Dynamic Deployment of Small Cells in TV White Spaces

Pablo Ameigeiras, David M. Gutierrez-Estevez, Jorge Navarro-Ortiz

***Abstract*—The operation of small cells in TV white spaces (TVWSs) represents a coexistence challenge due to their unplanned deployment, their heterogeneous transmission technologies, and the scarcity of TVWS channels in crowded cities. Whenever a new small cell is switched on, a spectrum reassignment of already deployed small cells can be used to avoid high interference and enable coexistence. However, as users may turn on and off their small cells at times, these reassignments may lead to frequent reconfigurations of already deployed small cells. For this reason, a solution named Small Cell Dynamic Deployment (SCDD) is designed to reassign TVWS channels only to the small cells in the neighborhood of the new one. A channel allocation is proposed for SCDD formulated as an exact potential game. Its exact potential function is the sum of the average capacity of the small cells considered in the game. Results show that SCDD requires a few channel reconfigurations of already deployed small cells because the channel assignment outside the neighborhood of the new cell remains unchanged. However, SCDD provides a performance similar to the case in which the allocation may modify the channels of all small cells already deployed.**

***Index Terms*—TV White Space, small cell, channel allocation, potential games, game couplings.**

I. INTRODUCTION

ACCORDING to the latest global mobile data traffic reports [1][2], mobile data traffic is experiencing an unprecedented growth, and it is expected to keep growing at a high rate in the coming years. This explosive growth has accentuated the spectrum shortage for commercial mobile radio services [3].

One way that can contribute to deal with the spectrum scarcity problem is the usage of TV white spaces (TVWSs). This allows unlicensed operation in a TV channel that is not being used by any licensed service at a particular location and at a particular time. In September 2010, the Federal Communications Commission (FCC) released revised rules regarding access to the TVWSs in the USA [4] including the regulation of a TV band database that provides information on the available channels at a given location.

Another partial solution to deal with the traffic growth and the spectrum scarcity is traffic offloading through small cells and Wi-Fi access points (APs) [5]. Small cells are low-power operator-managed wireless access points that operate in licensed spectrum [6].

In this paper, we concentrate on the operation of small cells in the TVWSs as a mechanism to offload traffic from mobile networks. One of the main challenges for networks operating in TVWSs is their coexistence [7]. Very different network technologies such as IEEE 802.11af [8], ECMA 392 [9], IEEE 802.22 [10] and 3G or 4G mobile technologies could potentially benefit from operating in the TV band spectrum. However, these technologies present very heterogeneous characteristics including network architecture, device category, transmission power, operational bandwidth, modulation and coding schemes, and especially medium access techniques (e.g. TDMA, CSMA/CA, and OFDMA). Hence, the simultaneous operation of these systems in the same channels is challenging. In this respect, the IEEE 802 LAN/MAN standards committee approved the P802.19.1 standardization project to develop a standard for coexistence among dissimilar or independently operated wireless devices and networks [11]. IEEE 802.19.1 system proposes an architecture for either discovery service or management service. With the first service, the system detects neighbor networks which may cause harmful interference. With the second, the system decides operational parameters for the coexisting networks [12].

Several researchers have investigated coexistence in TVWSs. In [13], Filin et.al. evaluated the performance of the discovery service of the IEEE 802.19.1 coexistence system at the early stages of the standard development. Similarly, in [14] Wang et.al. designed a registration signaling procedure for the IEEE 802.19.1 system, and evaluated its impact on the channel selection decisions taken by TVWS networks for coexistence. Their results already point out the potential benefits of the IEEE 802.19.1 system's discovery service. In [15], Jankuloska et.al. proposed a joint power and channel allocation algorithm for Wi-Fi-like systems operating in TVWSs. Their solution is based on a Nash Bargaining, and it aims at maximizing the number of supported users. In [16], Ye et.al. expressed the coexistence in TVWSs as a total interference minimization problem. They used Simulated Annealing (SA) to deal with the optimization problem and provided an algorithm to

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automatically choose the parameters required for SA.

We refer the reader to [17]-[19] and references therein for a comprehensive overview of spectrum allocation algorithms in general cognitive radio networks. In [20], Peng et.al. proposed to reduce the spectrum allocation problem to a variant of the graph coloring problem. However, solutions based on graph coloring are optimized for a fixed topology, and need to be refreshed up on topology changes. In [21], Cao et.al. proposed a distributed approach to spectrum allocation for networks with frequent topology changes, such as mobile ad-hoc networks. The same authors further delved into distributed spectrum management architectures in [22],[23]. Although the solutions in [21]-[23] are interesting, they assume a simplistic interference model in which interference is merely modeled with a binary geometry metric.

In this paper we aim at designing a coexistence solution for a network of unplanned small cells in TVWSs by means of channel allocation. As users may turn on and off their small cells at times [24], channel reassignments may lead to frequent reconfigurations of already deployed small cells. These reconfigurations would ultimately degrade the quality experienced by the end users. For this reason, we propose a Small Cell Dynamic Deployment (SCDD) solution, which reassigns TVWS channels to only the small cells in the neighborhood of a recently switched on small cell. The channels of the small cells outside the neighborhood are unaffected. The goal is to reduce the number of already deployed small cells that require a channel reassignment each time a new small cell is deployed.

In combination with the limited neighborhood concept for dynamic deployment, SCDD includes a novel channel allocation algorithm that incorporates an interference model much more accurate than the one in [21]-[23]. We formulate the coexistence problem as a maximization of the average network capacity. To solve it, we propose a non-cooperative game in which the players are the small cells. The utility function associated to each small cell includes its average capacity and the corresponding degradation caused to the rest of the small cells due to interference. We will demonstrate that this game is an exact potential game, and its exact potential function is equal to the average network capacity. Therefore, in this game the small cells are jointly trying to maximize the average network capacity. To be able to incorporate the average cell capacity in the utility function of our game, we base our solution on the IEEE 802.19.1 architecture. Our proposed SCDD solution could be incorporated in a practical implementation platform as the one presented in [25].

The main contributions of this paper are the following:

- The proposal of a novel exact potential game to enable coexistence through channel allocation for small cells in TVWSs. The exact potential function of the game is the average network capacity.
- The proposal of applying channel allocation in a small neighborhood around a new small cell instead of considering all small cells already deployed. In addition, the demonstration that the exact potential property of the proposed game is preserved when the channel allocation of the small cells outside the neighborhood is fixed.

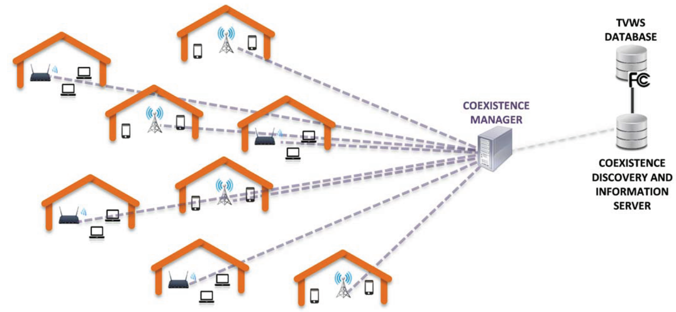

Fig. 1. IEEE 802.19.1 system architecture.

- We show that if the limited neighborhood includes a sufficient number of small cells, the proposed channel allocation game provides a performance similar to the case in which the game is applied to all the small cells.
- The proposal of an adaptive algorithm that dynamically estimates the required number of small cells and reassigns TVWS channels to them. This algorithm achieves a very low number of TVWS channel reallocations in the already deployed small cells whereas the performance degradation is almost negligible compared to spectrum reassignment applied to the whole network.

Although our solution has been specifically designed for TVWSs, if similar coexistence information is available for other systems (e.g. with dedicated spectrum), our solution would also be applicable in such systems.

The rest of the paper is organized as follows. The system model is presented in section II. The general problem formulation and the exact potential game for channel allocation are explained in section III. The proposed channel allocation using a limited neighborhood is presented in section IV. The SCDD solution and its dynamic estimation of the neighborhood size is described in section V. The performance results are presented in section VI. Finally, the main conclusions are drawn in section VII.

II. SYSTEM MODEL

LET us consider a network of small cells that operate in TV white spaces. These small cells provide wireless access to users located within their coverage area. Hereafter, we will refer to these small cells simply as nodes. Let us denote the overall set of nodes in the network as $L = \{1, ..., l\}$. These nodes are controlled by a Coexistence Manager CM[1] [12], which provides operational parameters to its network nodes to enable coexistence (see Fig. 1). For the sake of readability, we have included a table with all the variables used in this paper in Appendix B.

The network nodes are assumed to operate as portable mode II devices, so that they can signal their geolocation. However, the user's terminals may operate as portable mode I devices,

[1]In order to improve scalability, several CMs can be used to control a large deployment of small cells. We assume that the involved CMs would cooperate as if they were only one CM by using the *centralized decision making* [26]. It shall also be noticed that the required signaling for registration and reconfiguration commands is low [26].

which may not have geolocation capability. The Coexistence Manager has information about the location of each node $i \in L$, but it has no information about the locations of the user's terminals. We assume that user's terminals are uniformly distributed within the nodes coverage area. We also assume that both the nodes and the terminal devices transmit at a fixed power on each channel. Note that the maximum transmission power is 40mW in adjacent TV channels and 100mW in non-adjacent ones.

The Coexistence Manager CM has information about the set of TVWS channels $K = \{1, ..., k\}$ available for operation based on signaling from a TVWS database. We represent the set of channels used by node $i \in L$ with a vector $\bar{s_i} = \{c_1, ..., c_j, ..., c_n\}^T$ where $\forall j \; c_j \in K$. We also assume that each node has a number of available radios that limits the maximum number of channels used by the node, i.e. $\forall i, |\bar{s_i}| \leq r_i$, where r_i is the number of node i's available radios. We define function $\delta(\bar{s_i}, c)$ for node i and channel c as:

$$\delta(\bar{s_i}, c) = \begin{cases} 0, & \text{if } c \in \bar{s_i} \\ 1, & \text{if } c \notin \bar{s_i} \end{cases} \tag{1}$$

Additionally, let S_i be the set of all possible channel combinations available to node i, and S the channel combination space, that is the Cartesian product of each node's set of possible channel combinations, i.e. $S = S_1 \times ... \times S_i \times ... \times S_l$.

Let us assume that each node $i \in L$ has a coverage region R_i, and a user served by node i on channel c and located at a position $(x, y) \in R_i$ has a signal to interference ratio γ_i^c:

$$\gamma_i^c(x, y, \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) = \frac{g_i(x, y) \cdot P_{ic}}{I_c(x, y) + \sum_{\forall j \in L, j \neq i} g_j(x, y) \cdot P_{jc} \cdot \delta(\bar{s_j}, c)} \tag{2}$$

where I_c denotes the primary to secondary interference on channel c plus thermal noise, P_{ic} the transmission power of node i on channel c, and $g_i(x, y)$ the path gain from node i to position (x, y).

To simplify our model, we disregard the effect of fast fading in the signal to interference ratio. Nevertheless, γ_i^c is not constant in our system because the path gain from each node g_i vary with the user location due to the effects of deterministic path loss and shadow fading. Likewise, the primary to secondary interference also vary with the user location. Futhermore, γ_i^c also depends on the set of channels $\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}$ used by the nodes in L.

The maximum data rate of a user served by node i on a channel c can be calculated by the Shannon capacity as $V_i^c = W_u \; log_2(1 + \gamma_i^c(x, y, \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}))$, where W_u is the fraction of the channel bandwidth assigned by the node to the user. As γ_i^c is not constant, V_i^c is not constant too. Then, we compute its average $E[V_i^c] = \int_0^\infty W_u \; log_2(1 + \gamma_i^c) \; p_{\gamma_i^c}(\gamma_i^c / \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) d\gamma_i^c$ over positions $(x, y) \in R_i$ and assuming a fixed channel allocation $\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}$. $p_{\gamma_i^c}(\gamma_i^c / \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l})$ denotes the conditional probability density function of the signal to interference ratio of a user served by node i on channel c, located in positions $(x, y) \in R_i$, and conditioned to channel allocation $\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}$. We define the average capacity of node i over region R_i on channel c as the sum of the average maximum data rates of all users served by this node on this channel [27]. Considering a TDMA, FDMA, or TDMA-FDMA system in which all users are allocated the same bandwidth, $E[V_i^c]$ becomes the same for all users served by node i. In that case, the average capacity of node i over region R_i on channel c can be expressed as:

$$\begin{aligned} f(i, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) &= \sum_{\forall u \in i} E[V_i^c] = \\ \sum_{\forall u \in i} \int_0^\infty W_u \; log_2(1 + \gamma_i^c) \; p_{\gamma_i^c}(\gamma_i^c / \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) d\gamma_i^c &= \\ \int_0^\infty W \; log_2(1 + \gamma_i^c) \; p_{\gamma_i^c}(\gamma_i^c / \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) d\gamma_i^c & \end{aligned} \tag{3}$$

where $u \in i$ denotes that the user is served by node i, and W is the bandwidth of channel c.

We assume that CM has information about the transmission power and the location of a each node $\in L$. Hence, it can evaluate the signal level received from each node at a given position (e.g. based on Radio Environment Maps). We assume that CM can use such information to estimate the average capacity $f(i, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l})$ by integrating the Shannon capacity $W \; log_2(1 + \gamma_i^c(x, y, \bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}))$ over the coverage region R_i.

III. Problem Formulation: Enabling Coexistence by Channel Allocation

In this section we make a proposal to provide coexistence between the nodes $\in L$ through the execution of a novel channel allocation algorithm. We mathematically formulate the coexistence problem as a channel allocation problem in which the objective is the maximization of the average network capacity. We compute the average network capacity as the sum of the average node capacity of all nodes $\in L$:

$$\Omega_L = \sum_{\forall j \in L} \sum_{\forall c \in K} f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \tag{4}$$

Then, the coexistence problem is formulated as follows:

$$\begin{aligned} &\max_{\bar{s_i}, \forall i \in L} \Omega_L \\ &\text{subject to } |\bar{s_i}| \leq r_i, \forall i \in L \end{aligned} \tag{5}$$

This optimization problem is equivalent to a 0-1 integer program in which the variables are equal to $\delta(\bar{s_i}, c)$. Therefore, it belongs to the class of NP-hard problems [28]. All the exact algorithms for its solution have an exponential complexity. For that reason, next we propose a game theoretical approach to deal with this coexistence problem. We consider the following non-cooperative game $\Gamma_L = \langle L, \{S_i\} \forall i \in L, \{U_i\} \forall i \in L \rangle$, where each node i is a player with a strategy space S_i and a utility function U_i. Let us define the utility function as:

$$
\begin{aligned}
U_i =& \sum_{\forall c \in K} \delta(\bar{s_i}, c) \cdot \Big[f(i, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \\
& - \sum_{\forall j \in L, j \neq i} \Big(f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \\
& - f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \Big) \cdot \delta(\bar{s_j}, c) \Big] \\
=& \sum_{\forall c \in K} \delta(\bar{s_i}, c) \cdot \Big[\sum_{\forall j \in L} f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \\
& - \sum_{\forall j \in L, j \neq i} f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big]
\end{aligned} \tag{6}
$$

The objective of utility function U_i is to evaluate the potential average network capacity benefit if node i transmits on channel c. For this, we include in U_i the average capacity achieved by node i if transmits on channel c. Additionally, we subtract in U_i the degradation of the average capacity of the rest of nodes transmitting on channel c if node i uses this channel. This measure is computed over all channels on which node i transmits. Then, we can have the following theorem:

Theorem 1: The non-cooperative game $\Gamma_L = \langle L, \{S_i\} \forall i \in L, \{U_i\} \forall i \in L \rangle$ is an exact potential game, and its exact potential function is:

$$P_L(\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) = \Omega_L \tag{7}$$

That is, the exact potential function $P_L : S \rightarrow \mathbb{R}$ satisfies $\forall i \in L$ and $\bar{s_i}, \bar{s_i}' \in S_i$:

$$U_i(\bar{s_i}, \overline{s_{-i}}) - U_i(\bar{s_i}', \overline{s_{-i}}) = P_L(\bar{s_i}, \overline{s_{-i}}) - P_L(\bar{s_i}', \overline{s_{-i}}) \tag{8}$$

where $\overline{s_{-i}}$ is the set of strategies selected by players $\in L$ except player i.

Proof: The proof of Theorem 1 is given in Appendix A. ■

Since our proposed game has an exact potential and the number of players and their strategies is finite, the game has the *finite improvement path* (FIP) property. This means that, regardless of the order of the play and the initial condition of the game, the myopic learning process based on one-sided better reply dynamic converges to a Nash equilibrium [29]. Moreover, in a potential game players try to jointly maximize the potential function P_L [29]. Hence, in game Γ_L players try to maximize the average network capacity, which was the objective of the coexistence problem formulation.

IV. Channel Allocation on a Limited Neighborhood

The consumer deployment of small cells in TV White Spaces is expected to lead to a frequently changing interference scenario due to users turning on and off their small cells. To accommodate a new small cell, the strategy of applying channel allocation to all small cells previously deployed in the network would possibly trigger the reconfiguration of a large number of small cells. With such a strategy, the channels assigned to the new small cell would affect the channels of its neighbors, which, in turn, could affect the channels of their neighbors, propagating hence the changes. For that reason, in this section we propose a new optimization problem in which only the new small cell and a set of neighbor small cells are subject to change their channels (see Fig. 2). The channels of the small cells outside the neighborhood remain unaffected.

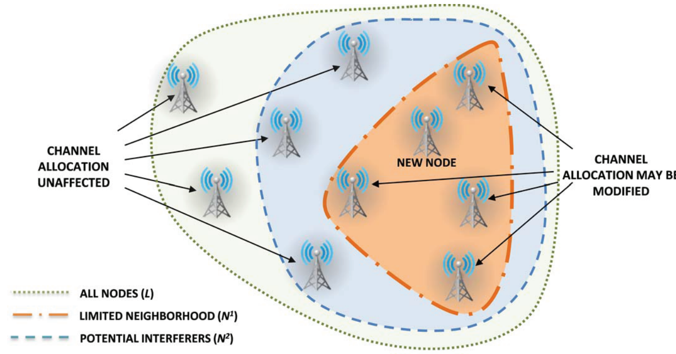

Fig. 2. Channel allocation on a limited neighborhood.

Let us n denote the new node to be switched on. Let us assume that CM partitions the set of nodes $\in L$ in three disjoint subgroups. First, set N^1 includes the node n and its neighbor nodes. These nodes are subject to change their channels. Second, set N^2 represents the nodes outside the neighborhood of node n, thus $N^1 \cap N^2 = \emptyset$. These nodes do not change their channel, but their level of interference may influence the channel assignment of nodes $\in N^1$. We define set N as $N = N^1 \cup N^2$. Third, the remaining nodes $\in L \setminus N$, are ignored in the problem formulation since those are considered to have a negligible interference impact on the channel assignment of nodes $\in N^1$.

In this scenario we compute the average network capacity of the nodes $\in N$ as:

$$\Omega_N = \sum_{\forall j \in N} \sum_{\forall c \in K} f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_n}) \cdot \delta(\bar{s_j}, c) \tag{9}$$

With these assumptions, the coexistence problem is now formulated as follows:

$$
\begin{aligned}
& \max_{\bar{s_i}, \forall i \in N^1} \Omega_N \\
& \text{subject to } |\bar{s_i}| \leq r_i, \forall i \in N^1
\end{aligned} \tag{10}
$$

Let us consider the game $\Gamma_N = \langle N, \{S_i\} \forall i \in N, \{U_i\} \forall i \in N \rangle$. Γ_N is equal to the game Γ_L presented in section III but just comprising the nodes $\in N$. Therefore, its potential function is $P_N(\bar{s_1}, ..., \bar{s_i}, ...\bar{s_n}) = \Omega_N$. In addition, the utility function U_i for this game only considers the nodes $\in N$. To solve (10), we have to restrict the game Γ_N to be played only by the nodes $\in N^1$. For this, we apply to Γ_N the concept of game-couplings presented in [30].

Definition 1: Γ_N is a $(N^1, ..., N^J)$-coupling of $\mathcal{P}$-games if:

- $N^1, ..., N^J$ are groups partitioning N, i.e. $N^j \cap N^{j\prime} = \emptyset, \forall j, j' \setminus 1 \leq j \leq j' \leq J$, and $N^1 \cup ... \cup N^J = N$. The groups are fixed: no player can choose its group.
- For any j and any fixed vector $s^{-j} \in S^{-j}$ of players in N^{-j}, the sub-game $\Gamma_N|_{N^{-j} \leftarrow s^{-j}}$ (played by N^j) has property $\mathcal{P}$. N^{-j} is the set of nodes $\notin N^j$, and $S^{-j} = \times_{i \in N^{-j}} S_i$ is the strategy space of the nodes $\in N^{-j}$.

$\Gamma_N|_{N^{-j}\leftarrow s^{-j}}$ denotes the sub-game of Γ_N (played by N^j) that arises when the strategies of the nodes outside group j are fixed according to s^{-j}.

We consider Γ_N as a (N^1, N^2)-coupling of games in which N^1 and N^2 are the nodes inside and outside the neighborhood respectively. We denote as $S^1 = \times_{i\in N^1}\bar{s}_i$ and $S^2 = \times_{i\in N^2}\bar{s}_i$ the strategy space of the nodes N^1 and N^2 respectively. $\Gamma_N|_{N^2\leftarrow s^2}$ denotes the sub-game played only by the nodes $\in N^1$, in which the strategies of the nodes $\in N^2$ are fixed according to their selected strategies $s^2 \in S^2$.

To solve (10), we aim at verifying that the sub-game $\Gamma_N|_{N^2\leftarrow s^2}$ is an exact potential game.

Theorem 2: The sub-game $\Gamma_N|_{N^2\leftarrow s^2}$ is an exact potential game.

Proof: As P_N is the potential function of Γ_N then, by definition, it satisfies:

$$U_i(\bar{s}_i, \overline{s_{-i}}) - U_i(\bar{s}_i', \overline{s_{-i}}) = P_N(\bar{s}_i, \overline{s_{-i}}) - P_N(\bar{s}_i', \overline{s_{-i}}), \quad \forall i \in N \tag{11}$$

If we consider any node $i \in N_1$, then $\overline{s_{-i}} = s^1_{-i} \cup s^2$, where s^1_{-i} is the set of strategies selected by players $\in N^1$ except player i. Then, from (11) it follows that:

$$\begin{aligned} U_i(\bar{s}_i, \overline{s_{-i}}) - U_i(\bar{s}_i', \overline{s_{-i}}) &= \\ & U_i(\bar{s}_i, s^1_{-i}, s^2) - U_i(\bar{s}_i', s^1_{-i}, s^2) = \\ & P_N(\bar{s}_i, s^1_{-i}, s^2) - P_N(\bar{s}_i', s^1_{-i}, s^2) = \\ & P_N|_{N^2\leftarrow s^2}(\bar{s}_i, s^1_{-i}) - P_N|_{N^2\leftarrow s^2}(\bar{s}_i', s^1_{-i}), \\ & \forall i \in N_1 \end{aligned} \tag{12}$$

where $P_N|_{N^2\leftarrow s^2}$ is the potential function for the game $\Gamma_N|_{N^2\leftarrow s^2}$. Since $\Gamma_N|_{N^2\leftarrow s^2}$ is an exact potential game, the exact potential property of the channel allocation game is preserved when the channel allocation of the nodes outside the neighborhood is fixed. ■

V. Small Cell Dynamic Deployment Solution

IN this section we propose an adaptive solution that provides the set of small cells composing the limited neighborhood and their assigned channels. This solution is named Small Cell Dynamic Deployment (SCDD). The Coexistence Manager is assumed to apply SCDD whenever a new node is switched on. The goal of SCDD is to reduce the number of small cel reconfigurations while achieving an interesting performance. For this goal, SCDD evaluates the performance of the game $\Gamma_N|_{N^2\leftarrow s^2}$ proposed in section IV for various sets of neighbor nodes. SCDD selects the smallest neighborhood that yields a performance similar to applying channel allocation in a large set of neighbor nodes. The performance is measured in terms of average network capacity and network throughput at 5% outage. The latter performance indicator is defined next.

Let us V_i denote the maximum data rate of a user located at a position $(x, y) \in R_i$ and served by node i on all its channels. We calculate it as:

$$\begin{aligned} V_i(x, y, \bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n) &= \\ \sum_{\forall c\in K} V_i^c(x, y, \bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n) \cdot \delta(\bar{s}_i, c) &= \\ \sum_{\forall c\in K} W_u \; log_2(1 + \gamma_i^c(x, y, \bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n)) \cdot \delta(\bar{s}_i, c) \end{aligned} \tag{13}$$

As in the case of γ_i^c, V_i varies with user locations $(x, y) \in R_i$ and the set of channels $\bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n$ used by the nodes in N. Considering a fixed channel allocation, the random variable V_i has a conditional probability density function $\mathrm{p}_{V_i}(v_i/\bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n)$.

We further define a random variable V that denotes the maximum data rate of a user located at position $(x, y) \in R$, where $R = \bigcup_{\forall i} R_i$. $V = V_i$ if the user is located in region R_i and served by node i. Let $p_V(v/\bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n)$ denote the conditional probability density function of V conditioned to channel allocation $\bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n$. It can be derived as follows:

$$p_V(v/\bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n) = \sum_{\forall i\in N} p_{V_i}(v/u \in i, \bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n) \cdot p(u \in i) \tag{14}$$

where $u \in i$ denotes that the user is located at a position $(x, y) \in R_i$ and served by node i, and $p(u \in i)$ is the probability of that event. Then, we denote Ψ as the network throughput at 5% outage, and define it as the value that fulfills:

$$\int_0^{\Psi} p_V(v/\bar{s}_1, ..., \bar{s}_i, ...\bar{s}_n) dv = 0.05 \tag{15}$$

We also assume that CM can estimate the network throughput at 5% outage based on its information of the received signal level from each node.

Next, we describe the SCDD algorithm. Whenever a new node n is to be deployed, the algorithm starts by selecting the set of all its interfering nodes N. For this, the algorithm includes in set N each node i which satisfies that the interference level to node n on some channel exceeds a given threshold, i.e. $P_i \cdot g_{ni} > interference\_threshold$. g_{ni} denotes the path gain from node i to node n, and P_i is the maximum transmission power of node i on any channel, i.e. $P_i = \max_{\forall c\in K}\{P_{ic}\}$.

Next, the set of neighbor nodes N^1 is to be derived. To derive N^1, the SCDD algorithm sorts the nodes $\in N$ by descending level of path gain g_{ni}. The resulting set $\hat{N}$ fulfills $\forall \hat{i}, \hat{j} \in \hat{N}$, $\hat{i} < \hat{j} \Rightarrow g_{n\hat{i}} \geq g_{n\hat{j}}$. Then, the algorithm creates set N^1 composed of the first num_of_neigh nodes of set $\hat{N}$ and the new node n.

The algorithm iteratively considers different sets N^1 with different number of neighbors. It starts with max_{N^1} neighbors, and it later reduces them down to min_{N^1} in steps of $\Delta_{num\_of\_neigh}$. In each iteration, the algorithm derives the set $N^2 = N \setminus N^1$, plays the sub-game $\Gamma_N|_{N^2\leftarrow s^2}$, and extracts the statistics. Additionally, the algorithm identifies the maximum average network capacity Ω^* and the maximum network throughput at 5% outage Ψ^* across all iterations.

After all iterations, the algorithm selects the optimum number of neighbors as the minimum number for which the average network capacity and the network throughput at 5% outage exceed a certain threshold thr relative to Ω^* and Ψ^*.

VI. Performance results

In this section we present the results of the performance evaluation of the potential games presented in sections III and IV, and the SCDD solution presented in section V.

SCDD Algorithm

1. Select N
  $N \leftarrow \{i \setminus P_i \cdot g_{ni} > interference\_threshold\}$

2. Sort nodes in N **by descending** g_{ni}
  $\hat{N} \leftarrow N$
  for $\forall \hat{\imath}, \hat{\jmath} \in \hat{N}$ **do**
    $g_{n\hat{\imath}} \geq g_{n\hat{\jmath}} \Rightarrow \hat{\imath} < \hat{\jmath}$
  end for

3. Iteratively determine N^1 **and play sub-game** $\Gamma_N|_{N^2 \leftarrow s^2}$
  $\Omega^* = 0,\ \Psi^* = 0$
  for $num\_of\_neigh = max_{N^1} : \Delta_{num\_of\_neigh} : min_{N^1}$ **do**
    $N^1 \leftarrow \left\{\hat{\imath} \setminus \hat{\imath} \in \hat{N}, \hat{\imath} \leq num\_of\_neigh\right\}$
    $N^1 \leftarrow N^1 \cup \{n\}$
    $N^2 \leftarrow \hat{N} \setminus N^1$
    Play sub-game $\Gamma_N|_{N^2 \leftarrow s^2}$ and extract statistics Ω and Ψ
    $\Omega(num\_of\_neigh) = \Omega$
    $\Psi(num\_of\_neigh) = \Psi$
    if $\Omega > \Omega^*$ **then** $\Omega^* = \Omega$ **end if**
    if $\Psi > \Psi^*$ **then** $\Psi^* = \Psi$ **end if**
  end for

4. Select N^1_{opt}
  $num_of_neigh_{opt} \leftarrow$min$\{num_of_neigh \setminus$
    $\Omega(num\_of\_neigh) > thr \cdot \Omega^*$ AND
    $\Psi(num\_of\_neigh) > thr \cdot \Psi^*\}$
  $N^1_{opt} \leftarrow \left\{\{n\} \cup \left\{\hat{\imath} \setminus \hat{\imath} \in \hat{N}, \hat{\imath} \leq num\_of\_neigh_{opt}\right\}\right\}$

A. Simulation setup

For the performance evaluation, we have implemented a network simulator in MatLab [31] that computes the path loss, the signal level and the average node capacity. The considered scenario and the simulation setup are described next.

The deployment area is a square of size 200m x 200m. A set of 20 nodes are randomly distributed over the area. The path loss is computed according to the model for suburban areas presented in [32]. The model includes a Gaussian random variable that accounts for the shadowing effects at the indoor receiver. We set its standard deviation equal to 6.4dB as described in [32] for suburban residence. Available channels are assumed to be adjacent TVWS channels, so the transmission power of all nodes is set to 40mW. The primary to secondary interference plus the thermal noise is assumed to be -100dBm for all considered channels. We consider a maximum number of available TVWS channels equal to 10, which is in consonance with the values provided by Hessar and Roy in [33]. They report an average number of available TVWS channels equal to 7.3 for portable/personal devices in areas with a minimum population density of 1000 persons per square mile. They also report an average number of available TVWS channels ranging from almost zero to 9.8 in selected cities of the United States. Each node i is assumed to serve a circular crown region R_i. This region is centered at the node location and has inner and outer radiuses of 2m and 20m respectively. The area of the deployment scenario is divided into pixels of size 1m x 1m. The simulator calculates the signal to interference ratio and the Shannon capacity in the pixels of region R_i $\forall i$. From this, the simulator computes the average node capacity for all nodes, and the network throughput at 5% outage. Nodes are assumed to have a number of available radios that limits the maximum number of channels they can transmit on. In our simulations we consider 1, 2 and 4 radios per node, which is similar to the frequency segments defined in IEEE 802.11af [8] and could be applicable to LTE-A small cells using several component carriers. Nodes are not allowed to select a strategy with less than 1 radio. 50 snapshots have been simulated for each analyzed case and the results have been averaged from all snapshots. Snapshots are network instances with different node locations in the deployment area.

We consider a static and a dynamic deployment scenario. In the static scenario, at the beginning of each snapshot all nodes are deployed together and a random channel assignment is carried out. Then, the game is executed. The initial channel assignment for the game is the random channel assignment. In the dynamic scenario, the deployment starts without any network node. Later, nodes are deployed one by one until completing all nodes. Each time a new node is deployed, the set of neighbor nodes N^1 is derived and the game is executed. The initial channel assignment for the game is the one provided by the game when the previous node was deployed.

B. Analysis of the performance results

1) Channel allocation applied to all network nodes: In this subsection we present the results obtained in the evaluation of the potential game presented in section III. A static deployment scenario is considered. We evaluated the cases of number of available channels = [2,3,4,5,6,7,8,9,10] and number of available radios = [1,2,4]. Fig. 3 and 4 depict the average network capacity and the network throughput at 5% outage respectively.

For comparison purposes, we implemented Peng's solution for spectrum assignment [20], which is based on a graph coloring approach. We utilized the centralized Max-Sum-Reward rule since it has a similar objective to our solution, i.e. to maximize the total throughput in the system. We assume two nodes interfere with each other if they are located within a distance of 40m, that is, twice the radius of the node coverage region. Peng's channel reward is constant over all nodes in our considered scenario. We refer to this solution as 'Graph'. The performance of the random channel assignment (referred as 'Random') is also included for comparison.

The results show that the game yields an increased average network capacity with a higher number of available channels or a higher number of available radios. Similarly, the game produces an increased network throughput at 5% outage with a higher number of available channels. However, it presents a reduced network throughput at 5% outage with a higher number of available radios. This is because a larger number of available radios implies a higher frequency reuse, and therefore an increased interference level especially at the edge of the region served by each node. Although the bandwidth of the node also increases with the number of radios, the first effect is predominant when the number of available channels is relatively low.

Another observation is that a player (node) may select a number of channels lower than its number of available radios (see Fig. 5). This is controlled by the utility function U_i

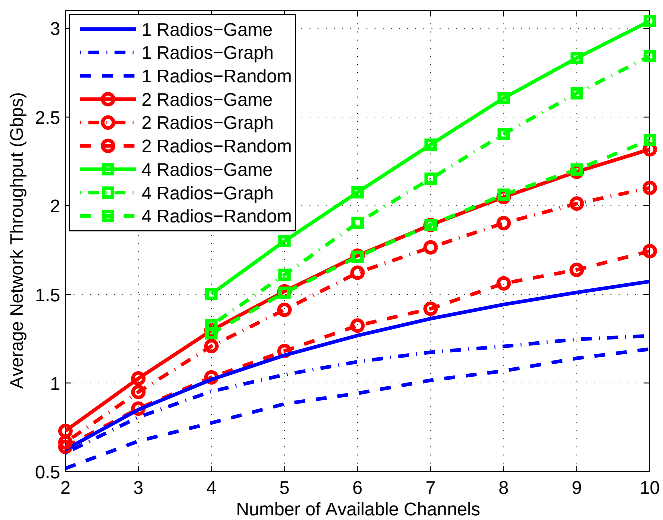

Fig. 3. Performance of channel allocation applied to all nodes of the network: average network capacity.

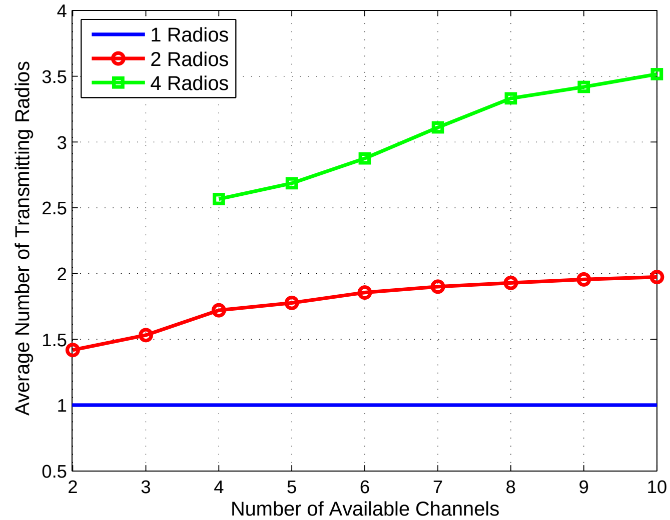

Fig. 5. Performance of channel allocation applied to all nodes of the network: average number of transmitting radios.

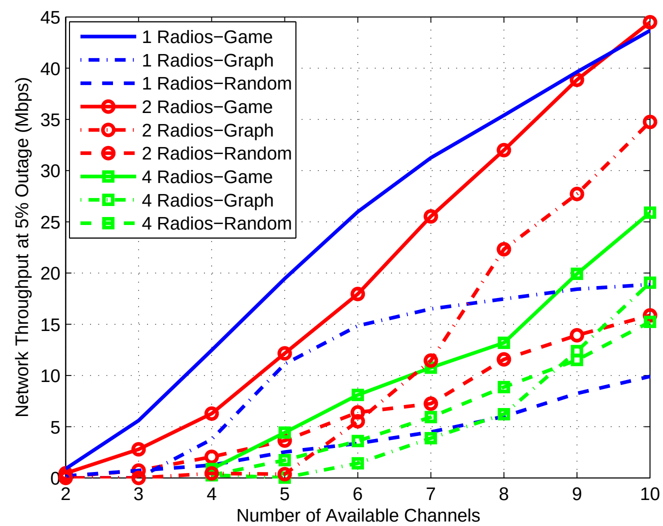

Fig. 4. Performance of channel allocation applied to all nodes of the network: network throughput at 5% outage.

TABLE I
PERFORMANCE OF CHANNEL ALLOCATION IN STATIC AND DYNAMIC DEPLOYMENTS: NETWORK THROUGHPUT (MBPS) AT 5% OUTAGE.

Available radios	Deployment scenario	Available Channels			
		2	4	6	8
4 radios	static	-	0.9	8.1	13.2
	dynamic	-	1.4	6.8	10.2
2 radios	static	0.5	6.3	17.9	32
	dynamic	0.5	5.6	16.2	27.7
1 radio	static	0.9	12.5	26	35.4
	dynamic	0.9	12.5	26	35.7

defined in (6). If a node transmits on a large number of channels it causes a degradation of the average capacity of the rest of the nodes of the network due to increased interference. If this degradation is larger than the average capacity obtained by this node, then it reduces the number of selected channels to improve its utility. The effect is more pronounced for a larger number of available radios or lower number of available channels.

As it can be observed, the performance (both the average network capacity and the network throughput at 5% outage) of our solution outperforms both the graph coloring and random approaches. Moreover, for certain snapshots, the graph coloring approach switches off some nodes, which degrades the performance, and in particular the network throughput at 5% outage.

2) Channel allocation in a limited neighborhood: In this subsection we present the results obtained in the evaluation of the potential game presented in section IV. For this evaluation a dynamic deployment scenario is considered. We evaluated the cases of number of available channels = [2,4,6,8,10], number of available radios = [1,2,4], and number of neighbors = [0,2,4,6,8,10].

The obtained results present the following trends: 1) For all evaluated cases of available channels and radios, the network throughput at 5% outage is considerably reduced for a very low number of neighbors (see Fig. 7 for the case of 4 available radios). However, increasing the number of neighbors beyond a certain value does not yield any benefit in terms of network throughput at 5% outage. On the other hand, for all evaluated cases of available channels and radios, the effect of the number of neighbors on the average network throughput is minor (see Fig. 6 for the case of 4 available radios). 2) For all evaluated cases of available channels and radios, the usage of a sufficient number of neighbors provides a performance similar to the case when the neighborhood includes all nodes already deployed (see the triangles on the right of Fig. 6 and 7). 3) Although not depicted here, the effect of the reduction of the network throughput at 5% outage in a small neighborhood is less pronounced with fewer available radios.

Additionally, the performance differences between the static

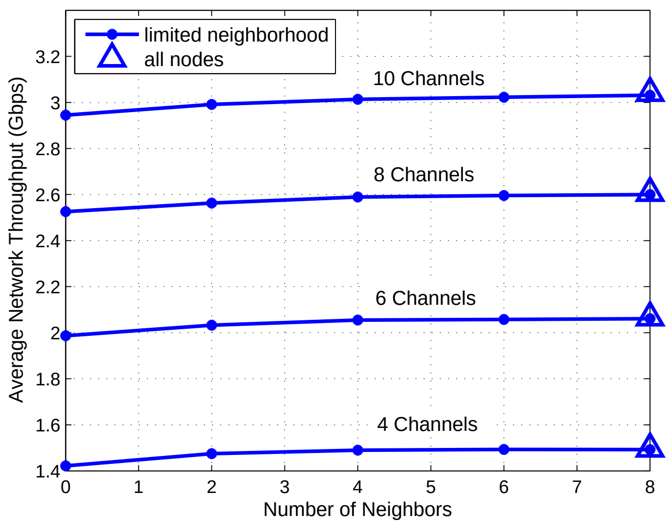

Fig. 6. Performance of channel allocation applied in a limited neighborhood: average network capacity. Number of available radios r_i equals 4.

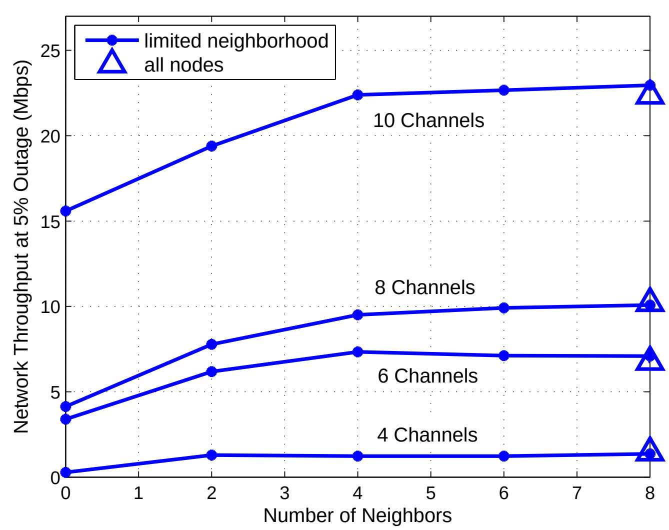

Fig. 7. Performance of channel allocation applied in a limited neighborhood: network throughput at 5% outage. Number of available radios r_i equals 4.

and dynamic deployment scenarios are also worth mentioning. See the comparison of the network throughput at 5% outage for both cases in Table I. In these results the dynamic deployment case considers a neighborhood composed of all nodes already deployed. The static scenario provides an improved network throughput at 5% outage over the dynamic one for a large number of available channels with two and especially four radios. In the dynamic scenario nodes are deployed one by one, and the non-cooperative game is executed each time a new node is deployed. As in each iteration the game uses an initial channel assignment equal to the one obtained in the previous iteration, the quality of the obtained Nash Equilibrium is affected.

3) Channel allocation in an adaptive neighborhood: In this subsection we present the results obtained in the evaluation of the SCDD solution. For this evaluation a dynamic deployment

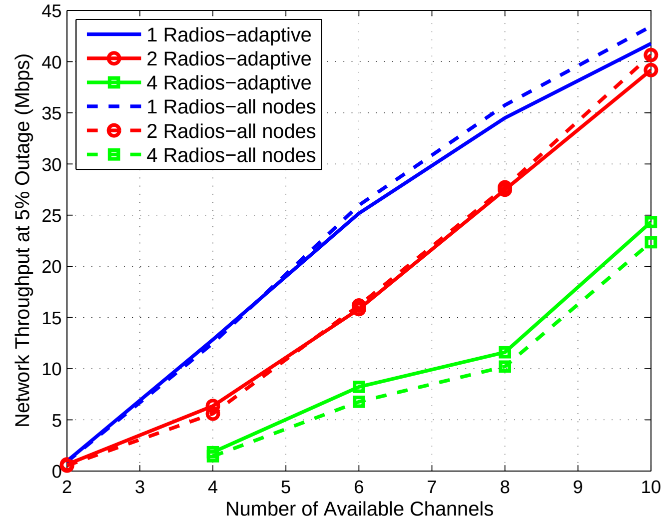

Fig. 8. Performance of channel allocation applied in an adaptive neighborhood: network throughput (Mbps) at 5% outage. Number of available radios r_i equals 4.

scenario is considered. Each time a new node is deployed, the SCDD solution is applied to obtain the limited neighborhood and the assigned channels. For comparison purposes, we also evaluate the strategy in which channel allocation is applied to all nodes already deployed. We evaluated the cases of number of available channels = [2,4,6,8,10], and number of available radios = [1,2,4]. For the evaluation of SCDD, we set $min_{N^1} = 0$, $max_{N^1} = 10$, $\Delta_{num\_of\_neigh} = 2$, and $thr = 90\%$.

For all evaluated number of channels and radios, SCDD and channel allocation applied to all nodes provide very similar average network capacity. The improvement considering all nodes over SCDD is at most 1.2%. The comparison in terms of network throughput at 5% outage is depicted in Fig. 8. For this performance indicator, the improvement considering all nodes over SCDD is at most 4.1%. However, for some evaluated cases SCDD yields an improved network throughput at 5% outage over the case with all nodes. This is because SCDD uses the network throughput at 5% outage as an indicator to select the optimum neighborhood (see section V).

Additionally, SCDD yields a low number of reconfigurations of already deployed nodes (see Table II). For all evaluated cases, on average less than one extra node requires channel reconfiguration when a new node is deployed. The average is computed over all 50 snapshots. In worst case, the maximum number of channel reallocations of extra nodes is lower than 3.6 for all considered number of channels and radios. The maximum is also averaged over all 50 snapshots.

From these results we conclude that SCDD achieves a similar performance compared to the case of channel allocation with all nodes while producing a very limited number of channel reconfigurations.

VII. Conclusions

In this paper we present a solution that enables coexistence for the dynamic deployment of unplanned small cells in

TABLE II
AVERAGE NUMBER OF RECONFIGURATIONS OF ALREADY DEPLOYED NODES PER NEW NODE USING AN ADAPTIVE NEIGHBORHOOD.

Available radios		Available Channels 2	4	6	8	10
4 radios	mean $(\times 10^{-1})$	-	7.4	9.1	8.3	9.7
	max	-	2.5	3.0	3.0	3.6
2 radios	mean $(\times 10^{-1})$	4.6	6.2	5.2	4.3	3.8
	max	1.7	2.5	2.4	2.2	2.3
1 radio	mean $(\times 10^{-1})$	3.2	3.2	2.2	0.9	1.3
	max	1.7	1.9	1.7	1.6	1.6

TVWSs. Each time a new small cell is deployed, the proposed SCDD solution assigns channels only to the new small cell and the small cells in its neighborhood. The channels of the small cells outside its neighborhood remain unaffected.

For channel allocation we develop an exact potential game in which the potential function is the average network capacity. We demonstrate that the exact potential property of the proposed game is preserved when the channel allocation is applied only to the small cells in the neighborhood of the new one. The evaluation of this potential game has provided an interesting performance in terms of average network capacity and network throughput at 5% outage for all tested cases of available channels and radios.

Additionally, the proposed SCDD solution adaptively selects the set of small cells that compose the limited neighborhood. The results showed that the proposed solution requires a few channel reconfigurations in already deployed nodes. However, its performance was similar to a spectrum reassignment applied to all network nodes.

APPENDIX A
PROOF OF THEOREM 1

Let us proof that the function $P_L(\bar{s_1},...,\bar{s_i},...\bar{s_l})$ defined in (7) satisfies $\forall i \in L$ and $\bar{s_i}, \bar{s_i}' \in S_i$:

$$U_i(\bar{s_i},\overline{s_{-i}}) - U_i(\bar{s_i}',\overline{s_{-i}}) = P_L(\bar{s_i},\overline{s_{-i}}) - P_L(\bar{s_i}',\overline{s_{-i}}) \quad (16)$$

and it is therefore an exact potential function.

We first develop the left side of equality (16):

$$\begin{aligned}
&U_i(\bar{s_i}=\{c_1,...,c_n\},\overline{s_{-i}}) - U_i(\bar{s_i}'=\{c_1',...,c_m'\},\overline{s_{-i}}) = \\
&\sum_{\forall c\in K}\delta(\bar{s_i},c)\cdot\sum_{j=1}^{l} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&-\sum_{\forall c\in K}\delta(\bar{s_i},c)\cdot\sum_{j=1,j\neq i}^{l} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&-\sum_{\forall c\in K}\delta(\bar{s_i}',c)\cdot\sum_{j=1}^{l} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&+\sum_{\forall c\in K}\delta(\bar{s_i}',c)\cdot\sum_{j=1,j\neq i}^{l} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c) = \\
&=\sum_{\forall c\in \bar{s_i}\setminus(\bar{s_i}\cap\bar{s_i}')}\Big(\sum_{j=1}^{l} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&+\sum_{\forall c\in \bar{s_i}\cap\bar{s_i}'}\Big(\sum_{j=1}^{l} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{\forall c\in \bar{s_i}\setminus(\bar{s_i}\cap\bar{s_i}')}\Big(\sum_{j=1,j\neq i}^{l} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{\forall c\in \bar{s_i}\cap\bar{s_i}'}\Big(\sum_{j=1,j\neq i}^{l} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{\forall c\in \bar{s_i}'\setminus(\bar{s_i}\cap\bar{s_i}')}\Big(\sum_{j=1}^{l} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{\forall c\in \bar{s_i}\cap\bar{s_i}'}\Big(\sum_{j=1}^{l}\cdot f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&+\sum_{\forall c\in \bar{s_i}'\setminus(\bar{s_i}\cap\bar{s_i}')}\Big(\sum_{j=1,j\neq i}^{l} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&+\sum_{\forall c\in \bar{s_i}\cap\bar{s_i}'}\Big(\sum_{j=1,j\neq i}^{l} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) = \\
&\sum_{j=1}^{l}\Big(\sum_{\forall c\in \bar{s_i}\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{j=1,j\neq i}^{l}\Big(\sum_{\forall c\in \bar{s_i}\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{j=1}^{l}\Big(\sum_{\forall c\in \bar{s_i}'\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&+\sum_{j=1,j\neq i}^{l}\Big(\sum_{\forall c\in \bar{s_i}'\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\varnothing,...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big)
\end{aligned}$$

Now, we develop the right side of equality (16):

$$\begin{aligned}
&P_L(\bar{s_i}=\{c_1,...,c_n\},\overline{s_{-i}}) - P_L(\bar{s_i}'=\{c_1',...,c_m'\},\overline{s_{-i}}) = \\
&\sum_{j=1}^{l}\sum_{\forall c\in K} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&-\sum_{j=1}^{l}\sum_{\forall c\in K} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c) = \\
&\sum_{j=1}^{l}\Big(\sum_{\forall c\in \bar{s_i}\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&\quad+\sum_{\forall c\in \bar{s_i}'\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&\quad+\sum_{\substack{\forall c\in \bar{s_i}\cap\bar{s_i}' \\ \forall c\in K\setminus(\bar{s_i}\cup\bar{s_i}')}} f(j,c/\bar{s_1},...,\bar{s_i},...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big) \\
&-\sum_{j=1}^{l}\Big(\sum_{\forall c\in \bar{s_i}\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&\quad+\sum_{\forall c\in \bar{s_i}'\setminus(\bar{s_i}\cap\bar{s_i}')} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c) \\
&\quad+\sum_{\substack{\forall c\in \bar{s_i}\cap\bar{s_i}' \\ \forall c\in K\setminus(\bar{s_i}\cap\bar{s_i}')}} f(j,c/\bar{s_1},...,\bar{s_i}',...\bar{s_l})\cdot\delta(\bar{s_j},c)\Big)
\end{aligned}$$

We can observe that the function $f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) = f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c)$, $\forall c \in \bar{s_i}' \setminus (\bar{s_i} \cap \bar{s_i}')$. Similarly, the function $f(j, c/\bar{s_1}, ..., \bar{s_i}', ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) = f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c)$, $\forall c \in \bar{s_i} \setminus (\bar{s_i} \cap \bar{s_i}')$. Additionally, $f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) = f(j, c/\bar{s_1}, ..., \bar{s_i}', ...\bar{s_l}) \cdot \delta(\bar{s_j}, c)$, $\forall c \in \bar{s_i} \cap \bar{s_i}'$ and $\forall c \in K \setminus (\bar{s_i} \cap \bar{s_i}')$. Therefore, we can rewrite the right side of (16) as:

$$\sum_{j=1}^{l} \Big(\sum_{\forall c \in \bar{s_i} \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$
$$+ \sum_{j=1}^{l} \Big(\sum_{\forall c \in \bar{s_i}' \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$
$$- \sum_{j=1}^{l} \Big(\sum_{\forall c \in \bar{s_i} \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$
$$- \sum_{j=1}^{l} \Big(\sum_{\forall c \in \bar{s_i}' \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \bar{s_i}', ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$

But $f(i, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_i}, c)$ is equal to zero $\forall c \in \bar{s_i}' \setminus (\bar{s_i} \cap \bar{s_i}')$. Similarly $f(i, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_i}', c)$ is also equal to zero $\forall c \in \bar{s_i} \setminus (\bar{s_i} \cap \bar{s_i}')$. Then, we have:

$$\sum_{j=1}^{l} \Big(\sum_{\forall c \in \bar{s_i} \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$
$$+ \sum_{j=1, j \neq i}^{l} \Big(\sum_{\forall c \in \bar{s_i}' \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$
$$- \sum_{j=1, j \neq i}^{l} \Big(\sum_{\forall c \in \bar{s_i} \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \varnothing, ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$
$$- \sum_{j=1}^{l} \Big(\sum_{\forall c \in \bar{s_i}' \setminus (\bar{s_i} \cap \bar{s_i}')} f(j, c/\bar{s_1}, ..., \bar{s_i}', ...\bar{s_l}) \cdot \delta(\bar{s_j}, c) \Big)$$

which is therefore equal to the left side of (16).

Appendix B
Some notations used in this paper

For the sake of readability, Table III includes the variables used in this paper.

TABLE III
NOTATIONS USED IN THIS PAPER.

c_j	channel j
$f(i, c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l})$	average capacity of node i on channel c
$g_i(x, y)$	path gain from node i
g_{ni}	path gain from node i to node n
$I_c(x, y)$	primary to secondary interference on channel c plus thermal noise
K	set of TVWS channels available for operation
L	set of nodes in the network
max_{N^1}, min_{N^1}	maximum / minimum number of neighbors for the SCDD solution
N	nodes that may affect the channel allocation ($N = N^1 \cup N^2$)
N^1, N^2	nodes in the neighborhood / outside the neigborhood
N^{-j}	group of nodes that $\notin N^j$ (j=1,2)
$p_V(v/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_n})$	conditional probability density function of V
$p_{\gamma_i^c}(\gamma_i^c/\bar{s_1}, ..., \bar{s_i}, ...\bar{s_l})$	conditional probability density function of γ_i^c
P_{ic}	transmission power of node i on channel c
P_i	maximum transmission power of node i
$P_N(\bar{s_1}, ..., \bar{s_i}, ...\bar{s_n})$	exact potential function of the non-cooperative game Γ_N
r_i	number of node i's available radios
R_i, R	coverage region of node i, $R = \bigcup_{\forall i} R_i$
$\bar{s_i}$	set of channels used by node i
$\overline{s_{-i}}$	set of channels used by all nodes except node i
S_i	channel combination space of node i
s^j	set of channels used by nodes $\in N^j$ (j=1,2)
S^j	channel combination space of nodes $\in N^j$ (j=1,2)
S	channel combination space for all nodes
U_i	utility function of node i
V	maximum data rate of a user located at a position $(x, y) \in R$
$V_i(x, y, \bar{s_1}, ..., \bar{s_i}, ...\bar{s_n})$	maximum data rate of a user served by node i
W	channel bandwidth
$\gamma_i^c(x, y, \bar{s_1}, ..., \bar{s_i}, ...\bar{s_n})$	signal to interference ratio of a user served by node i on channel c
Γ_N	non-cooperative game (players are nodes $\in N$)
$\Gamma_N\vert_{N^2 \leftarrow s^2}$	sub-game played only by nodes in the neighborhood with fixed strategies for nodes outside the neighborhood
$\delta(\bar{s_i}, c)$	v function that indicates whether node i uses channel c or not
Ψ	network throughput at 5% outage
Ω	average network capacity

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