Mathematical Models for Social Changes and Criminology - 2

a) Continuous social models

b) Basic mathematical criminology

Mario PRIMICERIO

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The integro-differential equation for social dynamics

In the previous lecture we have discussed the following

$$\frac{\partial n(x,t)}{\partial t} = -n(x,t)\int_0^1 \gamma(x,y,t)dy + \int_0^1 \gamma(y,x,t)n(y,t)dy, \quad x \in (0,1), t > 0,$$

$$n(x,0) = n_0(x), \quad x \in (0,1).$$

and assumed that the “scattering kernel” γ might be dependent on the total wealth

$$W(t) = \int_0^X \pi(x,t)n(x,t)dx$$

Possible generalizations

1) Taking **age** into account; this will make the model more complicated from the formal (and numerical) point of view but does not change the mathematical essence of the problem.
2) Considering **x** as a vector-valued variable (e.g. social condition and political position) and define the transition probability accordingly
3) Including, for example a class of criminals

Just a few words about criminals

To incorporate criminals into the model we have to introduce two more mechanisms:
(i) the “recruitment” and “retirement” rate
(ii) the effect on the common richness.

The former will also affect the equation for $n(x,a,t)$.

The second will be different (in some cases irrelevant) according to the type of crime that we want to consider.

Something that cannot be incorporated….

Space dependence.

Pursuit problems etc.

Important for planning strategy of security forces.

Back to the basic case: an approximation with a PDE

We start from the integro-differential equation

$$\frac{\partial n(x,t)}{\partial t} = -n(x,t)\int_0^1 \gamma(x,y,t)dy + \int_0^1 \gamma(y,x,t)n(y,t)dy, \quad x \in (0,1), t > 0,$$

$$n(x,0) = n_0(x), \quad x \in (0,1).$$

and we assume that the kernel γ has ***a short range***

in the sense that it is “negligibly small” for $|x - y| < \delta$
(with $\delta << 1$). For simplicity assume it does not depend on t.

An approximation with a PDE - 2

We expand n(y,t) in the second integral around y=x and write:

$$\int_0^1 \gamma(y,x)n(y,t)dy \cong n(x,t)\int_0^1 \gamma(y,x)dy + \int_0^1 (y-x)\gamma(y,x)dy + 1/2\int_0^1 (y-x)^2\gamma(y,x)dy$$

Consequently the equation becomes

$$n_t = a(x)n_{xx} + b(x)n_x + c(x)$$

An approximation with a PDE

So, we have a parabolic equation in which, for the case of a kernel just depending on (x-y), the term b(x) measures the “skewness” of the kernel, thus inducing a sort of “drift” in the social situation.

Note that, if the social mobility depends on the total wealth, we have an interesting PDE with the coefficients that are given functionals of the solution.

Some features of the model

Assume the kernel has a compact support (order δ, of course) with respect to each variable and e.g. has constant values in the strip above and below the diagonal.

Then, the term $c(x)$ vanishes for x in $(\delta, 1-\delta)$.

This means that the problem has a sort of "boundary layer" that has to be analyzed by matching techniques.

Some open problem

- How to formulate boundary conditions (the equation is not in divergence form)
- How to guarantee that the total dimension of the population is maintained.
- etc.

Now we turn our attention to criminology

But, first we have to clarify which is the goal of our analysis.

No wind is favorable for the helmsman who does not know the route

Ningun viento es favorable para el timonel que no sabe la ruta.

Basic mathematical criminology

AIMS OF MATHEMATICAL CRIMINOLOGY

- identify mathematical methodologies to predict how demographic, social and economic factors can affect criminality, e.g. on the scale of a single urban community
- provide insight into the dynamics of diffusion and development (with respect to space and time coordinates) of criminal behaviour
- how the most important socio-economic mechanisms could be framed for use in relevant mathematical models
- how it is possible to identify strategies to fight criminality

RECENT EXPERIENCE

- A workshop in Firenze supported by the European Union (project New and Emerging Themes in Applied Math.)
- A joint Spanish-Italian research project between Firenze and Madrid (Miguel A. HERRERO, Juan-Carlos NUÑO)
- A special issue of the European J. of Applied Mathematics (EJAM)
- Workshops organized by the UCLA group (A.Bertozzi, P.&J. Brantingham), the PIMS in Vancouver
- Activity of EHESS (H.Bertestycki) in Paris, of CEAMOS (R.Manasevich) in Chile

Modelling criminality

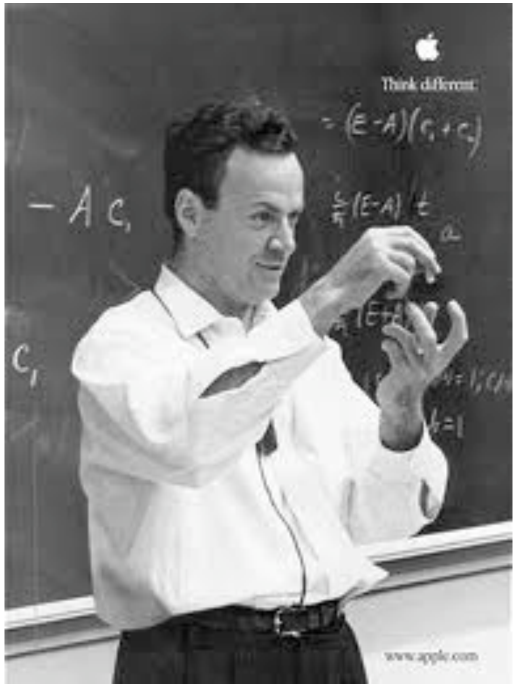
- **Observable:**

  number of crimes (of a given type) per unit time as a function of time and position

- **State variables:**
  - social situation (age and income distribution, mobility)
  - school, housing, social and urban segregation
  - topology of targets, repeated victimisation
  - crime patterns , crime organization etc.

- **Control functions:**
  - police forces and strategy
  - social control
  - law enforcement
  - social policy, etc.

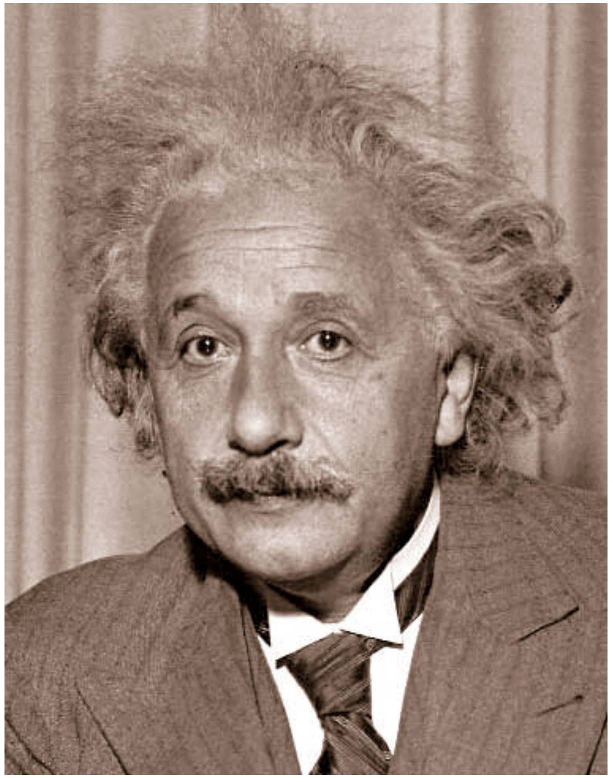
How to proceed ?



- “Transform the problem into one you can solve.”

Richard P. Feynman

Einstein's "golden rule"



- Things are to be made as simple as possible.
- But not simpler

More seriously….

- Use the available **data** with care.
- Consider each mechanism (i.e. the mutual influence of factors) separately.
- Characterize the type of illegal behaviour you want to study as precisely as possible.
- Disaggregating different types of illegal behaviour is not enough
- Of course data related to different crimes (burglary, pickpocketing, drug smuggling, aggression etc.) should be separated, but even within the same class of crimes mechanisms could be different.

An example

- Consider car theft.
- According to Marcus FELSON ("*Crime and Nature*" Sage Publications, 2006) one should consider several types of car theft (with different *modus operandi*, time patterns, offender patterns, etc.) according to the goal of the crime, that could be:
  - For transportation
  - Parts chopping
  - Stealing contents
  - For export
  - Joyriding
  - For another felony

Advantages of modelling

- Although mathematical models might be unable to make quantitative predictions *(e.g. number of burglaries that will happen in a given spatial domain over next month),* they may suggest the qualitative behaviour of a given social system and simulate how the outcomes change when the control functions are changed.

- Thus, they can be used to plan strategies to contrast criminality, to employ available resources in an optimal way.

- They can suggest how to structure and use the

Mathematical models for criminality

There are different approaches to model the evolution and influence of criminality in a society.

The main classes are:

- Agent-based models
- Models based on game theory
- Population dynamics

Models based on game theory

- Mostly used, under many variants, by economists.
- Roughly speaking, the starting point is the evaluation of costs and benefits of crime for each of the *"players"* of the game.
- It includes:

social loss function

social loss from offenses (number and produced harm)

cost of deterrence factors (apprehension and conviction)

probability of punishment per offense

Models based on game theory

For instance, the Nash equilibrium can be sought, in order to answer the following questions:

* how many offenses should be permitted ?
* how many offenders should go unpunished ?

Of course, I decided to skip this part of the lecture….

In any case, I believe that the different approaches should be used in a coordinated way!

Agent-based models

- In principle, an agent-based model should start from the analysis of the behaviour of the criminal (as well as of the victim, the police etc.) and from the knowledge of the external conditions.
- It should consider e.g. the specific *modus operandi*,

  - guess the offender's journey to crime,
  - guess the journey after crime, etc.

Agent-based models

- Then, make a statistics or simulate several scenarios etc.
- It could be useful to study the multiplicative effect of any single crime.
- It has been used (Shane Johnson) to simulate effects of repeated victimization and to map the **hot spots** of crime for a given community.

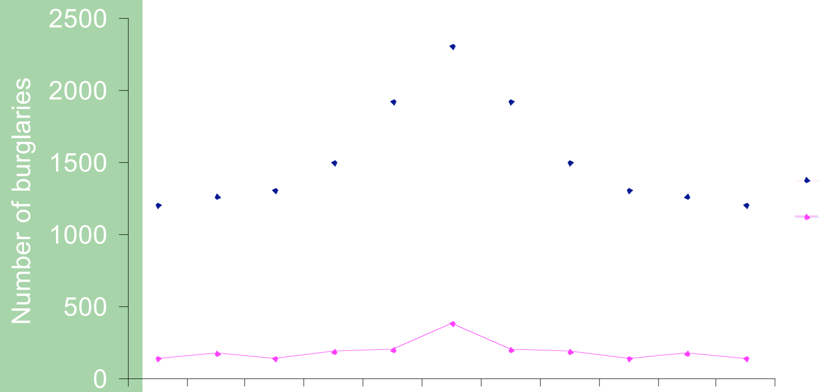
Crime Mapping as Anticipation

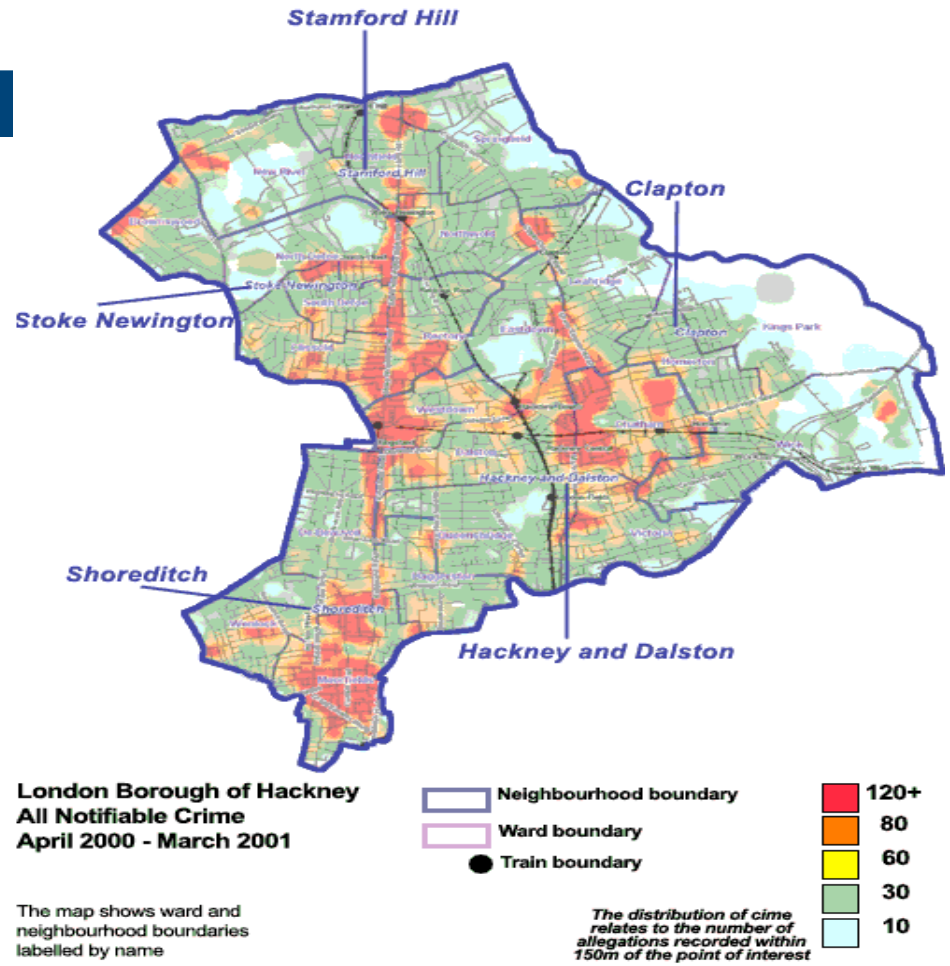
Shane Johnson, Jill Dando Institute

Hot-Spots of Crime

&

Neighbour effects for all housing





Agent-based models (an example)

- Each individual i of a closed society is characterized by two time-dependent state variables: a monthly wage W_i and a honesty level H_i .

- At each time step (month) the number of criminal attempts A(m) is defined as a function of the state variables of the population (e.g. average W and H, number of “intrinsic” criminals” with $H<H_{min}$).

- For each of the A(m) a couple (k,v) of criminal/victim is drawn at random and the probability that the crime is committed is defined in terms of H_k and of W_k. The wealth stolen S is proportional to W_v.

An example (2)

- The effect on W and H of k and v is defined, as well as a probability of punishment, depending on the importance S of the offense, e.g.

$$\pi(S) = \frac{p_1}{1 + [(p_1 - p_0)/p_0]e^{-S/\overline{W}}}$$

Crime punished implies a given increase in honesty level H_i for each i different from k. If crime is unpunished honesty level decreases for all individuals (even more for k!).

Simulations show that the values of the coefficients in the above formula influence dramatically the long-term status of the society.

Methods of population dynamics in criminology

- Probably this is the oldest approach to a mathematical modellization of evolution of criminality in a society

- **Two main categories:**
- “ecological” models
- “epidemiological” models

“Ecological” models

classical example: the “triangle”
(wolves, sheeps, and sheepdogs)

Essentially a predator-prey system with three sub-populations:

Mathematical model: 3 coupled (nonlinear) ODEs

Equilibria and their character (stability, attractivness, etc.)

A special case

Louis G. Vargo: *A note of crime control*, bullettin of mathematical biophysics, volume 28, 1966

$$\begin{cases} \dfrac{dC}{dt} = aC - bCG + A \\ \dfrac{dG}{dt} = -\alpha C + \beta CG + B \end{cases}$$

(C are the criminals, G are the guards)

Vargo's example

The constant terms on the r.h.s. represent the influence of the "external world". Vargo assumes $A = 0$ and B plays the role of a bifurcation parameter.

Setting e.g. $a=\beta=2$ and $\alpha=b=1$, the criminal-free equilibrium point is unstable for $B<2$.

The non-trivial equilibrium is asymptotically stable for $B<2$ and changes character when B crosses a critical value $-8+4\sqrt{2}$

"Ecological" models - 2

Besides of **predator-prey** dynamics, the mechanism of **symbiosis** could be used to model some kind of crimes:

drug smugglers S(t) ⟷ **drug producers P(t)**

Ricardo Azevedo Araujo, Tito Belchior S. Moreira: *A dynamic model of production and traffic of drugs*, Economics Letter 82, pp 371-376, 2004

$$\begin{cases} \dfrac{dS}{dt} = -aS + K(N - P - S)P \\ \dfrac{dP}{dt} = -bP + H(N - P - S)S \end{cases}$$

Araujo’s model: some results

It is immediately seen that

$$KN > a \quad \text{and} \quad HN > b$$

guarantee the existence of a nontrivial equilibrium and the Poincaré-Bendixsson theorem ensures that it is stable and no limit sycles exist.

Some intuitive qualitative behaviour of the solutions can be easily seen.

“Epidemiological” models

Among many similar papers, we can quote

Ormerod,Mounfield,Smith: *Non-linear modelling of burglary and violent crime in the UK*, Volterra Consulting Ltd, 2001

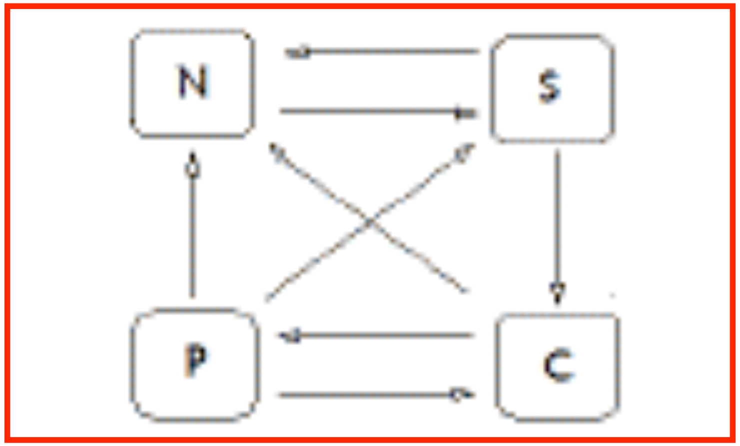
The population is divided in 4 subpopulations:

C = criminals

P = prisoners

S = susceptible (potential C)

N = non-susceptible citizens



The system of ODE's

The transitions between the compartments are defined according to the system of ODE's:

$$\begin{cases} \dfrac{dN}{dt} = -\theta N + \mu S + (A + BN)C + \gamma P + \pi\varphi_2 P \\[2ex] \dfrac{dS}{dt} = \theta N - \mu S - \alpha S - \lambda SC - \gamma P \\[2ex] \dfrac{dC}{dt} = \alpha S + \lambda SC - (A + BN)C - \varphi_1 P + (1 - \pi)\varphi_2 P \\[2ex] \dfrac{dP}{dt} = \varphi_1 C - \varphi_2 P \end{cases}$$

Ormerod’s model

- Calibration of the model using historical data (burglary).
- Discussion of equilibria
- Simulation of different scenarios corresponding to different setups of the parameters (policy)

It can be taken as an example of the possible use (and of the limits) of models based on population dynamics