



XI Jornadas de
Teoría de Anillos

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A Graded Criterion in the Classification of Cofinite Homogeneous Ideals

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XI Jornadas de Teoría de Anillos

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Notas basadas en el artículo

“A Graded Criterion in the Classification of Cofinite Homogeneous Ideals” (en revisión)

En colaboración con **Pascual Jara Martínez** y
Javier Lobillo Borrero



¿Qué vamos a estudiar?

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Álgebras cocientes del tipo $\mathcal{A} = k\langle X \rangle / \mathcal{I}$ donde

- k cuerpo
- $X = \{x_1, \dots, x_n\}$ conjunto finito de variables
- $k\langle X \rangle$ álgebra libre asociativa sobre el conjunto X
- $\mathcal{I} \trianglelefteq k\langle X \rangle$ ideal bilátero *homogéneo*

Ejemplo

- $k = \mathbb{R}$
- $X = \{x, y\}, x > y$
- $k\langle X \rangle$ álgebra libre en dos variables
- $\mathcal{I} = \langle x^2yx - y^2x^2, xyxy - y^3x, xy^2x - yxy^2 \rangle$



¿Por qué escogemos esta presentación?

- Realizar operaciones con los elementos $\bar{f}, \bar{g} \in k\langle X \rangle / \mathcal{I}$
- Necesitamos elegir *adecuadamente* representantes en cada clase



Bases de Gröbner-Shirshov

Problema (ambiente no conmutativo)

- Pueden ser infinitas (no lema de Dickson)
- No existe método de construcción *razonable*

Ideal **homogéneo** \Rightarrow **NO** aseguramos finitud
SÍ reducciones homogéneas
SÍ trabajar hasta el grado que queramos



Situación general del problema

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$\mathcal{A} = k\langle X \rangle / \mathcal{I}$, \mathcal{I} homogéneo

Proposición

Bajo estas condiciones, \mathcal{A} es un álgebra *graduada*

- $\mathcal{A} = \bigoplus \mathcal{A}_i, i \in \mathbb{Z}$
- $\mathcal{A}_i \cdot \mathcal{A}_j \subseteq \mathcal{A}_{i+j}$
- $\mathcal{A}_0 = k$
- k -base de $\mathcal{A}_j = \{\text{palabras irreducibles de } \mathcal{A}_j\}$

Parametrización ideales homogéneos

- a número de indeterminadas
- b longitud de los monomios
- c número de relaciones que conforman cada ideal



Cómo estudiar estas familias de ideales

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Observaciones

(a, b, c) fijos \Rightarrow n° finito de casos

- $(2, 2, 2) \rightarrow 45$ casos
- $(2, 3, 4) \rightarrow 58905$ casos
- $(2, 4, 3) \rightarrow 420040$ casos
- $(3, 2, 4) \rightarrow 148995$ casos

¿Cómo los estudiamos todos?

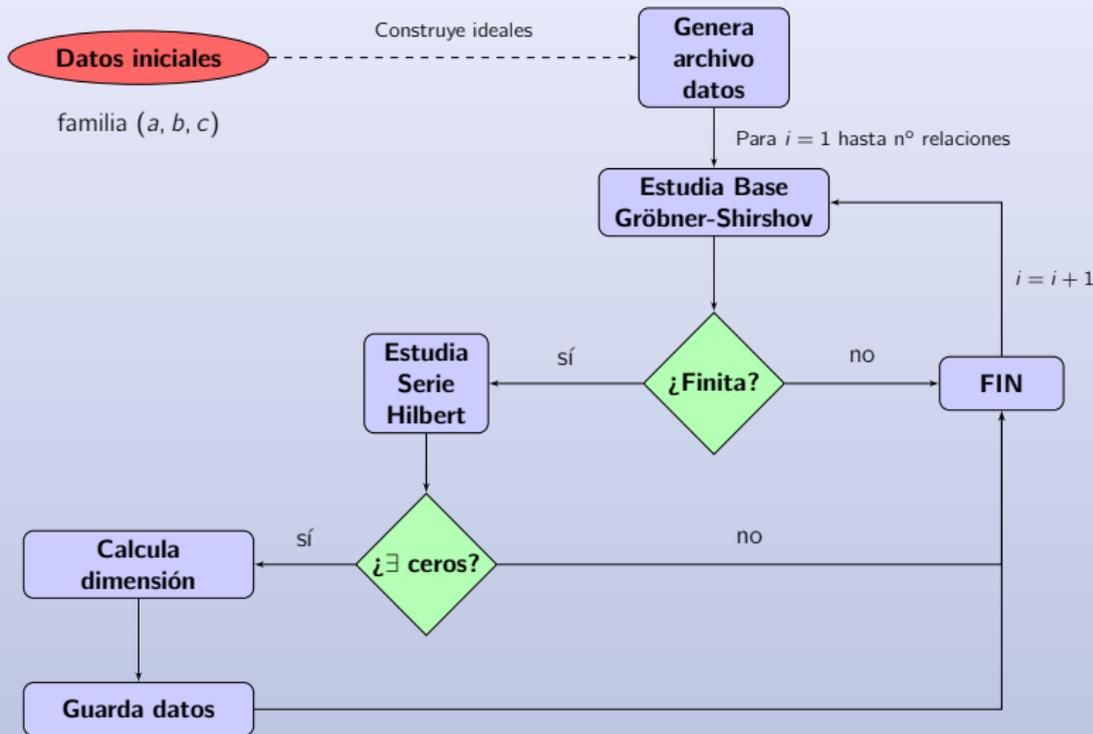
Solución

- Programar una rutina en C++
- Poder de cálculo del sistema Bergman

(Software libre y disponible para todos los usuarios)



Flujo del programa



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Pequeño ejemplo

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FAMILIA (2, 3, 4)

58905 casos \Rightarrow 10142 ideales cofinitos
dimensión finita máxima \Rightarrow 25

- 288 ideales que generan álgebras de dimensión 11
- 2446 ideales que generan álgebras de dimensión 12
- 3578 ideales que generan álgebras de dimensión 13
- 1246 ideales que generan álgebras de dimensión 14
- 2146 ideales que generan álgebras de dimensión 15
- 92 ideales que generan álgebras de dimensión 16
- 174 ideales que generan álgebras de dimensión 17
- 88 ideales que generan álgebras de dimensión 18
- 8 ideales que generan álgebras de dimensión 19
- 72 ideales que generan álgebras de dimensión 21
- 4 ideales que generan álgebras de dimensión 25

$$\mathcal{J}_1 = \langle xxx, xyx, xxx - xyx, xyy - yyy \rangle$$

$$\mathcal{J}_2 = \langle xxx, xyx, xxx - xyx, yyx - yyy \rangle$$

$$\mathcal{J}_3 = \langle xxx, xxx - xyx, xxx - yyx, xxx - yyy \rangle$$



Buscando isomorfismos

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Ejecutamos programa $\Rightarrow L$ dimensión finita máxima en cada familia

Primer *resultado*

Si un álgebra de la familia (a, b, c) tiene dimensión superior a L , es infinito dimensional

OBJETIVO

Quién (o quiénes) es el
álgebra maximal en
cada familia

\Rightarrow

SOLUCIÓN

Estudiar cuándo dos
de estas álgebras
son isomorfas



Primeros filtros

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- Dimensión
- Serie de Hilbert (invariante álgebras graduadas)

¿Cómo buscamos isomorfismos entre dos álgebras? (Shirayanagi,91)

$\mathcal{A} = k\langle X \rangle / \mathcal{I}_{\mathcal{A}}, \mathcal{B} = k\langle X \rangle / \mathcal{I}_{\mathcal{B}}$. Construyo $\varphi : \mathcal{A} \rightarrow \mathcal{B}$

- $X = \{x, y\}$ generadores
- Base de $\mathcal{B} = \{b_1, b_2, \dots, b_m\}$
- $x \mapsto \alpha_1 b_1 + \dots + \alpha_m b_m$
- $y \mapsto \beta_1 b_1 + \dots + \beta_m b_m$
- Calculo $M = M_{\varphi}$ y el determinante asociado $\det(M_{\varphi})$
- Comprobar que φ sea compatible con $\mathcal{I}_{\mathcal{A}}$ y $\det(M_{\varphi}) \neq 0$

Y, ¿qué hacemos si $\dim(\mathcal{A}) = \dim(\mathcal{B}) = 24$?



Isomorfismos graduados

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IDEA

Simplificar un poco la situación \Rightarrow buscar isomorfismo graduado

$$\begin{aligned}\tilde{\varphi} : \mathcal{A} &\rightarrow \mathcal{B} \\ x &\mapsto ax + by \\ y &\mapsto cx + dy\end{aligned}$$

$$\boxed{\mathcal{A}_1 \cong \mathcal{B}_1} + \boxed{\mathcal{A}_2 \cong \mathcal{B}_2} + \cdots + \boxed{\mathcal{A}_n \cong \mathcal{B}_n} = \boxed{\mathcal{A} \cong \mathcal{B}}$$

Comprobación en grado n

- Calculo elementos irreducibles en \mathcal{A}_n
- Calculo la matriz de $\tilde{\varphi}_n$
- $\det(M_{\tilde{\varphi}_n}) \neq 0 \Rightarrow$ condiciones sobre $\{a, b, c, d\}$
- Calculo $\tilde{\varphi}(\mathcal{I}_{\mathcal{A}_n}) \Rightarrow$ condiciones sobre $\{a, b, c, d\}$
- Compruebo compatibilidades



Ejemplo Isomorfismo Graduado

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Ideales

$$\mathcal{I}_A = \langle xxx, xyx, xxx - yxy, xyy - yyy \rangle$$

$$\mathcal{I}_B = \langle yxy, yyy, xxx - yxx, yxy - yyy \rangle$$

Comprobación grado 1

Comprobación grado 2

Comprobación grado 3

Relaciones del ideal

- $\text{Dim}(\mathcal{A}_1) = \text{Dim}(\mathcal{B}) = 25$

- $\mathcal{H} = 1 + 2x + 4x^2 + 5x^3 + 5x^4 + 4x^5 + 3x^6 + x^7$

$$\begin{cases} \varphi(x) = ax + by \\ \varphi(y) = cx + dy \end{cases}$$

elementos normales $\Rightarrow \{xx, xy, yx, yy\}$

matriz asociada $\tilde{M}_2 = \begin{pmatrix} aa & ab & ab & bb \\ ac & ad & bc & bd \\ ac & bc & ad & bd \\ cc & cd & cd & dd \end{pmatrix}$

$$\det(\tilde{M}_2) \neq 0 \Leftrightarrow ad - bc \neq 0$$

elementos normales $\{xxy, yxx, yxy, yyx, yyy\}$

matriz asociada

$$\begin{pmatrix} aad & abc & abd & aac + abc & bbc \\ abc & abc & bbc & aac + aad & abd \end{pmatrix}$$

Condición Grado 123+ Relaciones FINAL

$$ad - bc$$

$$\det(\tilde{M}_3) \neq 0 \Leftrightarrow bc(c+d)(bc-ad) \neq 0$$

$a = 0, \quad b = c \neq 0, \quad d = 0$



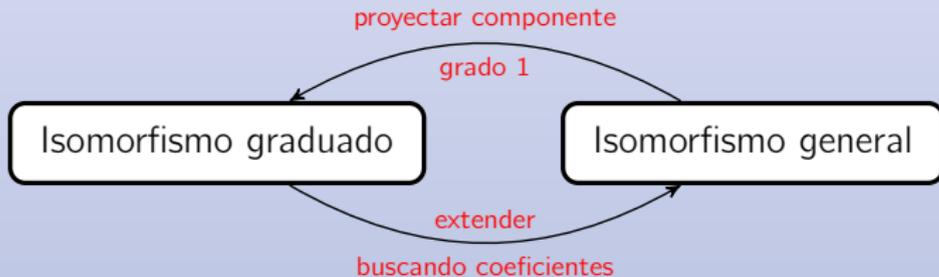
¿Es suficiente con esto?

¿isomorfismos graduados \ll isomorfismos en general?

Teorema (CJL11)

$$\mathcal{A} = k\langle X \rangle / \mathcal{I}, \mathcal{B} = k\langle X \rangle / \mathcal{I}'$$

Si $\exists \varphi : \mathcal{A} \xrightarrow{\cong} \mathcal{B} \Rightarrow \tilde{\varphi} = \pi_1 \circ \varphi$ define un *isomorfismo graduado* entre \mathcal{A} y \mathcal{B}





Comparación procedimientos

Álgebras de la familia (2, 3, 3) de dimensión 21

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$$\mathcal{I}_A = \langle xxx, yxy, yxx - yyy \rangle$$

Isomorfismo graduado

- 4 coeficientes

$$\varphi(x) = ax + by$$

$$\varphi(y) = cx + dy$$

- Determinante grado 15

$$-bc^4(bc - ad)^5$$

- 11 elementos en el ideal

$$\{a^2b, ab^2, a^3 + ab^2, acd, bcd, ad^2, ac^2 + bcd, abc - c^2d, a^2d - c^2d, a^2c + b^2c - c^3 - cd^2, abd - cd^2\}$$

$$a = 0 \quad b = \pm c \neq 0 \quad d = 0$$



$$\mathcal{I}_B = \langle xyx, yyy, xxx - xyy \rangle$$

Isomorfismo general

- 42 coeficientes

$$\begin{aligned} \varphi(x) = & a_1 + a_2x + a_3y + a_4xx + a_5xy + a_6yx + \\ & a_7yy + a_8xxy + a_9xyy + a_{10}yxx + \\ & a_{11}yxy + a_{12}yyx + a_{13}xyyx + a_{14}yxyx + \\ & a_{15}yxxy + a_{16}yyxx + a_{17}yyxy + a_{18}yxyyx + \\ & a_{19}yxyxy + a_{20}yxyxy + a_{21}yxyxyx \end{aligned}$$

$$\begin{aligned} \varphi(y) = & b_1 + b_2x + b_3y + b_4xx + b_5xy + b_6yx + \\ & b_7yy + b_8xxy + b_9xyy + b_{10}yxx + \\ & b_{11}yxy + b_{12}yyx + b_{13}xyyx + b_{14}yxyx + \\ & b_{15}yxxy + b_{16}yyxx + b_{17}yyxy + b_{18}yxyyx + \\ & b_{19}yxyxy + b_{20}yxyxy + b_{21}yxyxyx \end{aligned}$$

- Determinante grado 76

$$-a_3^{10}b_2^{16}(a_3b_2 - a_2b_3)^{17}(b_2^2 + 2b_3^2)^3$$

- 63 elementos en el ideal

$$\begin{aligned} \{ & 2a_1a_2b_1 + a_1^2b_2 - 3b_1^2b_2, a_1^2b_1 - \\ & b_1b_2^2, 2a_1a_3b_1 + a_1^2b_3 - 3b_1^2b_3, a_2^2b_1 + \\ & 2a_1a_4b_1 + 2a_1a_2b_2 - 3b_1b_2^2 + a_1^2b_4 - \\ & 2b_1^2b_2, a_2^2b_2 + 2a_1a_3b_2 + 2a_1a_4b_2 - \\ & 2a_1a_2b_3, a_1^2b_3 + 2a_1a_3b_3 - 3b_1^2b_3, a_2^2b_3 + \\ & 2a_1a_4b_3 + 2a_1a_2b_4 - 3b_1b_2^2 + a_1^2b_4 - \\ & 2b_1^2b_2, a_2^2b_4 + 2a_1a_3b_4 + 2a_1a_4b_4 - \\ & 2a_1a_2b_5, a_1^2b_5 + 2a_1a_3b_5 - 3b_1^2b_5, a_2^2b_5 + \\ & 2a_1a_4b_5 + 2a_1a_2b_6 - 3b_1b_2^2 + a_1^2b_6 - \\ & 2b_1^2b_2, a_2^2b_6 + 2a_1a_3b_6 + 2a_1a_4b_6 - \\ & 2a_1a_2b_7, a_1^2b_7 + 2a_1a_3b_7 - 3b_1^2b_7, a_2^2b_7 + \\ & 2a_1a_4b_7 + 2a_1a_2b_8 - 3b_1b_2^2 + a_1^2b_8 - \\ & 2b_1^2b_2, a_2^2b_8 + 2a_1a_3b_8 + 2a_1a_4b_8 - \\ & 2a_1a_2b_9, a_1^2b_9 + 2a_1a_3b_9 - 3b_1^2b_9, a_2^2b_9 + \\ & 2a_1a_4b_9 + 2a_1a_2b_{10} - 3b_1b_2^2 + a_1^2b_{10} - \\ & 2b_1^2b_2, a_2^2b_{10} + 2a_1a_3b_{10} + 2a_1a_4b_{10} - \\ & 2a_1a_2b_{11}, a_1^2b_{11} + 2a_1a_3b_{11} - 3b_1^2b_{11}, a_2^2b_{11} + \\ & 2a_1a_4b_{11} + 2a_1a_2b_{12} - 3b_1b_2^2 + a_1^2b_{12} - \\ & 2b_1^2b_2, a_2^2b_{12} + 2a_1a_3b_{12} + 2a_1a_4b_{12} - \\ & 2a_1a_2b_{13}, a_1^2b_{13} + 2a_1a_3b_{13} - 3b_1^2b_{13}, a_2^2b_{13} + \\ & 2a_1a_4b_{13} + 2a_1a_2b_{14} - 3b_1b_2^2 + a_1^2b_{14} - \\ & 2b_1^2b_2, a_2^2b_{14} + 2a_1a_3b_{14} + 2a_1a_4b_{14} - \\ & 2a_1a_2b_{15}, a_1^2b_{15} + 2a_1a_3b_{15} - 3b_1^2b_{15}, a_2^2b_{15} + \\ & 2a_1a_4b_{15} + 2a_1a_2b_{16} - 3b_1b_2^2 + a_1^2b_{16} - \\ & 2b_1^2b_2, a_2^2b_{16} + 2a_1a_3b_{16} + 2a_1a_4b_{16} - \\ & 2a_1a_2b_{17}, a_1^2b_{17} + 2a_1a_3b_{17} - 3b_1^2b_{17}, a_2^2b_{17} + \\ & 2a_1a_4b_{17} + 2a_1a_2b_{18} - 3b_1b_2^2 + a_1^2b_{18} - \\ & 2b_1^2b_2, a_2^2b_{18} + 2a_1a_3b_{18} + 2a_1a_4b_{18} - \\ & 2a_1a_2b_{19}, a_1^2b_{19} + 2a_1a_3b_{19} - 3b_1^2b_{19}, a_2^2b_{19} + \\ & 2a_1a_4b_{19} + 2a_1a_2b_{20} - 3b_1b_2^2 + a_1^2b_{20} - \\ & 2b_1^2b_2, a_2^2b_{20} + 2a_1a_3b_{20} + 2a_1a_4b_{20} - \\ & 2a_1a_2b_{21}, a_1^2b_{21} + 2a_1a_3b_{21} - 3b_1^2b_{21}, a_2^2b_{21} + \\ & 2a_1a_4b_{21} + 2a_1a_2b_{22} - 3b_1b_2^2 + a_1^2b_{22} - \\ & 2b_1^2b_2, a_2^2b_{22} + 2a_1a_3b_{22} + 2a_1a_4b_{22} - \\ & 2a_1a_2b_{23}, a_1^2b_{23} + 2a_1a_3b_{23} - 3b_1^2b_{23}, a_2^2b_{23} + \\ & 2a_1a_4b_{23} + 2a_1a_2b_{24} - 3b_1b_2^2 + a_1^2b_{24} - \\ & 2b_1^2b_2, a_2^2b_{24} + 2a_1a_3b_{24} + 2a_1a_4b_{24} - \\ & 2a_1a_2b_{25}, a_1^2b_{25} + 2a_1a_3b_{25} - 3b_1^2b_{25}, a_2^2b_{25} + \\ & 2a_1a_4b_{25} + 2a_1a_2b_{26} - 3b_1b_2^2 + a_1^2b_{26} - \\ & 2b_1^2b_2, a_2^2b_{26} + 2a_1a_3b_{26} + 2a_1a_4b_{26} - \\ & 2a_1a_2b_{27}, a_1^2b_{27} + 2a_1a_3b_{27} - 3b_1^2b_{27}, a_2^2b_{27} + \\ & 2a_1a_4b_{27} + 2a_1a_2b_{28} - 3b_1b_2^2 + a_1^2b_{28} - \\ & 2b_1^2b_2, a_2^2b_{28} + 2a_1a_3b_{28} + 2a_1a_4b_{28} - \\ & 2a_1a_2b_{29}, a_1^2b_{29} + 2a_1a_3b_{29} - 3b_1^2b_{29}, a_2^2b_{29} + \\ & 2a_1a_4b_{29} + 2a_1a_2b_{30} - 3b_1b_2^2 + a_1^2b_{30} - \\ & 2b_1^2b_2, a_2^2b_{30} + 2a_1a_3b_{30} + 2a_1a_4b_{30} - \\ & 2a_1a_2b_{31}, a_1^2b_{31} + 2a_1a_3b_{31} - 3b_1^2b_{31}, a_2^2b_{31} + \\ & 2a_1a_4b_{31} + 2a_1a_2b_{32} - 3b_1b_2^2 + a_1^2b_{32} - \\ & 2b_1^2b_2, a_2^2b_{32} + 2a_1a_3b_{32} + 2a_1a_4b_{32} - \\ & 2a_1a_2b_{33}, a_1^2b_{33} + 2a_1a_3b_{33} - 3b_1^2b_{33}, a_2^2b_{33} + \\ & 2a_1a_4b_{33} + 2a_1a_2b_{34} - 3b_1b_2^2 + a_1^2b_{34} - \\ & 2b_1^2b_2, a_2^2b_{34} + 2a_1a_3b_{34} + 2a_1a_4b_{34} - \\ & 2a_1a_2b_{35}, a_1^2b_{35} + 2a_1a_3b_{35} - 3b_1^2b_{35}, a_2^2b_{35} + \\ & 2a_1a_4b_{35} + 2a_1a_2b_{36} - 3b_1b_2^2 + a_1^2b_{36} - \\ & 2b_1^2b_2, a_2^2b_{36} + 2a_1a_3b_{36} + 2a_1a_4b_{36} - \\ & 2a_1a_2b_{37}, a_1^2b_{37} + 2a_1a_3b_{37} - 3b_1^2b_{37}, a_2^2b_{37} + \\ & 2a_1a_4b_{37} + 2a_1a_2b_{38} - 3b_1b_2^2 + a_1^2b_{38} - \\ & 2b_1^2b_2, a_2^2b_{38} + 2a_1a_3b_{38} + 2a_1a_4b_{38} - \\ & 2a_1a_2b_{39}, a_1^2b_{39} + 2a_1a_3b_{39} - 3b_1^2b_{39}, a_2^2b_{39} + \\ & 2a_1a_4b_{39} + 2a_1a_2b_{40} - 3b_1b_2^2 + a_1^2b_{40} - \\ & 2b_1^2b_2, a_2^2b_{40} + 2a_1a_3b_{40} + 2a_1a_4b_{40} - \\ & 2a_1a_2b_{41}, a_1^2b_{41} + 2a_1a_3b_{41} - 3b_1^2b_{41}, a_2^2b_{41} + \\ & 2a_1a_4b_{41} + 2a_1a_2b_{42} - 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