FisyMat **Programa de Doctorado en Física y Matemáticas**

A FisyMat activity in the framework of the 2013 Mathematics of Planet Earth Initiative

Escuela "Biomat 2013": MATHEMATICS OF PLANET EARTH:

EVOLUTION AND COOPERATION
IN SOCIAL SCIENCES AND BIOMEDICINE

Granada, June 17 -21, 2013

Directors: Miguel. A. Herrero (U. Complutense), Juan Soler (U. de Granada)

Secretary: Juanjo Nieto (U. de Granada)

Lecturers:
Tomás Alarcón (Centre de Recerca Matematica, Barcelona, Spain), **Mario Primicerio** (Dipartimento di Matematica "Ulisse Dini", Università degli Studi di Firenze, Italy), **Koby Rubinstein** (Department of Mathematics, Technion Institute of Technology, Israel), **Karl Sigmund** (Faculty for Mathematics University of Vienna, Austria), **Corina Tarnina** (Princeton University, USA)

A FisyMat activity in the framework of the 2013 Mathematics of Planet Earth Initiative

Programme:

1. AN INTRODUCTION TO STOCHASTIC METHODS IN MATHEMATICAL BIOLOGY (Tomás Alarcón, Centre de Recerca Matematica, Barcelona, Spain)

This short course aims to provide a general overview of stochastic modelling techniques and, in particular, their application to biological problems. In particular the aims of the course are (i) show the students when and why noise is important and the effects it may have on the dynamics of the system which cannot be accounted for by purely deterministic models, (ii) make students familiar with the relevant asymptotic and numerical techniques to study stochastic models, and (iii) expose the students to recent developments in stochastic multiscale models of the dynamics of cell populations.

The course is organised as follows:

1.- Motivation and introduction. This first part aims to motivate why and when stochastic effects are relevant by means of several examples. We will then go on to introduce the basic concepts and notation essential for the rest of the course, in particular, the formulation of the master equation.

2.- Asymptotic techniques. Stochastic models can rarely be solved analytically in closed form and, therefore, we often need to resort to asymptotic and numerical methods for their analysis. This part of the course is devoted to introduce two asymptotic techniques of the WKB type.

3.- Numerical techniques. Here we introduce two numerical (Monte Carlo) methods for the numerical simulation of the master equation, namely, Gillespie's algorithm, also known as the stochastic simulation algorithm (SSA) , and the τ-leap method

4.- Stochastic multiscale modelling of cellular populations. Here, recent developments in the area of multiscale stochastic modelling will be introduced. We shall explain how models of individual cell response to external stimulus can be coupled to a master equation for the cellular population, thus giving rise to a emergent properties at the level of the population dynamics.

2. MATHEMATICAL MODELS FOR SOCIAL CHANGES AND CRIMINOLOGY (Mario Primicerio, Dipartimento di Matematica, Università degli Studi di Firenze, Italy)

Mathematical models and methods are currently used in several countries for planning strategies aimed at contrasting and controlling criminality.
Since the transition from law-abiding to criminal behaviour is tightly related to the social dynamics of the population, the starting point (1st lecture) will be the modellization of the latter, either through compartmental models or by means of integro-differential equations. The dependence on age will be also taken in consideration.

There are three main branches of "mathematical criminology": agent-based models, game theory, population dynamics. We will give (2nd lecture) an introductory description of the use of mathematical methods, with particular emphasis on population dynamics. Some mathematical aspects will be discussed, together with numerical simulations: the case of a typical "triangle model" (criminals, targets, guards) and the different use of contrast policies (social promotion and repression) will be briefly discussed.

The 3rd lecture will be devoted to the discussion of space-dependent problems. In this context, a model based on simulation by means of cellular automata will be presented and its particular application to the case of contrasting tax evasion and to the case of inspections will be analyzed. A final fourth lecture will be devoted to discussing current and future reserach directions in the field.

In summary, this overview will show that mathematics can be instrumental in describing the possible scenarios associated with given policies, in simulating the effects of different choices, and thus in giving the public authorities important tools for implementing suitable choices in terms of social measures and in terms of police forces deployment. Another aim of this basic mini-course is to show that these topics can open a rather new and interesting area of investigation to applied mathematicians. All these aspects will be covered in a "panel discussion" with the participants.

3. MATHEMATICAL CHALLENGES IN MEDICINE (Koby Rubistein, Department of Mathematics, Technion Institute of Technology, Israel).

Lecture1. Mathematical models for the inner ear

Lecture 2. Innovative technology in visual optics

Lecture 3. Database analysis with application to cancer treatment

Lecture 4. Open to discussion with participants, questions about the first 3 talks and future directions..

4. ON THE EVOLUTION OF COOPERATION (Karl Sigmund, Faculty for Mathematics University of Vienna, Austria).

Lecture 1. Social learning and social dilemmas. This lecture introduces the basics of evolutionary game theory. Dynamical systems such as the replicator equation describe the evolution of social behaviour and the effects of social learning. This is illustrated by some of the simplest social dilemmas: situations where selfishness undermines the common good.

Lecture 2. The iterated Prisoner's Dilemma, This lecture deals with the archetype of social dilemmas, and introduces some of the basic strategies, such as Tit For Tat and Win-Stay, Lose-Shift. It shows how even in the simplest scenarios, the outcome of social learning can be surprisingly complex.

Lecture 3. Trust and Threat. This lecture deals with the effect of reputation, using simple economic experiments such as the Ultimatum Game and the Trust Game. It stresses the importance of social information, and shows how to guide human behaviour by incentives (carrot and stick).

A FisyMat activity in the framework of the 2013 Mathematics of Planet Earth Initiative

Lecture 4. Mutual Aid, Coercion and Compulsion. Here, we discuss interactions involving many players, and model the Mutual Aid Game and the Public Good Game. We show how sanctions can promote pro-social behaviour, and why voluntary participation can lead to the emergence of sanctioning institutions.

5. COOPERATION AND THE EVOLUTION OF SOCIAL BEHAVIOR: FROM MULTICELLULARITY TO EUSOCIALITY (Corina E. Tarnita, Princenton University, USA).

Lectures 1-2: Cooperation, Evolutionary Games and Population Structure. Evolutionary dynamics are strongly affected by population structure. The outcome of an evolutionary process in a well-mixed population can be very different from that in a structured population. There have been many attempts to study the effect of population structure on evolutionary and ecological dynamics. These approaches include spatial models in ecology, viscous populations, spatial games and games on graphs. In these lectures I will discuss the role of population structure in the evolution of cooperation and I will present results for both fixed networks as well as dynamical ones. Dynamical networks are those that change as a consequence of the evolutionary process. Finally, I will use the same mathematical tools to derive a general condition for strategy dominance in structured populations.

Lectures 3-4: Cooperation and construction. I will discuss two different types of construction and the implications each has for the understanding of cooperation. The two fundamental operations are ‘staying together’ and ‘coming together’ (aggregation). Staying together means that individuals form larger units by not separating after reproduction, while coming together means that independent individuals form aggregates. Staying together can lead to specialization and division of labor, but the developmental program must evolve in the basic unit. Coming together can be creative by combining units with different properties. Both operations have been identified in the context of multicellularity, but they have been treated very similarly. I will discuss existing approaches to the evolution of multicellularity, which rely on multilevel selection theory. I will then begin to describe a new theory that brings the question of primitive multicellularity on the same level of organization, rather than on multiple levels. I will then argue that staying together and coming together can be found not only in the construction of multicellulalrity, but in fact at every level of biological construction. I will them argue that they face different evolutionary problems and the distinction is particularly clear in the context of cooperation and defection. For staying together the stability of cooperation takes the form of a developmental error threshold, while coming together leads to evolutionary games and requires a mechanism for the evolution of cooperation (eg. Population structure as discussed in Lectures 1 and 2). I will then try to ask what determines which type of construction will be employed as well as what can be achieved with each type.