Hintikka and Frege on Quantifiers

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1. Introduction

Quantifiers have been one of the Philosophy of Logic trending topics of the past Century. Contemporary logic is quantification theory, and quantification theory is (or has been) Fregean logic. Hintikka stresses the essential role that the interpretation of quantifiers plays in the understanding of what is logic (see for instance, Hintikka and Sandu 1994, 113). Some of the shortcomings of Frege's logic, Hintikka claims (see for instance Hintikka 1977, 99, when he explains the ordering of quantifiers), directly depend on accidental aspects of the notation used, since the classical linear notation that stems from the Fregean account precludes the representation of patterns of dependency and independency among quantifiers and, in doing so, obscures the information conveyed by these patterns. Game Theoretical Semantics is a proposal about how to interpret quantified sentences, that defies the classical interpretation.

On the other hand, Game Theoretical Semantics (GTS henceforth) is an implementation of some Wittgensteinian intuitions, basically the understanding of meaning as derived from a set of rules in a particular kind of game. The game that confers quantifiers their meaning is, according to Hintikka, the game of searching and finding (see for instance Saarinen 1979, 8-ff., 333-ff.). Hintikka's intuition is also present in Peirce (cfr. Hilpinen 1982), although Hintikka's development doesn't arise as a development of Peirce's treatment.

Our aim is to highlight some of aspects of Hintikka's take on to the meaning of quantifiers and related issues, assess their originality and the improvement represented by them over Frege's position. In our assessment of the relations between Frege and Hintikka, we will not defy the obvious claim that both of them, Hintikka's and Frege's approaches, correctly explain the meaning of *some* uses of some quantifiers, nor the claim that there are manifest differences among them. One should expect that both theories fit the needs of the parcel of discourse whose analysis acted as their philosophical motivation, and not so well in parcels of discourse far from their original interests. Frege's motivation was primarily the foundations of arithmetic, Hintikka's motivation was the analysis of natural language sentences with more than one quantifier. Both, Hintikka and Frege, intend their proposals to be general and become *the* logic of language. And probably none of them is completely successful. Our purpose is to show that, when one goes into the details, Hintikka's approach is closer to Frege's than what one can infer from Hintikka's claims.

Hintikka has put forward some insights of outstanding value. Among them, (i) the identification of two notions of scope, (ii) the conflictive status of the Compositionality Principle, and (iii) the analysis of the information conveyed by quantifiers combination. The background of these claims is a specific theory of meaning that involves a specific analysis of truth. The theory of meaning supported by Hintikka stems from the Wittgensteinian characterization of language as a kind of game (see for instance, Saarinen 1979, 11-ff.). The pragmatic perspective of this approach involves a rejection of the standard Tarskian semantics (Hintikka 1994, 124). The two general aspects that the Wittgenstein-Hintikka approach add to the Tarskian approach are (a) the role of the agent, who has to make moves without necessarily having all the relevant information at her disposal, and (b) the dynamic aspect of the meaning of logical notions.

There are nevertheless some aspects of Hintikka's view that are open to further debate. The historical interpretation of Frege's view on quantifiers, in particular, and meaning, in general, is the feature we will focus on. We will also assess to which extent the Fregean treatment of quantifiers is incompatible with Hintikka's take on them. Our aim is to stress the relevance and depth of Hintikka's views, using Frege as a point of contrast.

2. Logical form and the logic of language.

According to Hintikka and Sandu (1994, 114), there have been three different accounts of quantifiers in the past century: the Fregean interpretation as "higher order predicates" (op. cit.), the sustitutional view that makes them meta-linguistic devices, and their own proposal of quantifiers as codifying choice functions. We will compare in this section the *higher-level* and *choice function* approaches with respect to the following question: what is the contribution of quantifiers to what is said? The answer to this question will crucially hinge on the notion of *logical form*. There is more to the use of this notion in recent linguistics and philosophy of language than meets the eye, as we will show throughout the following paragraphs.

Against classical quantification theory, *Frege's logic*, Hintikka has a general charge: that it cannot be *the* logic of language (see for instance in Saarinen 1979, 81-118). This surely doesn't mean that it cannot be the logic of *some* parcels of language for *some* purposes. Nor even Frege thought of his proposal as of general utility. "This ideography", he said on the Preface to (1879), "is a device invented for certain scientific purposes, and one must not condemn it because it is not suited to others" (Frege 1879/ van Heijenoort 1990, 6). He certainly hoped his *Beggriffsschrift* to be extendable to become a universal code, even though he was aware that the extension could not be "reached in one leap" (loc. cit), but through "a slow, step-by-step approximation" (loc. cit.). Hintikka, in turn, seems to assume the universal validity of his proposal, and this universality presupposes that quantifiers are a specific and homogeneous kind of conceptual device, that represent patterns of dependence and independence in the general game of searching and finding. "Actually", Hintikka & Sandu claim, "even though the point is not being appreciated by philosophers, logicians, and linguists, only one the three approaches has a realistic hope of being adequate without smuggling in ideas from other approaches" (Hintikka and Sandu 1994, 115). And they go on: "the real power of the logic of quantification lies in the interaction of quantifiers with each other, in other words, in the idea of dependent quantifier" (loc. cit.). IF-logic is the result of this intuition. Understanding the meaning of quantifiers means understanding their import in combination; the meaning of single quantifiers somehow derives from the information that their interaction conveys. If Frege advices in the Introduction to *Grundlagen* to ask for the meaning of a notion only in the context of a whole sentence, Hintikka suggests that, in the case of quantifiers, the favoured kind of sentence has to implement the basic form $(\forall x)$ $(\exists y)$ $S[x,y]$. The proposals put forward by Frege and Hintikka present patent divergences, and thus they cannot give both *the* logic of language.

But what does exactly mean to be "the logic of language"? Frege introduced the ideography that gives rise to contemporary logic by departing from grammatical intuitions. The bearers of logical properties and relations are not, in Frege's view, linguistic items. So, there is a sense in which Frege's logic cannot claim to be the logic of *language*, if "language" is understood as particular natural languages such as English. In a different sense, the universality he expected his ideography to reach suggests that he was seeking a universal characteristic to express any conceptual relation capable of being represented in (the scientific parcels) of any natural language. Frege intended his ideography to be the logic of *thought*. There are thus two possible interpretations of which is involved in being the logic of language, both have been developed in the past century, and both are relevant for the task of understanding what quantifiers are.

At first sight, the project of a general theory of quantifiers covering all cases faces the difficulty of explaining the proliferation of apparently incompatible accounts that linguists, mathematicians and logicians have developed (see, for instance, Westersthal (1989) and van der Does and van Eijck (1995), for a survey of the different proposals). In view of the diversity, two courses of action are available. The first one is to argue that linguists and logicians don't share their subject matter. Thus, the diversity of linguistic forms identified by some linguists doesn't conflict with the unified treatment that logicians such as Frege and Hintikka favour to explain the meaning of quantifiers. The second one would consist in rejecting the diversity stressed by linguists as merely apparent, and offering arguments for the thesis that that beneath diverse superficial structures, there is a unique linguistic form that is the concern of both logicians and linguists. Related to quantifiers, the debate pluralism vs. monism is recognizable amongst logicians, and also between linguists. Hintikka, on the logicians side, seems to defend that quantifiers are basically a single kind of concept. It is arguable that Frege, on the other hand, identified quantifiers of several kinds. Among linguists, some theorists claim that natural language quantifiers are identifiable by their syntactic category (as for instance Generalized Quantifier Theory), and some others reject this claim. The basic thesis that define Generalized Quantifier Theory (GQT) is that quantifiers are all NPs, all having the form "Det + Noun". Semantically, they are relations between subsets of the Universe. GQT has been very influential, and Westertahl, for instance, claims that "[it] seems fair to say that the dominance of predicate logic in these fields has obscured the fact that the quantifier expressions form a *syntactic category*" (Westerstahl 1989, 2, his emphasis). On the pluralist side, Loebner, for instance, has argued that "NPs are not the only quantificational devices [...]. The converse, however, does not hold either. The NPs treated in the literature can be divided into three subclasses, definite, indefinite and quantificational NPs in the narrower sense (leaving aside a fourth subclass of interrogative NPs)." (Loebner 1987, 181). Van der Does and van Eijck also say: "Explicit quantification can also be found outside noun phrases, in particular in adverbial modifiers. English has explicit adverbs of quantification which run over locations (*everywhere*, *somewhere*, *nowhere*), over periods of time (*always*, *sometimes*, *never*), and over states of affairs (*necessarily*, *possibly*, *impossibly*). Like noun phrase quantifiers, these standard adverbial quantifiers have non standard cousins: *often*, *seldom*, *at least five times*, *more than once*, *exactly twice*, and so on" (Van der Does and van Eijck 1995, 15)

The non-trivial relationships between the outcomes of linguistic theory and the outcomes of logical theory are sometimes obscured by the fact that both, linguists and logicians, debate about *logical form.* The expression "logical form" is ambiguous in the literature in the past century. Its meaning shifts between the Fregean notion of conceptual content, on the one hand, and the linguistic notions of grammatical structure and linguistic meaning, on the other. Thus, there are at least two different notions, all vindicating the label "logical form": conceptual content identified through inferential relations, and (apparent or deep) syntactic structure. Frege explains the object of his research as follows: "In logic, one must decide to regard equipollent propositions as differing only according to form. After the assertoric force with which they may have been uttered is subtracted, equipollent propositions have something in common in their content, and this is what I call the thought they express. This alone is of concern to logic. (...) Once we decide to take this step, we do away at a single stroke with a confused mass of useless distinctions and with the occasion for countless disputes which cannot for the most part be decided objectively. And we are given free rein to pursue proper logical analyses." (Frege 1980, VII/3 [xix/3], Frege to Husserl, 30.10-1.11.1906). Thoughts are the Fregean logical forms. Thoughts are inter-linguistic entities[1] that can be expressed through different linguistic clothes.

On the other hand, linguists use the expression "logical form" with a different purpose, related to particular languages. As Robyn Carston explains:

> "Having identified a particular acoustic (or visual) stimulus as linguistic, this system executes a series of deterministic grammatical computations, or mappings, resulting in an output representation, which is the semantic representation, or logical form, of the sentence or phrase employed in the utterance." (Carston 2002, 57).

Jason Stanley also endorses this use of the notion to refer to the result of a set of "deterministic processes":

[1] Sellars uses this term "inter-linguistic" to explain the status of propositions. Propositions are not linguistic but cannot live outsider some linguistic system or other. See, for instance (Sellars 1968, 100).

> "I will also assume, in this paper, that syntax associates with each occurrence of a natural language expression a lexically and perhaps also structurally disambiguated structure which differs from its apparent structure, and is the primary object of semantic interpretation. In accord with standard usage in syntax, I call such structures logical forms" (Stanley, 2000, 393).

These deterministic processes will have no option but to include certain context-dependent pre-semantic processes as well, like disambiguation and language-selection, since the logical form is "the primary object of semantic interpretation", but we will leave these nuances aside in what follows.

Let's call the former "conceptual logical form" and the latter "linguistic logical form". Conceptual logical forms and linguistic logical forms superficially seem to share some of their defining traits. They both are (or can be) semantically incomplete entities; they are in both cases the input of the processes that finally yield truth bearers; in both cases, they are said to be the bearers of logical properties and also to be made out of concepts; they both contain the items that we need to interpret to give a semantics for our language. A closer look, nevertheless, reveals that the two interpretations correspond to different theoretical entities. The difference is critical. Whereas conceptual logical forms acquire their status by stepping away from the syntactic structure of sentences, linguistic logical forms seek to be as close as possible to syntax. The understanding of logical forms as codifying inferential potential is clear in Frege (Frege 1879/van Heijenoort 1990, 6). In contrast, contemporary linguists –and philosophers of language to some extent, in spite of the diversity of their approaches, systematically manifest their respect for principles such as *Grammatical Constraint* or *Linguistic Direction* (Carston 2002, 185) –all of them referred originally to Jackendoff, that require minimal departure from the superficial syntactic structures of sentences. Contextualists such as Carston and Recanati, who include non-linguistically mandated components in logical forms, still support as a methodological desideratum some principle of linguistic direction. Recanati, for instance, says: "Despite of its merits, however, Davidson's theory is unacceptable. It blatantly violates a methodological constraint which philosophers of language of this century have too often ignored. Since the early days of 'ideal language philosophy', with a few exceptions (like Montague), logically-minded philosophers have been happy to posit 'logical forms' at variance with the superficial 'grammatical form' of the sentences there were dealing with. This policy was justified by a dubious ideology concerning the 'defects' of natural language — an ideology which so-called 'ordinary language philosophers' were quite right to criticize. Many philosophers still think the policy can be justified even though the ideology has been abandoned. However, it is the opposite policy which is justified" (Recanati 2000, 28).

Let us stress the difference explained above somewhat more crudely. Frege's notion of logical form is not *previous* to, or *independent* from, semantics in any way. Our intuitions about the meaning of the concepts involved in our thoughts, about the inferential potential of these thoughts, determine the logical representation associated with our expressions in Frege's ideography. Frege's logical form is *not* the output of certain "deterministic processes" resulting in "the object of semantic interpretation". Only from our 21st century point of view, used to consider Frege's approach to logical no more than an antecedent to the ulterior conjunction of first-order logic and Tarskian semantics, can this distinction be blurred.

A sharp distinction between the two notions of logical form is crucial for the task of sketching a map of the different accounts of quantifiers that have been put forward from the 19th century on. If *conceptual logical form* is independent from grammatical structure to a degree, then the different accounts of quantifiers at hand can be classified into two general classes: (i) those that, following Frege, consider quantifiers as concepts defined by their role, which is independent from the grammatical guise under which they appear in particular languages, and (ii) those that, following Montague (1974), Barwise and Cooper (1981), etc., see themselves as offering a theory of the quantification system of English (and other natural languages). Thus, Hintikka and Sandu are right when they say that: "Given such radically different looking approaches, one might be tempted to diagnose the state of quantification theory as a rampant case of split personality." (Hintikka & Sandu 1994, 115)

Frege's option is clear. Contemporary logic derives from the Fregean decision of abandoning the analysis of judgements in grammatical terms. For Frege, only judgeable content is relevant for logical relations. "A distinction between subject and predicate", it is read in *Begriffsschrift*, "does not occur in my way of representing a judgment. In order to justify this I remark that contents of two judgments may differ in two ways: either the consequences derivable from the first, when it is combined with certain other judgments, always follow also from the second, when it is combined with these same judgments, or this is not the case. The two propositions "The Greek defeated the Persians at Plataea" and "The Persians were defeated by the Greeks at Plataea" differ in the first way. Even if one can detect a slight difference in meaning, the agreement outweighs it. Now I call that part of content that is the same in both conceptual content.

Since it alone is of significance for out ideography, we need not introduce any distinction between propositions having the same conceptual content. [...] Now, all those peculiarities of ordinary language that result only from the interaction of speaker and listener –as when, for example, the speaker takes the expectations of the listener into account and seeks to put them on the right track even before the complete sentence is enunciated– have nothing that answers to them in my formula language, since in a judgment I consider only that which influences its possible consequences. Everything necessary for a correct inference is expressed in full, but what is not necessary is generally not indicated; nothing is left to guesswork." (Frege 1879/van Heijenoort 1990). In (Frege 1892/Geach & Black 1960, 66) Frege insists:

> "Grammarians view subordinate clauses as representatives of parts of sentences and divide them accordingly into noun clauses, adjective clauses, adverbial clauses. This might generate the supposition that the reference of a subordinate clause was not a truth value but rather of the same kind as the reference of a noun or adjective or adverb-in short, of a part of a sentence, whose sense was not a thought but only a part of a thought. Only a more thorough investigation can clarify the issue. In so doing, we shall not follow the grammatical categories strictly, but rather group together what is logically of the same kind."

And also in (Frege 1980, VII/4 [xix/6J], letter from Frege to Husserl, 9.12.1906):

> "It seems to me that an objective criterion is necessary for recognizing a thought again as the same, for without it logical analysis is impossible. Now it seems to me that the only possible means of deciding whether proposition A expresses the same thought as proposition B is the following, and here I assume that neither of the two propositions contains a logically self-evident component part in its sense. If both the assumption that the content of A is false and that of B true and the assumption that the content of A is true and that of B false lead to a logical contradiction, and if this can be established without knowing whether the content of A or B is true or false, and without requiring other than purely logical laws for this purpose, then nothing can belong to the content of A as far as it is capable of being judged true or false, which does not also belong to the content ofB; for there would be no reason at all for any such surplus in the content of B, and according to the presupposition above, such a surplus would not be logically self-evident either. In the same way, given our supposition, nothing can belong to the content of B, as far as it is capable of being judged true or false, except what also belongs to the content of A."

GQT is commonly understood as a development of Fregean theory of quantifiers. Hintikka (see for instance Hintikka & Sandu 1994, 114) shares this widespread view. There are some aspects of Frege's view that might motivate the association of Frege's quantification theory and GQT. Frege understands the existential quantifier as a particular case of the general kind of numerical expressions. GQT has extended the Fregean intuition, and it has classified all cases of expressions of quantity as quantifiers. Nevertheless, considered as general views on quantification concepts, the differences between Frege's view and GQT are deeper than their similarities. For Frege, the bearers of logical properties are *not* linguistic items. Quantifiers are defined by the function they perform in the propositions that contain them. And this fact draws a sharp divide between Frege's project and the project that stems from Mostowski (1957), and goes through Lindström (1966) and Montague (1974) to Barwise and Cooper (1981).

Hintikka's view on logical form does not look as clear cut as Frege's on this particular point. Hintikka insists on the idea that his task is to offer a better semantics for quantified *English* sentences, and one of the big spots of his Game theoretical treatment of quantifiers is represented by the "any-thesis", which undoubtedly is a rule for English (Hintikka 1977-8, 99). In (Saarinen 1979, 49-50) he mentions the results of Montague, Lakoff, and McCawley in the task of giving a theory of quantification for English sentences. And, although Hintikka considers all these views "seriously inadequate" (1974, 50), his proposal is understood as giving a correct account where the others have been astray, all belonging nevertheless to the same general project of giving the semantics of English. Jackendoff is also vindicated on footnote 24 (op. cit., 357). The Parallelism Thesis (see below) he accepts in (cfr. Hintikka 1983) is Hintikka's version of Jackendoff's intuition. If we accepted Hintikka's word at face value, then we should place his proposal within the general realm of mathematical (or at least, formal) *linguistics*, stepping out from the path inaugurating by Frege. Nevertheless, when one goes into the details, things are more complex. Some of the theses on which the originality and depth of Hintikka's approach rests are hardly compatible with any grammaticality constraint. Among these are the following: (i) the characterization of quantifiers as codifying choice functions, (ii) his support to branching quantification, and the identification of the linear ordering, and the Inside-Out Principle (IOP, see below) of semantic interpretation (Hintikka 1983) as sources of the inadequacy of first-order logic, (iii) the rejection of the absolute distinction between first-order and higher-order calculi, relying on the expressive power of IF-logic (Hintikka and Sandu 2007, section XVII, and 1998, 298) , and (iv) the analysis of a syntactic variety of modal expressions, and both of epistemic and doxastic transitive verbs, as some kind of quantifiers, and their treatment within Game Theoretical Semantics (see Saarinen 1979, 41). All these features of Hintikka's proposal clearly indicate that Hintikka is concerned with concepts, which in his case are defined following Wittgenstein by their role (use) in a conceptual system. The meaning of a concept is its role in the system to which it belongs, and the role of quantifiers is choosing "witness individuals". How they appear in particular natural languages is irrelevant.

Wittgenstein, one of Hintikka's *heroes* on the matter, was not primarily concerned with the meaning of English or German sentences. Wittgenstein's take on the attribution of mental states, for instance, encourages us not to be misguided by the transitive nature of the verbs that we usually use to express these concepts (cfr. Hacker 1996, 5 and ff; Hacker 2005, 246), and his arguments to depart from the traditional views on intentionality crucially depend on distinguishing concepts that come presented under the same linguistic guide in natural languages (see, for example, his arguments concerning the *duration* of mental states attribution in PI §638-640 on intention, PI §llxi on 'struck by', PG §12 on understanding, Z §78 on hope, etc.)

The thesis that meaning is use is naturally explained by conceptual role semantics as defended by Sellars, for instance (see Sellars 1968, chapter 2 and 3). At this point, Frege and Wittgenstein share their project on meaning, even though the ways in which they implement it might be incompatible. To this general project belong Russell's and Quine's views[2] and, to our mind, also a significant part of Hintikka's work on quantifiers. The project represented by Montague, Lindström, Barwise and Cooper, Lakoff, etc belongs to a different paradigm.

In a nutshell, the application of the sophisticated web of claims represented by Hintikka's account of quantifiers cannot be intended to cover only English sentences. If Hintikka's account is correct, it has to explain the job of these higher-level concepts also in Spanish and Finnish, for instance. The obvious rejoin that English grammar is similar enough to Spanish grammar as to allow the generalization would need a better argumentative effort and the explicit acknowledgement of something like the Chomskian thesis of the Universal Grammar. And Hintikka is well aware of the difficulty: "Such discrepancies between the quantifiers of logic and the quantifiers of natural language require explanations. The task of supplying them is neither a problem in formal logic alone nor yet a merely linguistic problem, it seems to me." (Saarinen 1979, 32). It is undoubtedly a problem, that still has to be solved.

[2] Although Hintikka explicitly place Russell on a par with Montague, and Quine on a par with Lakoff.

Even thus, sharing a general project for quantifiers doesn't guarantee the compatibility of quantification theory in Frege's logic, on the one hand, and Game Theoretical Semantics and in IF-logic, on the other. In fact, Hintikka considers that Frege was wrong in his characterization of standard quantifiers, and that his own proposal is a refutation of some core postulates of Frege's view. Assessing this claim is the aim of the next section.

3. Some Theses on Quantifiers

In Frege's logic, quantifiers are higher-level concepts. But what does exactly mean to qualify a concept as higher-level? A concept is higher-level, Frege says, if it is "used to assert something of a concept" (1884, § 52). *Being the negation of the number nought*, as Frege explains existence in (1884, § 53), is certainly a second level property. From a logico-syntactic point of view, arguments of higher-level concepts have to be lower-level functions, being functions non-saturated entities. The notion of higher-level concept has also been used with a slightly wider sense that departs from the basic Fregean paradigm. Russell (see, for example Russell 1919/1993, 146), for instance, considered propositional attitude verbs as codifying higher-level concepts, even though propositional attitude verbs have (or might have) complete propositions (i.e. non-saturated entities) as their arguments. Thus a general way of characterizing higher-level concepts from a logico-syntactic point of view is defining them as functions whose arguments are either concepts or propositions. Relevant xxth century logicians have assumed this general view that rests on the understanding of propositions as 0-adic predicables. C.J.F. Williams, for instance, says: "We should take more seriously the notion of a no-place predicable. (I use the term 'predicable' in the sense with which it was endowed by Peter Geach: namely, to describe an expression which can be used to predicate something of something, though it may not be being so used in the current context. Thus '___ is red and green all over' is a predicable, but not a predicate in the context 'Nothing is red and green all over'. The term 'no-place is borrowed from Arthur Prior, who attributes the original idea to Peirce.) A one-place predicable is what you get if you remove one name from a proposition; a two-place predicable is what you get if you remove two names from a proposition; and so on. It is not difficult to see what you get if you remove no names from a proposition: clearly, a no-place predicable. It is equally clear that what you have got is exactly the same as what you started with: a no-place predicable is just a proposition." (Williams 1992, 449). Being higher-level thus means that the device at issue produces predicables out of predicables. Quantifiers are a special kind of higher-level concept that yield (n–m)-adic predicables out of n-adic predicables, (n, m > 0) (see for instance Williams 1992, 450). This way of characterizing them relies on their status as binding devices. That quantifiers are binding devices only means that the number of the argument-places of the concept that is the argument of a quantifier exceeds the number of the argument-places of the expression resulting from attaching the quantifier to it. Understanding quantifiers as choice functions is compatible with taking them to be higher-level functions. Choice functions select an individual indirectly, by looking for instances of concepts, and this fact would be enough to qualify them as "higher-level".

The understanding of quantifiers as binding devices doesn't preclude that they possess other essential features. But this is certainly one of them. Thus, two theses that both Frege and Hintikka share are:

[T1] Quantifiers are higher-level predicables
[T2] Quantifiers are binding devices

Even as binding devices, quantifiers convey information of at least two distinguishable kinds. (i) A quantified sentence is *de jure* the expression of a general thought. Some of its instances may transmit *de facto* general thoughts as well, as when one uses singular terms as representatives of a class. This can be done by the use of free variables or by proforms in natural languages (see *Begriffsschrift*, §1, Frege's explanation of the meaning of variables). (ii) In quantified sentences the anaphoric links between some singular terms are exposed. Sense (i) fits the intuition that logical properties cannot discriminate among individuals: logical claims cannot be about particular objects, except in the sense of being about "witness individuals". Sense (ii) reports their function in discourse unification, by showing how the information about a realm is updated.

Hintikka has shown that besides their role in binding variables, they also are mechanisms to display dependence and independence relations among higher level predicables. This double role requires the identification of two notions of scope (see Hintikka 1997). *Begriffsschrift* was designed to represent in a conspicuous way the inferential relations between judgeable contents. Nevertheless, Fregean conceptual notation not only has the effect of taking to the surface otherwise hidden aspects of the information dealt with. It also has the opposite effect, that of hiding aspects of it. One of the outstanding results of Hintikka's GTS is the distinction between two different and independent notions of scope, intertwined in the standard notion that stems from Frege, and that has been assumed without discussion by standard logic ever since. Hintikka argues that quantifiers perform two distinguishable tasks, that don't always overlap. Quantifiers are binding devices that show anaphoric links, i.e. they inform about dependencies and independencies among singular terms. But, on the other hand, they also represent priority relations among them and with other logical concepts. Thus, Hintikka's approach has the merit of acknowledging quantifiers' functional complexity. The distinction between a binding scope and a priority scope is a decided improvement over Frege's logic, an improvement that extends standard logic to cover complex cases such as Donkey sentences, in which several quantifiers are at stake, and also sentences in which epistemic and modal operators appear together with quantifiers.

Some examples:

1. Every writer likes a book of his almost as much as every critic dislikes some book he has reviewed (Saarinen 1979, 62)
2. If John believes that no student of his class is truly gifted, then none of them is truly gifted.

To [T1] and [T2], Hintikka adds [HT3]

> [HT3] Combinations of quantifiers show priority relations among them.

And a further thesis connected to [HT3] is [HT4],

> [HT4] Combinations of quantifiers make a substantive contribution to the content expressed by the formula in which they occur, in the form of Skolem functions.

Unquestionably, [HT4] is completely alien to Frege's view. Assessing the status of [HT4] is not straightforward. Together with an objectual interpretation of quantifiers, [HT4] is true. If the role of existence under the scope of a universal quantifier can be represented as a Skolem function, then the semantic import of the formula in which both quantifiers appear includes existential quantification over functions. Nevertheless, the objectual view doesn't fit well within a Wittgenstenian semantic setting, as the one Hintikka is trying to promote. Defending an objectual view on quantifiers would be a step back from the pragmatist and dynamic conception of quantifiers as codifying rules of a game of searching and finding. The objectual view dovetails the semantic principles of Tarskian semantics, a position explicitly rejected by Hintikka, and as a matter of fact incompatible with the theoretical background of GTS and IF-Logic. If quantifiers codify rules, their meaning is exhausted by the *movement* of selecting an object. There is no movement linked to the objectual interpretation, which depends on the insights stemming from Tarskian semantics. We will not pursue this topic here.

As we mentioned at the beginning of this section, a particular predicable is higher-level from a semantic point of view if it codifies properties of concepts, as opposed to codify properties of objects. Because the information they help to convey relates to concepts and not to objects, quantifiers are the conceptual tools by means of which a conceptual system deals with generality, i.e. expresses general relations among concepts and their extensions. Thus, [T5]

[T5] Quantifiers are conceptual devices that help express general propositions,

is a further essential feature of Frege's quantifiers. Understanding quantifiers as expressing generality contrasts with the understanding of quantifiers as singular terms (or ingredients of complex singular terms), as it happens in the Aristotelian view, the view that Russell defended in *The Principles of Mathematics* (Russell, 1903 chapter V, § 60 ff.), and GQT. In *Grundlagen*, Frege says: "It is true that at first sight the proposition 'All whales are mammals" seems to be not about concepts but about animals; but if we ask which animal then are we speaking of, we are unable to point to any one in particular. [...]. As a general principle, it is impossible to speak of an object without in some way designating or naming it; [...] I suspect that 'indefinite object' is only another term for concept, and a poor one at that, being self-contradictory" (1884, § 47).

With respect to the dispute on whether quantifiers should be interpretated as general concepts or singular terms, Hintikka stands in between. Likewise, his positions seems half way through between the logical and the linguistic. In (Saarinen 1979, 31), after criticizing Quine's and Montague's proposals, he says: "The 'arbitrary choice' suggestion is an attempt to keep what is good in the analogy between quantifier phrases and singular terms and reject what is bad. What is bad is (among other things) that the analogy easily commits us to treating quantifier phrases as if they referred to constant individuals. What we have seen is enough to show that the 'arbitrary choice' idea does not avoid this difficulty and that it gives rise to others as well." Nevertheless, taking quantifiers to be expressions of generality is perfectly compatible with the notion of witness individual. In 1999, Hintikka writes: "What underlies this rule is the obvious meaning of the quantifier (∃x) in (∃x) S[x] .What it says is that there exists a "witness individual", call it b, such that S[b]." (Hintikka 1999, 284). And he goes on: "What is involved here is nothing stranger than the procedure of a judge who does not know the identity of an alleged perpetrator or does not want to divulge the name of a litigant and decides to refer to her or to him as "Jane Doe" or "Richard Roe". The instantiating terms like our "b" are nothing but logicians' "Jane Does" and "Richard Roes" (op. cit., 284-5).

Our proposal is that Frege and Hintikka share [T1], [T2], [T5]. That said, [HT3] is an genuine improvement over Frege's logic, and although not incompatible with the spirit of Frege's view, its concrete implementation in a first-order calculus is not possible, once [T2] has been added. [HT4] is a deeply original intuition whose compatibility with Hintikka's general approach is not obvious.

4. Existential and universal quantifiers in Frege's work

On Frege's view, Hintikka & Sandu say, "what, e.g., the existential quantifier does in a sentence like (1) (∃x) S [x] is to say that the complex or simple predicate S [x] is not empty" (Hintika and Sandu 1994, 114). Nevertheless, extension size is not the only property of concepts.

In *Begriffsschrift*, only generality is explicitly characterized (1879, § 10), whereas an explicit explanation of existence has to wait until *Grundlagen* (1884, § 46). In both cases, Frege recognizes that the judgement produced by the use of quantifiers gives information about concepts (and not about objects). In the case of the universal quantifier, the universal judgement *implies* some information about objects too: "From such a judgement, therefore, we can always deduce any number we like of judgements with less general content, by substituting something different each time for the Gothic letter" (Frege 1879/Geach & Black 1960, 16). Frege also gives an alternative explanation of the role of the universal quantifier: a quantified formula "signifies the judgement that the function is a fact whatever we take its argument to be" (1879, §10). Then, in *Begriffsschrift* there is a contextual definition of the meaning of the universal quantifier as a rule to produce less-general judgements: if the general judgement is asserted, any of its instances can be asserted too. "Being a fact" is the only predicable included in *Begriffsschrift*, and its connection with assertibility is also explicit in this work (see 1979

§ 2, and the introduction of the judgement-stroke). Frege's characterization is recognizable in Ramsey's: "Variable hypotheticals are not judgements but rules for judging. (...) This cannot be *negated* but it can be *disagreed* with by one who does not adopt it" (Ramsey 1929, 149). The Frege-Ramsey view of (some uses of) universal quantifiers as rules has had continuity in Ryle (Ryle 1950, 250) and Sellars (1968, 78), among others.

Thus, we can add to our list [FT6],

> [FT6] (Some uses of) universally quantified sentences don't express propositions but rules to produce propositions.

The negation of a universally quantified sentence, thus understood, "no for all *a*, X(*a*)", means that "we can find something, say Δ, such that X (Δ) is denied" (1879, § 12), or alternatively that "there are some things that not have the property X" (op. cit.). *Begriffsschrift*'s quantifiers are monadic devices, whereas the operator that occur in Ramsey's variable hypotheticals is binary. No wonder, given that Ramsey's operator is the combination of universal quantifier and conditional that is now the standard interpretation of universally quantified sentences: " 'If I meet a φ, I shall regard it as a ψ'" (Ramsey, *loc. cit.*).

Existence and numerical expressions convey information about the size of the extension of their arguments: "If I say", Frege explains, "'Venus has 0 moons', there simply does not exist any moon or agglomeration of moons for anything to be asserted of; but what happens is that a property is assigned to the concept 'moon of Venus', namely that of including nothing under it. If I say 'the King's carriage is drawn by four horses', then I assign the number four to the concept 'horse that draws the King's carriage" (Frege, 1884, § 46). The existential quantifier is, in *Grundlagen*, a monadic higher-level concept. Then we add a new thesis [FT7],

> [FT7] Affirmative existential sentences convey the information that some concept extension is not empty.

The explanation Frege offers in *Grundlagen* about how universally quantified sentences should be interpreted is different, though. The basic three possible relations between objects and concepts are (i) the falling of an object under a concept, (ii) the falling of concept under another of higher-level, and (iii) "the subordination of species to genus" (Frege 1884, § 53), i.e the subordination of a concept to another one. In this latter case, the quantified sentence convey the information that the extension of one of them is included in the extension of the other. This information is standardly represented by the use of the universal quantifier. Thus, there is a further thesis, [FT8],

[FT8] Binary universal quantifiers express concept subordination.

In "On Concept and Object", (Frege 1892 / Geach & Black 1960, 42-56), Frege insists on [FT8] and extends it to cover existentially quantified sentences: "It must be remarked", Frege says, "that the words 'all', 'any', 'no', 'some' are prefixed to concept-words. In universal and particular affirmative and negative sentences, we are expressing relations between concepts; we use this words to indicate the special kind of relation" (op. cit., 48). The adicity of a concept, i.e. the number of its argument-places, is an *essential* property. Thus, monadic quantifiers as characterized in (1879) and binary quantifiers as characterized in (1892) cannot be the same kind of concept. If quantifiers are higher-level *relations*, their meaning cannot be given by a monadic function (see, for instance, Westerstahl 1989, 12). [FT8] pushes forward the intuition that logic deals with general propositions and not with matters of fact, logic don't say anything about individuals but codifies conceptual relations.

Frege distinguishes between the relation of an object to a first-level concept that it *falls under*, and the relation of a first-level concept to a second-level concept that it *falls within* (see, for example, Frege 1892/ Geach & Black 1960, 50-51). Only the latter corresponds to the kind of relation that is established between a monadic quantifier and the first-level concepts needed to saturate it. Both relations, *fall under* and *fall within*, are sometimes grouped under a single label, *subsumption*, which also needs to be distinguished from the relation of *subordination* of a concept under a concept, marked by the presence of binary quantifier (see, for example, Frege 1895/ Geach & Black 1960, 93; Frege 1980, letter from Frege to Husserl 30.10-1.11.1906; Frege 1980, letter from Frege to Marty 29.8.1882). Perhaps one of the passages where this idea is most clearly expressed is the following one from the *Foundations of Geometry*:

> "Whatever is greater than 1 is a positive number. The first grammatical proposition actually takes the place of the subject, and the second contains the predicate belonging to it. From this it is also clear that logically speaking we have only a single proposition. Here we do not have a relation between thoughts, but the relation of subordination of the concept greater than 1 under the concept positive number." (Frege 1906/1984, 309)

Binary quantifiers, in this sense, do not contribute an extra component to the proposition expressed, as other first and second level concepts do. They express the relation of subordination between the concepts under their scope.

The interpretation of the existential operator as a numerical expression precludes the literal correction of the claim made in (1892) that makes it express "relations between concepts". Nevertheless, Frege's intuition is clear. In the case of particular affirmative sentences with two first level predicables, existence is predicated of their intersection, in particular negative sentences with two first level predicables the information conveyed is that the intersection is empty. In both cases, the intersection is a single set, and thus existence is attached to a single argument.

> [FT9] Affirmative existential sentences in which more than one first-level concept occur conveys the information that the intersection of their extensions is not empty.

[FT7], [FT8] and [FT9] is what is shown in the algebraic standard translation of quantified sentences:

(1) "$\exists$x Px" $\Rightarrow$ "$P \neq \varnothing$"

(2) "$\exists$x (Px & Qx)" $\Rightarrow$"$P \cap Q \neq \varnothing$"

(3) "$\forall$x (Px $\rightarrow$ Qx)" $\Rightarrow$"$P \subseteq Q$",

in which P and Q are predicables, and *P* and *Q* are their extensions.

Subordination of concepts doesn't have existential import, but extension size and non-exceptionality do. Thus existential import is related to the step from concepts to theirs extensions, which is linked to monary quantifiers, and is alien to binary quantifiers which express relations among concepts. Existential quantifiers being monary concepts, according to Frege, they always have existential import. Monary universal quantifiers might have existential import too, if they were used to assert that every individual in the universe possesses a particular property. This sense is related to the idea of assertibility of instances that is defended in *Begriffsschrift*.

The (alleged) inter-definition of the existential and universal quantifiers via negation obscures the fact that both kinds of quantifiers have uses in which they are more dissimilar than what the eye catches.

5. Compositionality

In 2001, Sandu and Hintikka (Sandu & Hintikka 2001) quite effectively argued that Tarskian semantics and truth-conditional semantics for intensional logics are unable to hold one of the three possible senses of compositionality –compositionality i:

> (Compositionality i) The applicability of a semantic attribute A to a complex expression E is completely determined by the applicability of this particular attribute A to the component expressions of E plus the way they are combined into A.
>
> (Compositionality ii) The applicability of A to E is determined by the applicability of all and sundry semantic attributes to the component expressions of E plus the structure of E in terms of these component expressions.
>
> (Compositionality iii) Is the result of adding some restrictions on the range of meaning attributes able to satisfy (compositionality ii).

Semantics for IF-languages, they conclude, cannot be made to accord with (compositionality iii), if certain "natural" restrictions are added, while it complies successfully with (compositionality ii). Compositionality as it is usually used to mark theoretical boundaries, that is (compositionality ii), does not seem to be here, then, a way to differentiate semantics for IF-languages from Tarskian semantics.

Hintikka's take on compositionality has not ever been that parsimonious, though. One of his main criticisms against the view that quantifiers are higher-level predicates is closely related to his views on compositionality. The theory of generalized quantifiers, widely considered as the best known development of Frege's analysis of quantifying expressions lacks the technical resources to provide an appropriate account for a number of cases, which turn out to be perfectly accounted for within IF-Logic's approach to quantifiers as choice functions (cfr. Hintikka 1983; Hintikka 1997; Hintikka 1994). Branching quantifiers and the two notions of scope, the conceptual tools that IF-Logic provides for the analysis of quantified expressions as choice functions, cast a shadow on context-independence, a crucial assumption of the higher-order view.

Context-dependence pose a problem for the higher-level view on quantifiers, as it is considered to be incompatible with compositionality. Compositionality's close relation with the higher-level view on quantifiers can be justified through two different paths:

(a) Frege's take on meaning, and correspondingly his analysis of quantifiers, requires compositionality (Hintikka 1983, 259-261).

(b) The theory of generalized quantifiers is developed by Tarski-type truth-definitions, relying heavily on compositionality (Hintikka 1994, 114).

The second path is of no use for the purposes of this paper, since it sheds no particular light on the relationship between Hintikka's and Frege's takes on quantifiers. We have mentioned that, in our view, GQT is built on non-Fregean grounds (and the same can be said of Tarskian semantics). Frege's idea of quantifiers as higher-level operators does not need to be implemented within a Tarskian framework. It is the first path that interests us here in order to determine the connection between compositionality and higher order functions.

Hintikka discusses Frege's endorsement of the principle of compositionality, while exploring the relations between language learnability and a family of close-but-different semantic principles (cfr. Hintikka 1983). Let us introduce the family of relevant notions in a list, before we proceed:

Learnability L: the meaning of a given complex expression E can be gathered from a finite number of clues in E. These clues have to be syntactical.

Compositionality C: the meaning of a complex expression is a function of the meanings of its constituent parts.

Compositionality C*:* the meaning of an expression is its contribution to the meaning of the larger complex expressions in which it can occur.

Principle of Context PC: words have meaning only in a context.

C*ontext-independence CI*: The meaning of an expression must not depend on the context In which It occurs.

Inside-out principle IOP: the proper direction of semantical analysis Is from the Inside out in a sentence or other complex expression.

Parallelism thesis PT: to each syntactical formation rule, telling us how a complex expression E is constructed from simpler ones, say e1. e2 ... en, there corresponds a semantical rule that tells how the meaning of E (the semantical object associated with E) depends on the meanings of those of the simpler Input expressions. e1, e2. . ., en.

The invariance thesis IT: When E is formed from certain simpler strings e1, e2, ..., en these will be constituent parts of W

> *Determinacy thesis DT*: the meaning of E must be completely determined by the meanings of the expressions E1, E2, ..., En from which it is constructed.

Hintikka reconstructs Frege's position in the following way: Frege is committed to C*, which is taken to be equivalent to C, but it needs to be complemented with PC and CI so as to reach a basic level of sub-sentential meanings, that can be used within the building-blocks model that Learnability seems to require. This interpretation of Frege's position is admittedly non-standard (Hintikka 1983, 261). As a matter of fact, it goes against a common assumption in the literature –the idea that there is a palpable tension between principles C and PC. Janssen's diachronic study of Frege's commitment to C shows that Frege can only be said to endorse C, if that at all can be said, as he leaves PC progressively aside (cfr. Janssen 2001). CP is supposed to grant certain amount of context-dependency, which seems to be incompatible with Hintikka's reconstruction of Frege's position.

Besides, the whole purpose of taking C* to serve a project to explain Learnability, which relies ultimately on an atomistic approach to meaning, seems clearly anti-Fregean:

> "A distinction between subject and predicate does not occur in my way of representing a judgment. In order to justify this I remark that contents of two judgments may differ in two ways: either the consequences derivable from the first, when it is combined with certain other judgments, always follow also from the second, when it is combined with these same judgments, [[and conversely,]] or this is not the case. The two propositions "The Greek defeated the Persians at Plataea" and "The Persians were defeated by the Greeks at Plataea" differ in the first way. Even if one can detect a slight difference in meaning, the agreement outweighs it. Now I call that part of content that is the same in both conceptual content. Since it alone is of significance for out ideography, we need not introduce any distinction between propositions having the same conceptual content." (Frege 1879/ van Heijenoort 1990, 12).

As it was later customary (see e. g. Carnap 1934/1937, 120), the primary bearers of content for Frege were complete judgments, items that can be declared true and false, that can be individuated through their inferential potential. It is in the context of this take on content that both C* and PC should be understood. This take on content could not be further away from the atomistic intuitions that motivate the connection between Learnability and Compositionality from Fodor on. Hintikka's reconstruction of Frege's position is not only questionable on historical grounds, but on conceptual grounds alone as well.

Nevertheless, even if one were to admit that Hintikka's reconstruction of Frege's position is without problems, more needs to be said to show why IF-Logic for quantifiers is superior to the higher order approach, with respect to compositionality. Let us begin by spelling out the connections between the principles related to compositionality, as established by Hintikka explicitly in (Hintikka 1983):

(1) *if* (*if* L *then* IT or DT) *then* (*if* L *then* PT)
(2) *if* (PT & IT & DT) *then* IOP
(3) *if* ¬ CI *then* ¬ IOP
(4) CI *iff* C
(5) *if* C *then* PC

(4) and (5) belong to Hintikka's historical reconstruction of Frege's position, so we'll focus on (1)-(3) now. Hintikka brilliantly argues that the analysis of a number of quantified expressions –amongst others in natural language– requires dropping IOP, the inside-out principle of analysis that characterizes, as we mentioned above, both Tarskian semantics, and possible-world semantics for intensional logics. Now, according to (2), PT can hold in a framework were IOP is sacrificed. Negating IOP would only imply that *either* PT *or* IT or DT have to be false. Hintikka is specially interested in retaining "some sort" of PT, as we saw in the previous section, due to his concerns about the relationship between logical form and grammar.

Dropping IOP is the characteristic mark of the tools provided by IF-Logic for the analysis of quantified expressions. Semantics for an IF-Language cannot proceed from the inside out, because the meaning of certain expressions can only be achieved by paying attention to certain other expressions, whose meaning interact with the meaning we are trying to determine *in the inside*. Allowing context-dependence, as IF-Logic does, makes it impossible to have an inside-out semantics. This is explicitly stated in (3).

Consider now the following two theses:

(6) if IOP then C
(7) if ¬IOP then ¬C

The key point here is that, contrary to what Hintikka seems to assume, from (3) and (4) one can only derive (6), but not (7). Only (7), nevertheless, seems to guarantee that IF-Logic's ability to give a proper analysis of certain quantified expressions –by dropping IOP– is going to imply some kind of difference concerning compositionality between IF-Logic and First-order logic. Frege's analysis of quantified expressions is obviously less powerful than the one provided by IF-Logic, but if our only evidence comes from the results of dropping IOP, then we cannot ground that difference on a distinct take on compositionality.

Hintikka's insistence on the superior explanatory power of IF-Logic with respect to a number of natural language expressions is accounted for by the fact that IF-Logic does not require an inside-out, left-to-right analysis, as first-order logic requires. This being the result of the context-dependent character of certain natural language expressions is a completely ungrounded assumption. Only if this were the case IF-Logic's dropping IOP could be taken to imply a difference concerning compositionality.

Of course, (4) can be disputed independently. Perhaps one of the most convincing arguments along these lines is the one provided by Pelletier, to defend that compositionality can be both not formally vauous (Pelletier 1998) and compatible with some sort of context-dependence (Pelletier 2003, 155):

> "Objections concerning the compositionality of a node in a 'logical form tree' merely on the grounds that it contains information that is not a part of any of the subnodes is therefore misguided, unless it can be shown in addition that this information is not due to the manner in which the parts are combined. Or to put the point somewhat differently: so long as there is a function which takes semantic information of the sort possessed by the parts and generates the semantic value of the whole, then this account does not violate compositionality. (It may be an implausible theory for other reasons. But it is nonetheless compositional.)"

But even if (4) is accepted, in order for C to constitute a real difference between these two different theories of quantifiers –higher order functions vs. choice functions–, an extra argument should be provided to defend that IF-Logic's dropping IOP, which concedes this alternative its explanatory superiority over first-order logic, can only be the result of context-dependence. To our knowledge, there is no argument to that effect that avoids begging the question.

Besides, as we saw through the explanation of [FT8] and [FT9] in the previous section, Frege's account of quantified expressions has some *dynamic* aspect to it. The distinction between monary and binary quantifiers determines their meaning, and this process cannot be done taking into account the quantified expression in isolation. In order to determine the semantic value of the quantifiers involved in a certainsentence, we need to determine whether our quantifiers are monadic or binary, since that difference crucially determines their meaning. This decision cannot be addressed just by looking at the quantifiers, it has to be done taking the whole sentence into consideration. Thus, if GTS' dropping IOP is due to the context-dependent nature of quantifiers, and therefore GTS shows the shortcomings of C, in the same vein should we say that Frege's description of the semantic value of quantifiers, as it depends on their adicity, and this is clearly context-dependent, is also incompatible with C. This latter take on the subject will be completely in accordance with Janssen's remarks on Frege and the principle of compositionality (Janssen 2001, see also Pelletier 2001). Hintikka claims that C "in practice" amounts to IOP (Hintikka 1994, 120), but Fregean logic's commitment to C "in practice" only comes together with Tarskian semantics. Our assessment of Frege's insights on quantifiers should not be automatically associated with our take on Tarskian practitioners.

To sum up, Hintikka's IF-Logic –together with GTS– beats first-order logic –and its Tarskian semantic counterpart– fair and square, since it provides a way to represent the content of several types of natural language constructions which first-order logic finds unavoidably troublesome. The theoretical feature that guarantees IF-Logic success at this respect is dropping IOP. Showing that Frege's view on quantifiers and Hintikka's view differ on their treatment of C is quite a different matter, one that requires further assumptions concerning i) the incompatibility between compositionality and context-dependence, and ii) the origin of IF-Logic's dispense of IOP, and iii) the *static* nature of Frege's treatmeant of quantifiers. Both assumptions turn out to be unnecessary if compositionality is no longer the center of heated debates concerning narrow-minded learnability for natural languages.

6. Conclusion

Hintikka's insights have definitely improved our understanding of quantifiers, and the explanatory success of the titanic framework built upon these grounds –IF-Logic and GTS– cannot be stressed enough. In order to fully assess Hintikka's rather complex views on the matter, nevertheless, it's also useful to point at those features of his position that were somewhat already present from the starting point of the contemporary take on quantifiers, ever since Frege laid the foundations of our discipline.

We have shown that Hintikka's approach is closer to Frege's than it is normally acknowledged on a number of issues concerning the concept of logical form, the second-order nature of Frege's quantifiers, the context-dependency of the meaning of quantifiers, and compositionality. This is the lesson to take home: Frege and Hintikka clearly took different paths to understand the meaning of quantified expressions, they simply parted ways later than many seem to think they had.

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