Plan of the course.

Program

Systems with many particles. Interacting particle systems: How to describe using a reduced number of variables the dynamics of a system of many particles in suitable asymptotic limits.

Questions to be considered for interacting particle systems.

(i) Mean-field theories. Oelschlager results. Derivation of the KS model taking as a starting point a particle model.

(ii) Self-interacting particle systems with non-diffusive interactions. (Othmer-Stevens model).

(iii) Hydrodynamic limits of stochastic systems. (Luckhaus-Triolo model).

(iv) Kinetic equations. Conditions for their validity.

(v) Some problems with correlations of order one.

Lecture 1:

Introduction: On the motion of brownian particles: Independent particles: Discrete equations describing the evolution of individual particles. Diffusion equation. (Solution of the discrete problem in some simple cases). Drifting.

Continuous stochastic processes. Brownian motion. (Wiener measure). Local times. Ray-Knight theorems.

Lecture 2:

Interacting particle systems interacting by means of pair-potentials. Mean field theories:

(i) Deterministic case. (1D case). The Porous Medium Equation as limit.

(ii) Stochastic case. Limit equation. (Including the higher dimensional case).

(iii) Steven's results. Derivation of the KS limit in the mean-field limit case.

(iv) Open problems: Oeschlager problem in the higher dimensional deterministic case.

Lecture 3:

Systems with non-diffusive interactions:

(i) Reinforced random walks. (Self-interacting particle systems).

(ii) Mean-field approach: The Othmer-Stevens model. (Derivation).

(iii) Some mathematical results for the Othmer-Stevens model.

(iv) Which limit of particles gives the Othmer-Stevens model?.

Lecture 4:

Hydrodynamic limits

(i) Basic ideas in hydrodynamic limits: Local equilibrium with slowly varying parameters.

(ii) Oeschlager problem: Deterministic case in the hydrodinamic regime.

(iii) The Luckhaus-Triolo model: A tumour model with two time scales.

Lecture 5:

Miscellaneous:

(i) On the limit of validity of the kinetic equations.

(a) Examples of kinetic models in biological problems: (Amoeba like movements, alignment models).

(b) Smallness of the correlations: A basic condition for the validity of the kinetic models. In gas dynamics such a condition is satisfied for (i) Weak interactions, (ii) Low densities.

(ii) Problems with correlations of order one: Reinforced random walks. (Balin Toth's results, Davies results, Pemantle's representation formula, random media).