledge teaching mathematics in the middle school, particularly using the example of working with 'Hands-On Equations'.

Marylin reports that her participation in the collaborative action research project opened her eyes to the many ways in which she can get involved in the education community. She is thankful for her own personal and professional growth and, pleased that she was able to play a minor role in the professional development of other colleagues.

4.4. *Issues of collaborative action research*

Action research tends to empower individuals in their battles to implement educational change (Carson, 1990). In the case of Marylin described herein, it is clear that Marylin was empowered by the fact that she was acting responsibly by investigating and documenting her change efforts. Through her documentation she gathered data in search of answers to her questions, 'How is a Hands-On approach to teaching algebra affecting my students' interest and confidence in algebra', 'How are their abilities to solve algebraic problems affected', and 'Will my efforts have a lasting effect?'

Although the ensuing action research project did not conclusively answer all of these questions, the experience confirmed Marylin's belief that a hands-on, active approach to mathematics is the best means of teaching mathematics. Reflections on the research also prompted Marylin to make a career-altering decision, placing herself in a position in which she would feel more comfortable teaching in the manner which she wholly supports. In a sense, action research provided Marylin with insights that empowered her to effect change beyond her original expectations.

Marylin's case furnishes evidence of the potential of action research to impact the development of mathematics teachers. However, her experience also exemplifies many challenges for action research. In addition, Marylin's case illustrates the complexity of issues related to collaborative action research including 'action research as a means of bridging the gap between theory and practice' and 'the distinction between reflective teaching and action research'.

4.4.1. *Challenges for action research in mathematics*

Noffke (1994) suggests there are four contemporary challenges for action research. First, to many, the only goal of action research has been to further personal and/or professional development. Thus, action research is often valued 'less for its role in the production of knowledge about curriculum, pedagogy, and the social contexts of schools, and more for its ability to help teachers grow in their self-awareness or in terms of their professional skills and dispositions' (p. 15). Marylin certainly developed both personally and professionally, but I believe her action research endeavor also provides insights into the disparity between the middle and high school mathematics programs and the differences in pedagogical practices that often exist. Marylin's research efforts, in being reported and shared through various media, add to the knowledge base on the social contexts of schools, particularly in the power-struggle over the 'right way' to teach algebra.

The second challenge, is the question of whether action research is 'real research' or whether it is a 'new form of research whose methods, methodology, and epistemology are now being clarified' (Noffke, 1994, p. 16). Rather than create a false dichotomy, perhaps it can be viewed as both, especially when you consider the case of Marylin.

Marylin's study can be viewed as legitimate research in that it followed the basic constructs of research: posing questions, developing a method through which one could investigate those questions, and analyzing, interpreting, and reporting research results. On the other hand, collaborative action research houses the additional benefits of having an immediate, recognizable impact on the mathematics classroom and contributing to professional development in a manner that is not generally afforded in traditional research efforts. The impact on Marylin's practice was evident, particularly in its influence on her decision to change grade levels.

The third challenge centers around the issue of 'the politics of knowledge production, the establishment of relationships and the breaking down of barriers of isolation within teaching and across schools and universities that action research invites' (Noffke, 1994, p. 16). Marylin felt as if she were in a state of isolation, unsure of her mathematics teaching and not believing she had colleagues with whom she could discuss her dilemma. By creating a partnership with Anne and posing research questions, Marylin made large strides in breaking down traditional roles held in research and pulled herself out of her perceived situation of isolation.

Other politics are involved in collaborative action research. Researchers and teachers come from different epistemological viewpoints. The mathematics teacher will engage in research to improve her teaching. The researcher, on the other hand, engages in research to create theory. The researcher, therefore, is typically more concerned with the research method than with the application of the outcomes. Conversely, the teacher is more concerned with designing a strategy which solves the day-to-day problems she faces.

In Marylin's case, she was very up front in expressing her belief that Anne was the researcher and she was the mathematics teacher. She constantly asked Anne questions like, 'How should we gather data about that?' Should I be writing this down in my reflective journal? 'Now, how can we analyze this?' Oddly enough, Anne noticed that Marylin offered as many suggestions on how to gather and analyze data as she did. However, each time Anne countered her comment that 'Anne is the researcher' by saying, 'We're both the researchers', she insisted, 'No, no, I'm just the teacher'.

From Anne's perspective, Marylin did most of the work in the collaboration. Although the two talked about what to collect, Marylin did the collecting. In addition, she did a considerable amount of the reflection upon and analysis of the data. In a strong sense Anne has felt more like a coach in the collaborative action research project, viewing Marylin more as the 'star quarterback'.

Finally, Noffke (1994) suggests that action research plays a role in social transformation in its concern for systemic change. I believe this is strongly tied to the third challenge regarding the issue of role changing within the research process. In addition to that, however, Noffke is suggesting that action research has the potential to impact wide-spread systemic change beyond change within a single classroom. She implies that the current challenge is to encourage educators to view action research as a vehicle to stimulate school-wide, corporation-wide, and nation-wide change. For Marylin, the goals of action research were modest. However, as her story is reported it holds the potential to having a larger impact on teachers and teaching beyond her own mathematics classroom.

4.4.2. *Bridging Gaps*

Noffke's (1994) third challenge hints at the potential for action research to play a role in bridging the gaps that traditionally exist between theory and practice, and between theoretician and practitioner. The notion of valuing 'teacher as researcher in the classroom' is growing among educational researchers (Cardelle-Elawar, 1993; Rafferty, 1995), partly because of its ability to not only provide access to classrooms for research but to also foster links between research and practice. Practicing teachers are drawn to action research because it allows them to view research as integrated with practice rather than as a process which is conducted separate from real classroom issues (Ross, 1984). Collaborative efforts leave a different impression on classroom teachers than do solely university-researcher investigations, primarily because there is the sense of ownership in the investigation and a feeling on the part of the teachers that they have been 'worked with' as opposed to 'worked on' (Lieberman, 1986).

The case of Marylin exemplifies the promise for action research in building connections between mathematics research and practice. Stemming from the practitioner's questions of concern, Marylin's study was connected to practice from its origins. Further, the resulting research had an impact on decisions made by the practitioner. Consequently, Marylin came to value the power of research and now finds herself reading research reports prepared by others through a new lens.

By broadening the definition of what constitutes research, action research invites a larger audience to take an interest in the theoretical discussions that result from mathematics educational research. Thus, the gap is bridged not only in moving researchers to investigate issues of immediate, practical interest to teachers, but also through exposing teachers to the power of research to inform researchers and practitioners alike.

4.4.3. *Is collaborative action research merely reflective teaching?*

It is fitting to close this discourse by returning to an earlier discussion of what characterizes action research. Some might raise the question, 'Is action research merely engagement in reflective teaching?' After all, reflective teaching involves critically examining one's practice and making instructional decisions based on those reflections. In action research, teachers are certainly engaged in reflective teaching. However, we contend that reflective teaching is a necessary but not sufficient component of action research.

Action research enables the mathematics teacher both to formulate and act upon her own concerns, thereby personally and professionally developing herself within and through her practice (Tripp, 1990). In this sense, action research fosters reflective practice. However, beyond developing a professional culture that engages in and values reflection, action research leads teachers to value research (Clift et al., 1988). Marylin, who Anne knows to have been a reflective practitioner for years by constantly questioning her practice and trying new strategies, found a whole new perspective on what it means to examine mathematics practice through action research. She employed a systematic means of documenting her efforts through on-going data collection and analysis. In addition, through action research she entered into the larger conversation regarding mathematics teaching practice by disseminating her findings to the mathematics education community. In short, action research goes beyond reflective practice

because of its dynamic and in-depth nature as well as its inextricable ties to the theory.

4.4.4. *Action research in mathematics teacher education*

Given that teacher educators hope that all teachers develop into reflective practitioners, and that action research can cultivate such development, the question that surfaces is, 'Should all teachers engage in action research?' Marylin and Anne believe that exposure to action research should be inherent in teacher education programs. Pre- and in-service teachers ought to be educated to the nature of action research and exposed to action research stories. Consequently, when invited to speak to undergraduate and graduate students, Marylin has been an advocate of and a model for action research.

However, it would be unrealistic to expect that all teachers would have an interest in taking on the rigors of action research. Nor is it reasonable to anticipate that all university researchers should be willing to or made to engage in collaborative action research projects. Although action research adds an interesting dimension to the larger body of research, providing a different lens through which educators can view classrooms and schools, we must generate knowledge through many types of research. Thus, as all researchers should not fit the same mold, neither should teachers. In this spirit, Marylin and Anne do not believe that all teachers will benefit from action research.

We conjecture that teachers who engage in action research are generally teachers who are at a critical juncture in their teaching practice and are in a state of mind where they are open to change. That state of mind might be that one's philosophy of teaching may be in a state of disequilibration, or perhaps the teacher is faced with a dilemma, as in Marylin's case, in which she is questioning her efforts to change. Not every mathematics teacher reaches this point of questioning or change. Thus, action research may not become a viable option for all teachers. However, for Marylin, and other mathematics teachers who are ready for change, action research is truly a vehicle for teacher development.

NOTES

1 'Hands-On Equations: Making Algebra Child's Play' was developed by Dr Henry Borenson, Ed. D. and can be obtained through: Borenson and Associates, P.O. Box 3328; Allentown, PA 18106 U.S.A.; (610)398-6908.

APPENDIX

EXAMPLES OF SURVEY QUESTIONS – PHASE ONE

How did you feel last August when you learned that all eighth graders would take algebra?

How do you now feel about algebra at the end of the school year?
Rate your knowledge of algebra? (Check one) Low – Medium – High – Explain your rating:
Rate your confidence in doing algebra? (Check one) Low – Medium – High – Explain your rating:
Using Dr Borenson's 'Hands-On' Equations method, solve the following equation with the 'pictorial' method and explain how you did it: $2x + x - x + 1 = x + 9$.
Which did you like better?: (Check one) Using the textbook – Working with manipulatives – Explain:
When did you learn more algebra?: (Check one) With the textbook – With manipulatives – Explain:
What would you change about the eighth-grade algebra program?

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