Short and medium-term evolution of shoreline undulations on curvilinear coasts

Alejandro López-Ruiz, Miguel Ortega-Sánchez⁎, Asunción Baquerizo, Miguel A. Losada
Instituto Interuniversitario de Investigación del Sistema Tierra en Andalucía (IISTA), Universidad de Granada, Avda. del Mediterráneo, s/n. 18006 Granada, Spain

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A B S T R A C T
This article proposes a one-line type model to explain the formation and evolution of shoreline undulations on circular or elliptic curvilinear coasts, such as littoral spits. The model takes into account the variation of the surf zone width stemming from the convergence and divergence of the waves propagating over a conical bathymetry with a small radius of curvature. The alongshore sediment transport varies with the angle formed by the wave crests and the coastline, as well as with surf zone width and sediment grain size. This model was applied to the shoreline undulations observed at the mouth of the River Guadalquivir (Gulf of Cádiz, Spain) and those at El Puntal Spit (Cantabrian Sea, Spain). In the first case, the model was forced with several sets of five-year wave climate simulations. At the El Puntal Spit, three different wave conditions, corresponding to the growth, saturation, and decay stages of the undulations, were simulated. In both cases, a net longitudinal growth of the spits was observed. Despite simplifications, the amplitude and wavelength of the shoreline undulations agree with the observations. Furthermore, both the zone and development time of the shoreline undulations can be estimated. The mechanism proposed for their generation and evolution may be complementary to other mechanisms, such as the instability mechanism of the coastline associated with high-angle waves.

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1. Introduction

Coast morphology has been widely studied by many fields of knowledge such as Geomorphology, Coastal Engineering or Physical Geography. Agents involved in coastal waters include winds, waves, tides, storms and currents, which together provide the energy which shapes and modifies a coastline by eroding, transporting and depositing sediment (Pethick, 1984; Bird, 2000). Among different types of coast, wave-dominated spits have one of the most variable morphologies, as they are constantly shaped by the forcing agents (Zhang et al., 2012). Besides the morphological long-term changes over a geologic timescales (i.e. they can shift landward or seaward), this type of coasts shows changes at event to engineering timescales that range from weeks to a few years, (Cowell and Thom, 1994). These short timescale changes are associated with the prevailing hydrodynamics and with human interventions.

The plan view of many sandy beaches is usually rectilinear or slightly curved. When the beach suddenly changes its orientation, as occurs at the tip of a spit, this frequently leads to one or two permanent or ephemeral undulations in the shoreline with spatial dimensions of 10¹–10³ m. These undulations can propagate towards the tip of the spit (Petersen et al., 2008).

Such oscillations can be observed on shorelines all over the world. For example, on Sylt Island in the southern North Sea (Germany) with dimensions of 500–1000 m (Fig. 1a; Lindhorst et al., 2010); at the spit on the west side of Cape Lookout (North Carolina, USA) with dimensions of 250–1000 m (Fig. 1b; Park and Wells, 2007); at the Arcay Spit (France) with lengths of 500–1000 m (Fig. 1c; Allard et al., 2008); and at Sandy Hook Spit (New York, USA) with lengths of 1000–2000 m (Fig. 1d; Nordstrom et al., 1978).

Two examples on the Spanish coast are the undulations found at Doñana (Fig. 1e), at the mouth of the River Guadalquivir (Gulf of Cádiz), and at the El Puntal Spit in Santander (Medellín et al., 2008, 2009). It seems that at Doñana these features have a wavelength, λ, of 500–2000 m and a crest–trough amplitude, a, of ≃150 m. In contrast, at El Puntal they have a λ of 125–150 m and a crest–trough amplitude of ≃15 m. In neither case has migration been observed, although in the case of El Puntal these features appear and disappear depending on the forcing conditions (Medellín et al., 2008). In the case of Doñana spit, the shoreline features are present in the available photographic evidence. These shoreline undulations are not only characteristic of the evolution of littoral spits, but they can also be observed between the tips of large-scale features as well as in the sudden change of orientation of Carchuna Beach (Granada) (Ortega-Sánchez et al., 2003, 2008) in the Alborán Sea (Fig. 1f).

These examples suggest the existence of shoreline undulations with wavelengths of the order of 10²–10³ m and amplitudes of ≃ λ/10, whose formation, evolution, and dimensions are linked to shoreline contours with a conical bathymetry and sudden changes in orientation with a
variation of the mean orientation, $\Delta\delta$, above $\pi/4 - \pi/3$ (Fig. 1b), as occurs on the front of the spits. The Doñana and El Puntal spits have the same ratio between the mean alignment radius of the coast, $R$ (Fig. 1a), and mean surf zone width, $b$, of $R/b = 1 - 10$. In storm conditions at the Doñana Spit (Figs. 1e, 6), $R \approx 2000$ m and $b \approx 600$ m, whereas at the El Puntal Spit $R \approx 300$ m and $b \approx 250$ m (Fig. 8).

Curvilinear shorelines (i.e. spits) are common features on river mouths and estuaries. These coastal systems are complex and many physical processes may coexist, including waves, tides and river floods (Komar, 1998). Sediment transport, sediment budgets and resulting morphological evolution of tidal inlets have already been widely studied by geomorphologists, coastal engineers and sedimentologists (Castelle et al., 2007). Sedimentary processes and tidal inlet morphology are related to the combined action (balance) of tidal currents and wave-induced currents. Variations in hydrodynamics (flood event, changes in offshore wave conditions, etc.) can generate changes in sediment transport, which will act on the sand volume transferred to the coast downstream, influencing accretion or erosion of adjacent beaches (D’Alpaos et al., 2010). This work is focused on the wave processes dominating the outer part of the spit shoreline where the shoreline undulations are generally found.

To our knowledge, the only reference to these shoreline undulations at spits is Petersen et al. (2008). Their theoretical and experimental study analyzed the shape, dimensions, and growth rate of an accumulating sandy spit, and complemented previous works, i.e. Carr (1965) and King (1972). The idealized case of a spit growing without change of form under a constant wave forcing was considered. They found in their experiments that as the spit develops the shoreline obtains an undulating character similar to the examples shown in Fig. 1. Some of these undulations appeared and vanished over time, but immediately up-drift of the tip of the spit, a more permanent protrusion appeared. As time passed, this protrusion became more pronounced until it eventually overtook the tip of the spit. To avoid the assumption of quasi-uniform conditions, they also used the two-dimensional (depth integrated) numerical model MIKE21 (developed by DHI) to characterize waves, hydrodynamics and sediment transport along the spit. Despite these numerical and experimental results, the physical processes that can generate the shoreline undulations were not studied in detail.

Other shoreline features studied in the literature that are similar to these undulations are the alongshore shoreline sand waves. Stewart and Davidson-Arnott (1988) and Davidson-Arnott and Heyningen (2003) referred to these large-scale features as kilometer-length plan-view humps with cross-shore amplitudes on the order of 100 m that migrate alongshore in the direction of net sediment transport, and are different from the bedforms known as sand waves (Blondeaux, 2001) and others alongshore-migrating features like oblique swash bars or sand bars (Pethick, 1984). They can also be defined as rhythmic shoreline features related to the high angle wave incidence theory (Ashton and Murray, 2006a). Given their similar geometries, the formation and evolution of these two large-scale shoreline features may be affected by similar physical processes or mechanisms.

Since Bruun (1954) identified shoreline sand waves propagating in the same direction as longshore transport on the Danish coastline, numerous articles have been published on these shoreline features. Bakker (1968), Verhagen (1989), Guillon et al. (1999) and Ruessink and Jeuken (2002) analyzed shoreline sand waves on the Dutch coastline in the North Sea. In this case, according to Ruessink and Jeuken (2002), the wavelength was about 3.5–10 km and the amplitudes were small, ranging from 20 to 50 m. Their celerity ranged from 0 to 0.2 km yr$^{-1}$.
Ashton et al. (2001) explained the presence of shoreline sand waves on rectilinear coastlines as the result of an instability mechanism based on the coupling between littoral drift and coastal morphology induced by high-angle waves (offshore waves approaching at an angle greater than 43° with respect to shoreline orientation). This seems to confirm the presence of these features on the Dutch coastline (Ashton et al., 2003; Falqués, 2006) and on the capes of the North Carolina coast.

Moreover, Ashton and Murray (2006b) stated that the instability mechanism might play an important role in the behavior of the Long Point shoreline sand waves observed by Stewart and Davidson-Arnott (1988) and Davidson-Arnott and Heyningen (2003). However, as Falqués and Calvete (2005) pointed out, the instability caused by high-angle waves has a preferred wavelength at which shoreline features initially develop. This wavelength, \( \lambda \), scales with the width of the surf zone, \( b \), but with a large factor \( \lambda/b \approx 40-150 \). This is typically about 4–15 km, i.e., one or two orders of magnitude larger than the spacing of surf zone rhythmic features. Nevertheless, given the similarity between both morphologies, the wave angle seems to play an important role in the development of spit undulations.

Medellín et al. (2008) analyzed coastline sand waves at the El Puntal Spit. The presence of only one or two shoreline sand waves, their smaller scale, and their non-propagating condition was different from the observations and predictions previously made at other sites, and were found to be more similar to the shoreline undulations observed at the Doñana Spit. According to Medellín et al. (2008), the formation and growth of shoreline sand waves at the El Puntal Spit occur when the predominant deep-water waves are of moderate obliquity (50°–60°) (in a high-angle wave context) and moderate significant wave height (0.1–0.5 m), and decay with very high wave obliquity (75°–85°) and higher waves (0.5–1.5 m).

The persistence of high-wave obliquity (\( \chi_b > 43° \)) at El Puntal seems to indicate that high-angle wave instability is a possible explanation for the formation of shoreline sand waves. However, Medellín et al. (2008) suggested that the distinctive local wave climate at El Puntal, characterized by small amplitude and long period waves (along with a very steep underlying bathymetry) can explain the much shorter scales in this case. In their opinion, these conditions along with persistent high wave obliquity appear to be the reason why the scales of the developing instability could be similar to the scales of the shoreline sand waves.

This review of the literature reflects that no conclusive explanation has been found for the formation of either the shoreline undulations at the rounded edge of spits or the shoreline sand waves on sandy beaches. The similarity between both shoreline features suggests that the wave climate (especially the wave angle) and the spit bathymetry may play an important role in the development of undulations, as affirmed by Ashton and Murray (2006b) for shoreline sand waves.

This article proposes a one-line type model (Payo et al., 2002) to explain the formation and evolution of shoreline undulations on the prograding fronts of sandy spits with significant changes in coastal orientation \( \Delta \theta > (\pi/4 - \pi/3) \) and a ratio between the mean alignment and the mean width of the surf zone \( \Gamma/b = 1–10 \). It will be shown that these shoreline undulations seem to be generated by the simultaneous effect of the local alongshore variation of energy (due to the morphology of the coastline) and the obliquity of incident waves. The model takes into account the variation in the surf zone width caused by the convergence or divergence of the waves propagating over a conical bathymetry of small radius of curvature. The alongshore sediment transport varies with the angle formed by the wave crests and coastline, with the surf zone width, and with the across shore variation of the sediment grain size. To complement previous experimental and numerical studies (Petersen et al., 2008), this model focuses on the physical processes. Despite simplifications, this model was applied to the Doñana and El Puntal spits and it was found to adequately reproduce the length as well as the amplitude of the undulations observed.

Section 2 provides a summary of the physical processes that occur in transitions from an almost straight to curvilinear coastal stretches. The variation of wave energy conditions is included in the formulation of longshore sediment transport after the average of the Inman and Bagnold (1963) expression. Section 3 presents the results of the study pertaining to the prograding front of circular and elliptical spits. Section 4 discusses the formulation proposed for longshore sediment transport. Finally, Section 5 gives the conclusions derived from this research.

2. Shoreline evolution in a transition from a straight to a curved coast

As waves approach the shore, they shoal and refract, typically reducing their speed (celerity) as their crests become more shore-parallel. There is a progressive increase in wave steepness and wave height/depth ratio until they eventually break onto the beach. As a result, the breaking wave angle and breaking wave height are interdependent. For straight coasts and small deep-water angles, this phenomenon does not induce the appearance of new shoreline features. However, when waves approach with large deep-water angles, the phenomenon is particularly pronounced and affects the coastline (Ashton and Murray, 2006a).

Along curved stretches of coast, the breaking wave height cannot be considered constant. Pocinki (1950) studied wave refraction on a circular island, and showed that in these circumstances, propagation produces a convergence of the rays on the coastal zone nearest to the direction of the incident wave train and a divergence in the opposite direction. Fig. 2 shows the evolution of the ratio of deep-water wave height and breaking wave height on Pocinki’s circular island for waves approaching from the left to the right. In addition, if the type of wave breaking does not change significantly (e.g. spilling and spilling-plunging breakers with Iribarren number \( I_c < 0.1 \)) and if the sediment size is almost constant, the surf zone width should change at the transition from a straight coast to a curvilinear one.

This longshore variation of energy also occurs at spit fronts where the rectilinear coastline becomes curved (see Fig. 3). In what follows, we have adopted a reference frame with the \( X \)-axis parallel to the straight stretch of the coast pointing towards the tip of the spit and the \( Y \)-axis pointing seaward. The angles of incidence were measured counterclockwise from the positive \( Y \)-axis. For waves approaching at small angles of incidence, wave energy concentrates in the zone where the change of coastal orientation occurs. Petersen et al. (2008) observed this effect and suggested that even if the angle of approach for the incoming waves at the base of a spit exceeded 45°, an area of erosion could actually be found along the spit. As the curvature of the

![Fig. 2. Wave propagation on Pocinki’s circular island: evolution of the ratio of deep-water wave height and breaking wave height for waves approaching from the left of the figure (Pocinki, 1950).](image-url)
spit decreases, the sediment transport seems to become more uniform and the erosion areas disappear. This result illustrates the effect of the curvature of the coast on the sediment transport. Furthermore, to compute the growth of the spit, these authors also considered surf zone width as a measurement of the wave energy gradient.

To properly characterize the wave energy variation along a schematic spit, the Ref-Dif model was used to generate different cases of wave propagation over an idealized conical bathymetry. The Ref-Dif is a parabolic, weakly non-linear numerical model that combines refraction and diffraction, and which incorporates shoaling, reflection, energy dissipation, wave breaking, and diffraction effects (Kirby and Dalrymple, 1983, 1994). Ref-Dif disregards reflection effects and, due to some limitations solving diffraction for high angles of incidence, the computational grids were carefully defined. It is therefore capable of accurately reproducing the phenomena under consideration, where shoaling and reflection are dominant (Ashton and Murray, 2006a).

For a high-angle wave approach (Fig. 3a), we observed a progressive decay in the surf zone width towards the tip of the spit. However, when waves arrive approximately normal to the rectilinear stretch, (i.e. with angles of incidence close to zero) or with negative values, the concentration of energy already observed by Petersen et al. (2008) produces a variation in the surf zone width that shows a maximum in the transition zone (Fig. 3b). The exact location of this maximum depends on the angle of incidence. The case shown in Fig. 3a would apply to the El Puntal Spit, whereas at Doñana both situations alternate, as depicted in the wave rose in Fig. 15. In both cases, there is a longshore variation in the sediment transport capacity. To evaluate the importance and significance of this alongshore gradient of the surf zone in the coastline morphology, the Inman and Bagnold (1963) sediment transport formula was adapted to include the variation of the surf zone width.

2.1. Longshore sediment transport and the sediment conservation equation

Our study analyzed the temporal evolution of a curvilinear stretch of the coast. For this purpose, a one-line type model was modified and used to account for the alongshore variation of the width of the surf zone. The reference frame was a curvilinear coordinate system \((s, y)\), with \(s\) parallel to the shoreline and \(y\) perpendicular cross-shore. The hypotheses adopted for the analysis were the following: (1) regular bathymetry with a mean beach slope within the surf zone remains constant along the domain; (2) since wave breaking is depth-limited and the surf zone is saturated, the wave height in the surf zone \(H(y)\), the longshore component of the radiation stress \(S_{xy}(y)\), and the wave energy flux \(E(y)c_g(y)\) are functions that decrease monotonically towards the coast and which are assumed to be continuous with continuous derivatives. Their values can be obtained with the Snel law and linear theory, as exemplified in other works such as Ashton and Murray (2006a); something usual in the definition of one-line type models; (3) sediment size monotonically varies across-shore along the beach according to Dean and Dalrymple (2001); and (4) the turbulent diffusion is negligible. Consequently, and ignoring the slope effects, the longshore current generated by the breaking waves and the associated longshore transport are both parallel to the bathymetry.

The longshore sediment transport equation used in this framework is the Inman and Bagnold (1963) formula:

\[
S_{st} = \frac{K}{(p_s - p)(g(1 - p))} \left( E_{cg} \right) \cos \theta \frac{V}{U_m}
\]

(1)

where \(S_{st}\) is the transport rate; \(V\) is the longshore current caused by the oblique incidence of the waves (Longuet-Higgins, 1970); \(U_m\) is the maximum horizontal orbital velocity of the waves evaluated at the breaker zone; \(p\) is the water density; \(p_s\) is the sediment particle density; \(g\) is the acceleration of gravity; and \(p\) is the porosity of the material. \(K\) is a dimensionless coefficient that depends on the grain size. The sediment diameter variation throughout the beach profile can be accounted for by calculating this coefficient using an across-shore linear expression according to the formula of del Valle et al. (1993). They found an exponential relationship between the coefficient in the CERC formula (USACE, 1984) and the sediment size represented by \(D_{50}\). According to their expression, and in order to obtain an analytical expression for the total sediment transport \(Q\), a linear law with the distance to the shoreline \(y\) was adopted:

\[
K(y) = A_1y + A_2
\]

(2)

This choice is valid for small variations of \(D_{50}\), for which the exponential law can be approximated with a linear function, after assuming a linear dependency of the sediment size on the distance to the shoreline. The angle formed by the wave crests and the coastline is defined as \(\theta = \alpha + \phi_e\), where \(\alpha\) is the angle of the waves and \(\phi_e\) is an angle to correct the Snel law in the curvilinear stretch of the coast that can be approximated by the shoreline angle \(\phi\) (see Appendix A). The wave angle at breaking is \(\phi_b\) (Fig. 4), and its value varies with \(s\), since \(b = b(s)\). This formulation is based on the physical framework chosen by Bagnold (1963) to model sand transport in the nearshore zone. This accounts for the combined effects of waves and currents, where waves place the sand in motion and the longshore current produces the net sand advection (Komar, 1998). The ratio \(E_{cg}/U_m\) is proportional to the mean stress exerted by the waves, and the transport is in the direction of the longshore current. This equation is more fundamental than other empirical correlations with the wave power estimations found in the literature.
The total sediment transport parallel to the coastline is obtained by averaging Eq. (1) per unit width of the surf zone:

$$Q(s) = \frac{1}{b(s)} \int_0^{\rho(s)} S_{uw}(s, y) dy$$

$$= \frac{1}{b(s)} \int_0^{\rho(s)} K \left(E_0(c_0(y)) \cos(\theta(s, y)) V(s, y) \right) dy$$

(Solving Eq. (3) (see Appendix A):

$$Q(s) = P_1(b(s)) \cdot \cos(2\phi(s)) + P_2(b(s)) \cdot \sin(2\phi(s))$$

where $P_1(b(s))$ and $P_2(b(s))$ are two polynomial functions dependent on the surf zone width and the characteristics of deep-water waves. These polynomials are modulated by trigonometric functions of the angle of the coastline.

If nearshore sediment is conserved, the following equation yields:

$$\frac{\partial \eta}{\partial t} = -\frac{1}{D} \frac{\partial Q}{\partial s}$$

where $\eta(s,t)$ is the position of the shoreline; $t$ is time; and $D$ is the closure depth (Payo et al., 2008). This equation was used to calculate the temporal evolution of shorelines for the prograding spit fronts shown in Section 3. Eq. (5) is solved with a finite differences scheme for Neumann-type boundary conditions.

When Eq. (4) is substituted in Eq. (5) and the chain rule is applied, this yields the following:

$$\frac{\partial \eta}{\partial t} = -\frac{1}{D} \left( P_2(b(s)) \cos(2\phi(s)) - P_1(b(s)) \sin(2\phi(s)) \right) \frac{\partial \phi(s)}{\partial s}$$

$$+ \left( \frac{\partial P_1(b(s))}{\partial b} \cos(2\phi(s)) + \frac{\partial P_2(b(s))}{\partial b} \sin(2\phi(s)) \right) \frac{\partial b}{\partial s}$$

In this equation, the first term in the right member is proportional to the curvature of the coastline, whereas the second term includes the effect of the variation of energy conditions along the coastline. The former dependence is similar to that previously obtained by Ozasa and Brampton (1980), whereas the latter dependence resembles the one given by Ashton and Murray (2006a).

2.2 Surf zone width-dependent sediment transport

In this section the proposed longshore sediment transport expression (Eq. (4)) is compared with the one in Ozasa and Brampton (1980) and the CERC formulation. Fig. 5 shows this comparison for three different shapes of the surf zone width. These shapes are represented in the lower left corner of each graph along with a schematic representation of the coastline. For cases a) and b), the maximum value of $b(s)$ is the same and its position varies from $\phi(s) = 0^\circ$ to $30^\circ$. The values of the corresponding deep-water wave angles are $\alpha_0 = 40^\circ$ and $\alpha_0 = -40^\circ$, respectively. These values are representative of the situations described in Fig. 3. Case c) is a hypothetical situation (constant b) defined to compare the behavior of the three formulations. For all of them, a slope tan$\beta = 0.005$, a wave period $T = 8$ s, and significant wave height in deep water $H_0 = 2.5$ m were defined. These characteristic wave energy conditions ensure that the longshore transport is capable of causing changes in the coastal morphology.
The relative alongshore sediment transport rate obtained with the three expressions shows a similar behavior with the maximum value reached at a similar shoreline position $\phi(s)$. Moreover, the maximum values of all the relationships move towards the tip of the spit as the position of the maximum breaker width shifts in the same direction. For constant $b$, the CERC and the Ozasa and Brampton (1980) expressions coincide since the Ozasa and Brampton (1980) formula is a modification of the CERC formula, which includes the breaking wave height gradient (see Falqués and Calvete, 2005). The modified Imnnan and Bagnold (1963) formula shows a similar trend, but the maximum transport is reached for a smaller shoreline angle of $\phi(s) = 38°$.

For cases (a) and (b), there is an inversion in the sediment transport direction for those points of the coastline at which $\alpha(s) + \phi(s) \geq 90°$. These points do not vary significantly in all of the three cases studied. This indicates that the sediment transport direction is mainly determined by the alignment of the coast and the deep-water wave angle. Finally, the results of the formulation for larger values of $\phi(s)$ should be used with care since the wave refraction was calculated using the Snell law. However, this zone is far away from the transition from a straight to a curvilinear coast, where shoreline undulations are expected to develop.

3. Undulations on the prograding spit front

The shape of the tip of a spit can be either circular (i.e. Sylt Spit, Fig. 1a) or elliptical (i.e. Cape Lookout Spit, Fig. 1c). In this work, two examples corresponding to a circular and an elliptical spit have been selected to study the evolution of the shoreline undulations: Doñana and El Puntal. Doñana characteristic dimensions are on the order of kilometers whereas El Puntal is on the order of hundreds of meters. Human influence during last decades has also been different at both sites. At the Guadalquivir estuary there has been a massive occupation of wetlands that affects the equilibrium of the system. On the contrary, no significant interventions were done at El Puntal since late 80s of the past century. For both sites there are available data from previous studies (Medellín et al. 2008, 2009; Díez-Minguito et al. 2012). The reasons why a spit develops a curvilinear or elliptical shape or it has different characteristic dimensions are out of the temporal scale of the processes analyzed in this work, as this paper deals with short (event) to medium-term evolution of shoreline features instead of the long-term evolution of the spit (Rodríguez-Ramírez et al., 1996). By applying the formulation proposed, this section studies how the shape and dimensions of the spit front influences the generation and evolution of the undulations of the coastline.

3.1. Circular spit front

The aerial photographs taken since 1956 show that the undulations at the Doñana Spit seem to be permanently present though their location, shape, and dimensions vary. Consequently, the generation and evolution of these undulations are not linked to the occurrence of a single storm event. Regarding the dimensions of the spit, it appears that at least a few years of climate forcing are required for these undulations to develop (Falqués and Calvete, 2005). Accordingly, we analyzed the evolution of the shoreline position after five years of wave climate forcing, using the one-line type model developed (Eq. (5)) as well as the methodology in Baquerizo and Losada (2008). This procedure assumes that since morphologic changes in coastal areas are cumulative processes, their response to a certain state is the initial condition for the next sea state. The process can be reproduced for a certain period of time by using the local climate as the forcing mechanism. The local climate is thus simulated as a series of consecutive states and as a morphodynamic model that runs under stationary input climatic conditions.

Wave climate conditions were simulated by using the methodology devised by Solari and Losada (2011). For significant wave height, $H_0$, a non-stationary parametric probability model is used for seasonal variability and a copula-based model for time dependence. The wave peak period and mean direction were obtained with a Vector Autoregressive (VAR) model on the basis of the simulated wave height and the available wave climate data (Solari and Losada, 2011).

The wave climate forcing was provided to the model as a series of three-hour sea states, the first of which started with the initial shoreline. In each subsequent wave state, the shoreline position used was that predicted for the previous state. For each sea state the surf zone width was calculated as proportional to the wave height, whereas the shape of the surf zone depended on the incoming wave direction. According to the results of the Ref–Dif propagations, a different shape was defined for each 10° interval of the deep-water wave angle (Section 2). Furthermore, it was assumed that during the sea state, the possible changes in bathymetry did not significantly affect wave propagation. After repeating the experiment $N$ times, a sample space consisting of $N=500$ equally likely outcomes of the shoreline was obtained and analyzed with probabilistic techniques.

This procedure was applied to a straight stretch of coastline, followed by a curved stretch (Fig. 6) of the same dimensions as the Doñana Spit front. The slope of the surf zone was $\tan \beta = 0.004$ and the sediment size was $D_0 = 0.17$ mm. For the wave climate simulations, we used the data obtained at hindcasting point WASA14718 (36.5° N, 7° W), provided by Puertos del Estado (Ministry of Public Works, Spain, WANA project).

Fig. 6 shows ten representative shorelines after 2.5 years of climate forcing, i.e. halfway through the five-year simulation. As can be observed, undulations are beginning to develop on all of the shorelines. The first undulation is located between longshore positions of 2500 and 3000 m, whereas the second undulation is located between longshore positions of 1000 and 2500 m. Although the location and wavelength of the features are almost deterministic, their amplitudes are variable. Moreover, this sequence of erosion–sedimentation stretches is similar to that obtained in laboratory experiments by Petersen et al. (2008) and to those observed at the Sylt Spit (Fig. 1a).

At the end of the simulations, the shoreline undulations are more pronounced although they remain in the same alongshore position (Fig. 7). The similarity between the shorelines obtained after 2.5 and 5 years of wave climate simulation, as well as the small variability between all the final shorelines, confirm that, in the short term, the model solutions tend to a quasi-steady geometry. Once again, the location and wavelength of the undulations are almost deterministic, and a certain variability is observed in their amplitudes, which range from 50 to 170 m. It was also found that the straight stretch of the coast was eroding, and that there was an accretion zone at the tip of the spit. In other words, the spit showed a net longitudinal growth.

3.2. Elliptical spit front

Medellín et al. (2008) identified two sand–wave formation events at the El Puntal spit (Santander, Northern Spain) with a duration of
approximately two months. Because of the similarities between those features and the ones observed at Doñana, the proposed one-line type model was applied to this spit. The wave climate for the phases of growth, saturation, and decay of these morphologies are different and are also influenced by the presence of the Magdalena Peninsula, which protects the prograding front of the spit from the severe wave conditions of the Cantabrian Sea (Medellín et al., 2008).

An initial coastline without undulations was defined with the same dimensions as the El Puntal spit and a slope of \(\tan \beta = 0.13\). Although the tidal range in the area is 3–4 m, a representative constant mean level was considered for the simulation. The model was run with the mean wave conditions that occurred during the phases of sand wave development described by Medellín et al. (2008), with a total simulation time of 60 days (~20 days/phase). In each stage, high-angle waves from the origin to the tip of the spit were observed. Thus, the surf zone width was defined without an energy concentration in the transition from the straight stretch of the coast to the curved stretch (Fig. 3a).

Fig. 8 shows the result after 20 days of simulation with a wave climate of \(H_0 = 0.8\) m, \(T_p = 12\) s, and \(\alpha_0 = 50^\circ\), which is characteristic of the growth stage of the sand waves. The sediment transport pattern has only one maximum followed by a minimum, which results in a simple shoreline undulation with a wavelength smaller than that at the Doñana spit.

Another 20-day simulation was performed using the final shoreline in Fig. 8 as the initial condition. In this case, the wave conditions defined were \(H_0 = 0.5\) m, \(T_p = 15\) s, and \(\alpha_0 = 60^\circ\), typical of the saturation stage of the sand waves. The result (Fig. 9) shows the development of two shoreline undulations with a wavelength of approximately 200 m and an amplitude of 15 m. These features are very similar to those observed by Medellín et al. (2008) during the two sand-wave development events that were recorded.

Finally, to study the possible disappearance of the shoreline undulations, we simulated the decay stage by forcing the system during 20 days and using the final shoreline obtained in Fig. 9 as the initial condition. The wave climate was defined as \(H_0 = 1\) m, \(T_p = 15\) s, and \(\alpha_0 = 70^\circ\), which were the most energetic and oblique conditions recorded during the sand-wave development events. The results (Fig. 10) show a shoreline without undulations. Furthermore, there is a net longshore growth of the spit and a reduction of its width.

3.3. Comparison with observations

Fig. 11 shows the comparison between model simulations and observations for the two study areas. In the case of Doñana, the geometry of the obtained coastlines is similar to that observed in aerial photographs of Doñana spit in 1956, before the intense human interventions in the estuary (Díez-Minguito et al., 2012) (Fig. 11a); however, there are some differences. Although these differences are, in part, caused by model simplifications, they are mainly due to the fact that the effect of the river floods on river mouth dynamics was ignored. More specifically, this boundary was treated like a sink of sediment in a lagoon where no currents move the sediment.

In the case of El Puntal spit, the available aerial image confirming the events observed by Medellín et al. (2008) in 2006 is compared with the results shown in Fig. 9. The differences between the model results and observations are again due to the model simplifications. For both study areas, the locations, the lengths and the amplitudes of the shoreline undulations are very similar to the observations.

4. Discussion

Eq. (6), which was obtained after considering sediment conservation, includes a term that depends on \(\partial b/\partial x\) as well as a new term that depends on the gradient of the energy conditions along the coast (\(\partial b/\partial s\)). Ashton and Murray (2006a) proposed a formulation equivalent to
Eq. (6), based on deep-water wave parameters. In their equation, under certain conditions a change in the diffusivity sign was produced, which led to the formation of new coastal morphologies. In a one-line model, Payo et al. (2008) also included a new term for the variation in wave height and direction of propagation along the coastline. In the formulation presented here, the term proportional to \( \frac{\partial b}{\partial s} \) is a function of \( T, \phi(s), b(s), \alpha_0 \) and represents the variation in wave energy conditions along the coastline. Unlike the other formulations, Eq. (6) includes a term that depends on the gradient of the shoreline angle, in which no assumption or simplifications of the curvature are made.

If it is assumed that \( \tan \phi(s) = \frac{\partial \eta}{\partial s} \), the first term in Eq. (6) is similar to the diffusive term of other formulations such as the Pelnard-Considéré (1956) equation. To facilitate the comparison of our model with the formulation of Ashton and Murray (2006a), the equivalent diffusivity, \( \Psi_p \), was obtained:

\[
\Psi_p = \left[ \frac{2}{1 + (\frac{\partial \eta}{\partial s})^2} \left( P_2(b(s)) \cos(2\phi(s)) - P_1(b(s)) \sin(2\phi(s)) \right) \right] \tag{7}
\]

For constant \( b \), the Ashton and Murray (2006a) diffusivity, written in the reference system of Fig. 4, is proportional to:

\[
\Psi_{AM} \sim \left[ \cos^{1/5}(\alpha_0 + \phi(s)) \left( \frac{6}{5} \right) \sin^2(\alpha_0 + \phi(s)) - \cos^2(\alpha_0 + \phi(s)) \right] \tag{8}
\]

Fig. 12 shows the dimensionless diffusivity for a curvilinear shoreline (with \( \phi(s) \) ranging from 0° to 90°), which was calculated with Eqs. (7) and (8) for a deep-water angle of \( \alpha_0 = 0° \). The behavior of the Ashton and Murray (2006a) expression and that of the proposed formulation are very similar and only differ in the zone with larger shoreline angles (close to \( \phi = 90° \)). As was mentioned in Section 2, the results in this part of the spit should be treated carefully.

The new term obtained in Eq. (6) depends on the gradient of \( b(s) \). To study the effect of this term on the evolution of the shoreline, we performed ten simulations with a five-year wave climate forcing for the Doñana spit (see Section 3.1). The surf zone width along the coast for each sea state (Fig. 5c) was assumed to have a constant value. Thus, the second term on the right-hand side of Eq. (6) vanishes. Although this situation is hypothetical (see Fig. 3 and Section 2), it
shows the importance of considering the gradient of the surf zone width. The results are depicted in Fig. 13 and can be compared with those in Fig. 7, where a variable surf zone width was considered. It can be observed that for a constant surf zone width, only one smooth undulation is generated. However, as shown in Fig. 7, the combined effect of a curved coast and an alongshore gradient in the surf zone width produces the development of two undulations with wavelengths and amplitudes similar to our observations. Consequently, this new term seems to play a decisive role in the formation and development of these features.

Moreover, the proposed expression for the longshore sediment transport allows inclusion of the variability of the grain size distribution along the beach profile. An exponential relationship between the sediment $D_{50}$ and the $K$ parameter of Eq. (1) was found by del Valle et al. (1993). If the sediment size is a function of the cross-shore coordinate $y$, then $K = K(y)$ and Eq. (3) can be integrated along the surf zone width after inserting the expression for $K(y)$. For example, if the sediment size monotonically varies cross-shore according to Dean and Dalrymple (2001), and if a linear law with the distance to the shoreline $y$ is adopted [$K(y) = A_1 y + A_2$], then it is possible to find an analytical solution for Eq. (3), (see Appendix A). This choice is valid for small variations of $D_{50}$ as the exponential law can be approximated by a linear function. For the case of constant $D_{50}$ along the beach profile, $A_1 = 0$ in the expression for the total longshore sediment transport (Eq. (4)).

Ashton and Murray (2006a) showed that the deep-water angle of wave approach strongly affects plan view coastal evolution. This gives rise to an antidiffusional high-angle wave instability for sufficiently oblique deep-water waves (with angles between wave crests and the shoreline trend larger than the value that maximizes longshore sediment transport, $\alpha_0 = 43^\circ$). The results obtained with the present formula indicate that for the general development of shoreline undulations, it is not strictly necessary for the incident wave angles to be very large (Fig. 14).

**Fig. 12.** Dimensionless diffusivity along a curvilinear shoreline for constant $b$ and $\alpha_0 = 0^\circ$ using the proposed formulation (Eq. (7)) and the Ashton and Murray (2006a) expression (Eq. (8)).

**Fig. 13.** Some final shorelines for Doñana spit after five years of wave climate forcing with a constant value of $b$ for each sea state (colored lines). The black line represents the initial shoreline.
The shape and width of the surf zone also influence and propitiate
the generation of shoreline undulations, even in conditions of low wave
obliquity or normal incidence on curvilinear shorelines. To analyze the
role that the shape of the surf zone width has in shoreline evolution, we
applied the present model to a spit with the same geometry as the circular
spit considered in Section 3.1 for different incoming wave
directions. Fig. 14 shows the distance of the final shoreline position to
the initial shoreline for waves approaching from $\alpha_0 = 40^\circ$, $\alpha_0 = 0^\circ$ and
$\alpha_0 = -40^\circ$ after five months of constant climate forcing ($H_0 = 2.5$ m;
$T_p = 12$ s). In each case, the shape of the surf zone changed (see
Section 2). The results show that undulations are formed for all the
incoming wave directions, although their amplitudes are different, with
the highest values obtained for normal incidence. Furthermore, the
shoreline locations where there is no significant movement after the
simulation are similar for all cases. It can therefore be concluded that for
curvilinear coastlines with a varying surf zone width, $b(s)$, undulations
can be generated under low-obliquity conditions.

The mechanism described in this research for the development of
shoreline undulations considers the recent shoreline evolution in
response to prevailing hydrodynamics and human interventions, but
does not attempt to describe the long-term evolution of the coast.
Therefore, it can be considered as a process-based coastal morphody-
namic model (Zhang et al., 2012). The analysis of the long-term
evolution would require the consideration of many other factors in the
model, as well as Holocene environmental changes (Rodríguez-Ramírez
et al., 1996).

This mechanism can coexist with other processes found in the
literature, such as the one proposed by Ashton et al. (2001) for the
formation of sand waves. Indeed, according to the proposed model,
the formation of shoreline undulations may produce sudden changes in
the orientation of the shoreline where waves approach obliquely, thus
creating the conditions that favor the occurrence of the instability
phenomenon and the development of sand-wave features. Reciprocally,
the sand waves in the bathymetric contours caused by the instability
phenomenon on rectilinear beaches are capable of creating energy
concentration zones that may trigger the mechanism proposed for
shoreline undulations (see Fig. 15a,b).

5. Conclusions

This article proposes a one-line type model to explain the
formation and evolution of shoreline undulations on coastlines with
significant changes in orientation $[\Delta \phi > (\pi/4 - \pi/3)]$ and a ratio
between the mean alignment radius of the shoreline and mean surf
zone width, $R/b = 1–10$, as occurs at the prograding front of littoral
spits. The model includes the variation of surf zone width caused by
the convergence or divergence of waves propagating over a conical
bathymetry with a small radius of curvature and sediment grain size
variation across the surf zone. In relation to this model, the following
conclusions can be drawn.

1. The modified longshore sediment transport obtained varies with the
angle formed by the wave crests and coastline as well as with the surf
zone width. After applying the sediment conservation equation, a
term that depends on the curvature of the shoreline and another
term that depends on the longshore gradient of the surf zone width
were obtained. The latter term plays an important role in the
formation of shoreline undulations on curvilinear coasts.

2. The model was applied to the Doñana Spit (Gulf of Cádiz, Spain)
using a simplified circular geometry with a mean alignment radius of
$R=2000$ m. The simulation of the shoreline evolution after five
years of wave climate simulation shows the formation of two
large-scale undulations with wavelengths and amplitudes similar
to the observations. The results of the 500 simulations performed
indicate that the wavelengths of the undulations are almost
deterministic, whereas their amplitudes show a random behavior.

3. In the case of the El Puntal Spit (Santander, Spain), a simplified elliptic
geometry with a mean alignment radius of $R=300$ m was used. The

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**Fig. 14.** Distance from the initial shoreline for a circular spit in the following cases: $\alpha_0 = 40^\circ$ (black line); $\alpha_0 = 0^\circ$ (gray line); and $\alpha_0 = -40^\circ$ (dashed line).

**Fig. 15.** Aerial photographs taken in 1956 of the shoreline undulations at the mouth of the River Guadalquivir (Southern Spain): (a) area where waves approach the coastline at a very oblique angle; (b) area where waves approach at a slightly oblique angle [the diagram in the corner of (a) shows the location of both zones and the wave rose characteristic of the area].
growth, saturation and decay stages of the sand wave development described by Medellín et al. (2008) were simulated. The results obtained show the formation of two shoreline undulations with the same geometric characteristics as the observations. The model is also capable of reproducing their disappearance.

4. In the two cases described in our study, it was observed that the sequence of sea state conditions simulated, which generated the undulations on the coastline, caused a net longitudinal growth of the spits. This growth can be attenuated by the effect of river discharges and tidal flow.

5. Unlike the instability mechanisms, our model does not require the presence of a previous perturbation of the shoreline for the formation of shoreline undulations since the curvilinear shape of the coast, and thus the existence of a gradient in wave energy conditions, is sufficient to trigger this process. However, the mechanism proposed in this article can be complementary to others such as the instability mechanism of Ashton et al. (2001). Indeed, the variations in shoreline morphology induced by the instability mechanism can generate longshore variations in surf zone width, which can initiate this process. On the other hand, the morphology obtained with our model can provide the obliquity necessary to activate the instability mechanism.

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Appendix A. Longshore sediment transport

Eq. (4) for the longshore transport is obtained by averaging the longshore transport sediment rate given by Inman and Bagnold (1963) over the width of the surf zone:

$$ S_{lt} = \frac{K}{(\rho_s - \rho_p) \rho_p g} \left( E_c \right)_b \cos \theta_b \frac{V}{u_m} $$

(A.1)

where $u_m$ is the orbital velocity of the particles on the seabed; $E$ is the wave energy; $c_g$ is the group celerity; $p$ is the bed porosity; $g$ is gravity; $V$ is the mean of the longshore current in the surf zone; and $\rho, \rho_p$ represent the water and sediment densities, respectively. The term $\theta$ represents the angle between the wave crests and the shoreline in the surf zone; the subscript $b$ indicates its value at the breaking location, and $K$ is a dimensionless coefficient. Komar and Inman (1970) compared this formulation with their field data for longshore sand transport and obtained $K=0.28$. In Eq. (4), the angle is written at each location $y$ of the beach profile as follows (Fig. 4):

$$ \theta(s, y) = \alpha(y) + \phi(s) $$

(A.2)

where $\alpha(y)$ is the wave angle obtained with the Snell law and $\phi(s)$ is a parameter introduced to correct the Snell wave refraction for the curvilinear stretch of the coast. As waves do not have sufficient time to adapt to the shoreline, it can be assumed that $\phi'(s)=\phi(s)$, with $\phi(s)$ being the shoreline angle defined in Fig. 4. After the analysis of the Ref-Dif results and since the shoreline undulations appear in the transition from a straight to a curvilinear coast, the assumption can be considered acceptable.

$\theta(y)$ can be expressed in terms of deep-water waves with the Snell law:

$$ k_0 \sin \alpha_0 = k \sin \alpha $$

(A.3)

where $k_0$ and $k$ are the wave numbers at deep water, $k_0=\frac{\omega}{c}$, and in shallow depths $k = \frac{\omega}{\sqrt{g s_0}}$, with $\sigma=2n/T$, which gives the following expression:

$$ \alpha(y) = \arcsin (c_1 \sqrt{y}) $$

(A.4)

where $c_1$:

$$ c_1 = \sigma \sin \alpha_0 \sqrt{\frac{\tan \beta}{g} $$

(A.5)

Supposing that the longshore current is generated only by the breaking waves, the calculation is the following (Longuet-Higgins, 1970):

$$ V(s, y) = \frac{5}{8} n g \tan^2 \beta \sin \theta(s, y) \frac{1}{\sqrt{g \tan \beta}} $$

(A.6)

where $f=\gamma$ is a friction coefficient and $\tan \beta f$ is assumed to be constant (Komar, 1998).

The wave energy is:

$$ E(y) = \frac{1}{8} \rho g H^2 = \frac{1}{8} \rho g (\gamma \tan \beta y)^2 $$

(A.7)

The group celerity is expressed as follows:

$$ c_g(y) = \frac{c}{2} \left( 1 + \frac{2kh}{\sin 2kh} \right) \sqrt{\gamma \tan \beta y} $$

(A.8)

where $c$ is the wave celerity. The orbital velocity of the particles at the sea bottom is:

$$ u_m(y) = \frac{1}{2} \gamma \sqrt{g \tan \beta y} $$

(A.9)

The sediment diameter variation throughout the beach profile can be accounted for by calculating the value of $K$ using the Eq. (2).

Finally, averaging Eq. (A.1) over the width of the surf zone, the following expression is obtained:

$$ Q(s) = \frac{1}{b(s)} \int_0^b \frac{1}{2} K_1 (A_1 y + A_2 y^{5/2}) \sin (2 \arcsin (c_1 \sqrt{y}) + 2 \phi(s)) \, dy $$

(A.10)

with:

$$ K_1 = \frac{5 \pi \rho p^2 g \tan^2 \beta \sqrt{g}}{32 (\rho_s - \rho_p) p f} $$

(A.11)

This integral has the following explicit solution:

$$ Q(s) = P_1 (b(s)) \cos (2 \phi(s)) + P_2 (b(s)) \sin (2 \phi(s)) $$

(A.12)

where $P_1 (b(s))$ and $P_2 (b(s))$ are polynomials dependent on the surf zone width $b(s)$ given by:

$$ P_1 (b) = \frac{K_2}{B} \left[ -2 (1 - c_1 b)^{1/2} (11 A_1 c_1 (16 + c_1 b (24 + 5c_1 b (6 + 7c_1 b))) + 2 (1 - c_1 b)^{1/2} 11 A_1 c_1 (A_1 (128 + c_1 b (192 + 5c_1 b (48 + 7c_1 b (8 + 9c_1 b)))) + 22 A_1 c_1 (16 + 12b A_1) \right] $$

(A.13)

and

$$ P_2 (b) = K_3 c_1^{5/2} \left[ 5b^{5/2} A_2 (99 - 154 c_1 b) - 7A_1 (-11 + 18c_1 b) \right] $$

(A.14)
with $K_2 = 2/3465$, $K_3 = K'/(2K_a c_0^2)$ and $b = b(s)$ as the breaking zone width.

References


