Tax and Transfer Programs, Retirement Behavior, and Work Hours Over the Life Cycle: Technical Appendix*

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1 The Bechmark Model Economy

We study an overlapping generations model economy with a continuum of heterogeneous households, a representative firm, and a government, This model economy builds on Díaz-Giménez and Díaz-Saavedra (2015) which we describe below.

1.1 Households

Households[1] in our model economy are heterogeneous and they differ in their age,, $j \in J$; in their education, $h \in H$; in their employment status, $e \in \mathcal{E}$; in their assets, $a \in A$; in their pension rights, $b \in B$, and in their pensions $p \in P$. Sets J, H, $\mathcal{E}$, A, B, and P are all finite sets which we describe below. We use $\mu_{j,h,e,a,b,p}$ to denote the measure of households of type (j, h, e, a, b, p). For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

Population. We assume that the model economy is populated by an exogenous measure of heterogeneous households, μ, whose sum is one.

Age. Every household enters the economy when it is 20 years old and it is forced to exit the economy at age 100. Consequently, $J = \{20, 21, \ldots, 100\}$. We also assume that household faces a conditional probability of surviving from age j to age $j+1$, which we denote by ψ_j. This probability depends on the age of the household but it does not depend on the household's education.

Education. We abstract from the education decision, and we assume that the education of every household is exogenously given. We consider three educational levels and, therefore, $H = \{1, 2, 3\}$. Educational level

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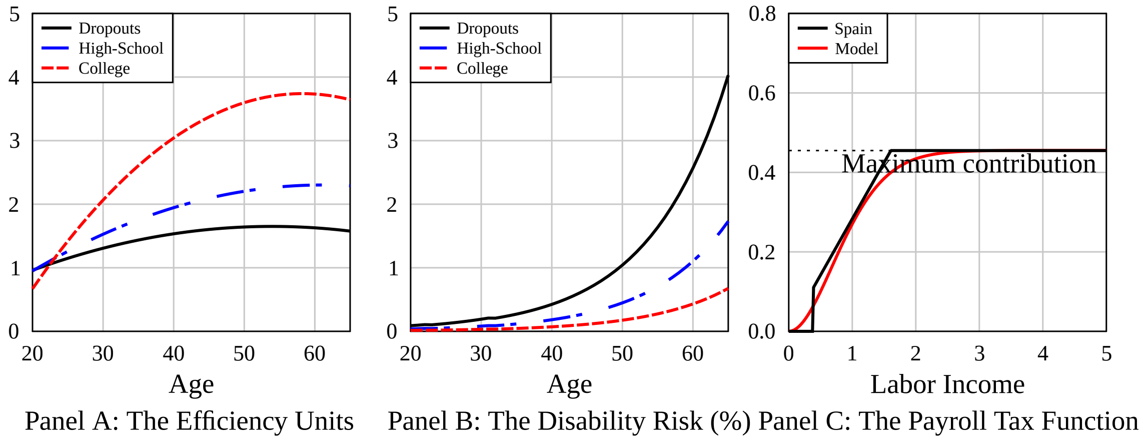
[1]To calibrate our model economy, we use data per person older than 20. Therefore our model economy households are really individual people.

Figure 1: The Endowment of Efficiency Labor Units, the Disability Risk, and the Payroll Tax Function*



* The horizontal axis of Panel C measures labor income as a proportion of Spanish GDP per person who was 20 or older. The vertical axis measures payroll taxes as a proportion of that same variable.

$h = 1$ denotes that the household has dropped out of high school; educational level $h = 2$ denotes that the household has completed high school but has not completed college; and educational level $h = 3$ denotes that the household has completed college.

Employment status. Households in the economy are either workers, retirees, or disabled households. We denote workers by ω, retirees by ρ, and disabled households by d. Consequently, $\mathcal{E} = \{\omega, \rho, d\}$. Every household enters the economy as a worker. The workers face a positive probability of becoming disabled at the end of each period of their working lives. And they decide whether to retire at the beginning of each period once they have reached the first retirement age, which we denote by R_0. In the model economy, both the disability shock and the retirement decision are irreversible and there is no mandatory retirement age.

Workers. Workers receive an endowment of efficiency labor units every period. This endowment has two components: a deterministic component, which we denote by ϵ_{jh}, and a stochastic idiosyncratic component, which we denote by s.

We use the deterministic component to characterize the life-cycle profile of earnings. This profile is different for each educational group, and we model it using quadratic functions on age of the form

$$\epsilon_{jh} = a_{1h} + a_{2h}j - a_{3h}j^2 \tag{1}$$

We choose this functional form because it allows us to represent the life-cycle profiles of the productivity of workers in a very parsimonious way. We represent the calibrated versions of these functions in Panel A of Figure 1.

We use the stochastic component of the endowment shock, s, to generate earnings and wealth inequality within the age cohorts. We assume that s is independent and identically distributed across the households, that it does not depend on the education level, and that it follows a first order, finite state Markov chain with conditional transition probabilities given by

$$\Gamma[s' \mid s] = \Pr\{s_{j+1} = s' \mid s_j = s\}, \text{ where } s, s' \in \omega = \{s_1, s_1, \ldots, s_n\}. \tag{2}$$

We assume that the process on s takes three values and, consequently, that $s \in \omega = \{s_1, s_2, s_3\}$. We make this assumption because it turns our that three states are sufficient to account for the Lorenz curves of the Spanish distributions of income and labor earnings in sufficient detail, and because we want to keep this process as parsimonious as possible[2].

Retirees. As we have already mentioned, workers who are R_0 years old or older decide whether remain in the labor force, or whether to retire and start collecting their retirement pension. They make this decision after they observe their endowment of efficiency labor units for the period. In our benchmark model economy retirement pensions are incompatible with labor earnings[3].

Disabled households. We assume that workers of age j and education level h face a probability φ_{jh} of becoming disabled from age $j+1$ onwards. The workers find out whether they have become disabled at the end of the period, once they have made their labor and consumption decisions. When a worker becomes disabled, she exits the labor market and it receives no further endowments of efficiency labor units, but she is entitled to receive a disability pension until she dies.

To determine the values of the probabilities of becoming disabled, we proceed in two stages. First we model the aggregate probability of becoming disabled. We denote it by q_j, and we assume that it is determined by the following function:

$$q_j = a_4 e^{(a_5 \times j)} \tag{3}$$

We choose this functional form because the number of disabled people in Spain increases more than proportionally with age, according to the *Boletín de Estadísticas Laborales* (2007).

Once we know the value of q_j we solve the following system of equations:

$$\left\{ \begin{array}{lcl} q_j \mu_j & = & \sum_h \varphi_{jh} \mu_{jh} \\ \varphi_{j2} & = & a_6 \varphi_{j1} \\ \varphi_{j3} & = & a_7 \varphi_{j1} \end{array} \right. \tag{4}$$

This procedure allows us to make the disability process dependent on the educational level as is the case in Spain. We represent the calibrated values for φ_{jh} in Panel B of Figure 1. [4]

Preferences. We assume that households derive utility from consumption, $c_{jh} \geq 0$, and from non-market uses of their time and that their preferences can be described by the following standard Cobb-Douglas expected utility function:

$$\max E \left\{ \sum_{j=20}^{100} \beta^{j-20} \, \psi_j \, \varphi_{jh} \, [c_{jh}^{\alpha} (1 - l_{jh})^{(1-\alpha)}]^{(1-\sigma)} / 1 - \sigma \right\} \tag{5}$$

where $0 < \beta$ is the time-discount factor; 1 is the normalized endowment of productive time; and $0 \leq l_{jh} \leq 1$ is labor. Consequently, $1 - l_{jh}$ is the amount of time that the households allocate to non-market activities.

1.2 Technology

We assume that aggregate output, Y, depends on aggregate capital, K, and on the aggregate labor input, L, through a constant returns to scale aggregate production function, $Y = f(K, L)$. We choose a standard

[2]We assume that there are no insurance markets for the stochastic component of the endowment shock.

[3]After the last reforms of Spanish social security, one no longer has to stop working to collect social security benefits. However, the eligibility requirements are tight.

[4]The data on disability can be found at www.empleo.gob.es/es/estadisticas.

Cobb-Douglas aggregate production function with capital share θ. Aggregate capital is obtained aggregating the capital stock owned by every household, and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every worker. We assume that capital depreciates geometrically at a constant rate, δ, and we use r and w to denote the prices of capital and of the efficiency units of labor before all taxes.

1.3 Government Policy

The government in our model economy taxes capital income, household income and consumption, and it confiscates unintentional bequests. It uses its revenues to consume, and to make transfers other than pensions. In addition, the government runs a pay-as-you-go pension system.

In this model economy the consolidated government and pension system budget constraint is

$$G + P + Z = T_a + T_s + T_y + T_c + E \tag{6}$$

where G denotes government consumption, P denotes pensions, Z denotes government transfers other than pensions, T_a, T_s, T_y, and T_c, denote the revenues collected by the capital income tax, the payroll tax, the household income tax, and the consumption tax, and E denotes unintentional bequests. I also assume that Z is thrown to the sea so that they create no distortions in the household decisions.

1.3.1 Taxes

Capital income taxes are described by the function

$$\tau_a(y^a) = a_8 y^a \tag{7}$$

where y^a denotes the income that the households obtain from their assets.

Household income taxes are described by the function

$$\tau_y(y^b) = a_9 \left\{ y^b - \left[a_{10} + (y^b)^{-a_{11}} \right])^{-1/a_{11}} \right\} \tag{8}$$

where the tax base is

$$y^b = y^a + y^l + p - \tau_a(y^a) - \tau_s(y^l) \tag{9}$$

where p is retirement or disability pension, y^l is labor income before taxes, and $\tau_s(y^l)$ are payroll taxes. Expression (8) is the function chosen by Gouveia and Strauss (1994) to model effective personal income taxes in the United States, and it is also the functional form chosen by Calonge and Conesa (2003) to model effective personal income taxes in Spain.

Consumption taxes are described by the function

$$\tau_c(c) = a_{12} c. \tag{10}$$

Finally, we assume that at the end of each period, once they have made their labor and consumption decisions, a share $(1 - \psi_j)$ of all households of age j die and that their assets are confiscated by the government.

1.3.2 The Pension System

In our benchmark model economy we choose the payroll tax and the pension system rules so that they replicate as closely as possible the *Régimen General de la Seguridad Social* of the Spanish pay-as-you-go pension system[5].

Payroll taxes. In Spain the payroll tax is capped and it has a tax-exempt minimum. In the model economy the payroll tax function is the following:

$$\tau_s(y^l) = \begin{cases} a_{13}\bar{y} - \left[a_{13}\bar{y}\left(1 + \frac{a_{14}y^l}{a_{13}\bar{y}}\right)^{-y^l/a_{13}\bar{y}}\right] & \text{if } j < R_1 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where parameter a_{13} is the cap of the payroll tax, $\bar{y}$ is per capita output *at market prices*, and R_1 is the full entitlement retirement age. This function allows us to replicate the Spanish payroll tax cap. In Panel C of Figure 1 we represent the payroll tax function for the calibrated values of a_{13} and a_{14}.

Retirement pensions. A household of age $j \geq R_0$, who chooses to retire, receives a retirement pension which is calculated according to the following formula, which replicates the main features of Spanish retirement pensions:

$$p = \phi(1.03)^v(1-\lambda_j)\left[\frac{1}{N_b}\sum_{t=j-N_b}^{j-1}\min\{a_{15}\bar{y}, y^l\}\right] \tag{12}$$

where the last expression on the right hand side is called *Regulatory Base.* In this expression 12, parameter N_b denotes the number of consecutive years immediately before retirement that are used to compute the retirement pensions; parameter $0 < \phi \leq 1$ denotes the pension system replacement rate; variable v denotes the number of years that the worker remains in the labor force after reaching the normal retirement age;[6] function $0 \leq \lambda_j < 1$ is the penalty paid for early retirement; and $a_{15}\bar{y}$ is the maximum covered earnings. We also model minimum and maximum retirement pensions. Formally, we require that $p_0 \leq p \leq p_m$, where p_0 denotes the minimum pension and p_m denotes the maximum pension.

The Spanish *Régimen General de la Seguridad Social*[7] establishes that the penalties for early retirement are a linear function of the retirement age. To replicate this rule, the choice for the early retirement penalty function is the following

$$\lambda_j = \begin{cases} a_{16} - a_{17}(j - R_0) & \text{if } j < R_1 \\ 0 & \text{if } j \geq R_1 \end{cases} \tag{13}$$

Finally, the Spanish pension replacement rate is a function of the number of years of contributions. The model economy abstracts from this feature because it would require an additional state variable.

Disability pensions. We model disability pensions explicitly for two reasons: because they represent a large share of all Spanish pensions (12.2 percent of all pensions in 2010), and because, in many cases, disability

[5] The Régimen General de la Seguridad Social is the most important pension program in the Spanish Social Security System. For instance, 82.1 percent of the affiliated workers and 54.9 percent of existing pensions belonged to this program in 2010. And we target the Régimen General de la Seguridad Social in place in 2010, as the effects of later reforms have not yet had time to show up in the data.

[6] This late retirement premium was introduced in the 2002 reform of the Spanish public pension system.

[7] The Spanish Régimen General de la Seguridad Social is the most important pension program in the Spanish Social Security System. For instance, 82.1 percent of the affiliated workers and 54.9 percent of existing pensions belonged to this program in 2010.

pensions are used as an alternative route to early retirement.[8] To replicate the current Spanish rules, we assume that there is a minimum disability pension which coincides with the minimum retirement pension. And that the disability pensions are 75 percent of the households' retirement claims. Formally, we compute the disability pensions as follows:

$$p = \max\{p_0, 0.75b\}. \tag{14}$$

Pension Rights and Pensions. We assume that the workers' pension rights belong to the discrete set $B = \{b_0, b_1, \ldots, b_m\}$. Let parameter N_b denote the number of years of contributions that are taken into account to calculate the pension. Then, when a worker's age is $R_0 - N_b < j < R_0$, the b_i record the average labor income earned by that worker since age $R_0 - N_b$. And when a worker is older than R_0, the b_i record the average labor income earned by that worker during the previous N_b years. We assume that $b_0 = 0$, and that $b_m = a_{15}\bar{y}$, where $a_{15}\bar{y}$, and as we said before, denotes the maximum earnings covered by the pension system. We also assume that $m = 9$ and that the spacing between two consecutive points on B is increasing.

We also assume that both the disability and retirement pensions belong to set $P = \{p_0, p_1, \ldots, p_m\}$. The rules of the pension system determine the mapping from pension rights into pensions, and workers take into account this mapping when they decide how much to work and when to retire. Since this mapping is single valued, and cardinality of the set of pension rights, B, was 10, $m = 9$ also for P. Finally, we assume that the distances between any two consecutive points in the pensions set is increasing.

1.4 The Households' Decision Problem

We assume that the households cannot borrow. They do so accumulating real assets, which we denote by a_t, and which take the form of productive capital[9]. Then, the households solve the following decision problem:

$$\max E \left\{ \sum_{j=20}^{100} \beta^{j-20}\, \psi_j\, \varphi_{jh}\, [c_{jh}^{\alpha}(1 - l_{jh})^{(1-\alpha)}]^{(1-\sigma)}/1 - \sigma \right\} \tag{15}$$

subject to

$$c_{jh} + a_{j+1h} + \tau_{jh} = y_{jh} + a_{jh} \tag{16}$$

and where

$$\tau_{jh} = \tau_a(y_{jh}^a) + \tau_s(y_{jh}^l) + \tau_y(y_{jh}^b) + \tau_c(c_{jh}) \tag{17}$$
$$y_{jh} = y_{jh}^a + y_{jh}^l + p_{jh} \tag{18}$$
$$y_{jh}^a = a_{jh} r \tag{19}$$
$$y_{jh}^l = w \epsilon_{jh} s l_{jh} \tag{20}$$
$$y_{jh}^b = y_{jh}^a + y_{jh}^l + p_{jh} - \tau_a(y^a) - \tau_s(y^l) \tag{21}$$

where $a_{jh} \in \mathcal{A}$, $p_{jh} \in P$, and $s \in \omega$. Notice that every household can earn capital income, only workers can earn labor income, and only retirees and disabled households receive pensions.

[8] See Boldrin et al. (1997) for an elaboration of this argument.

[9] For computational reasons we restrict the asset holdings to belong to the discrete set $\mathcal{A} = \{a_0, a_1, \ldots, a_n\}$. We choose $n = 99$, and assume that $a_0 = 0$, that $a_{99} = 75$, and that the spacing between any two consecutive points in set $\mathcal{A}$ is constant.

1.5 Definition of Stationary Equilibrium

Let $j \in J$, $h \in H$, $e \in \mathcal{E}$, $a \in \mathcal{A}$, $b \in B$, and $p \in P$, and let $\mu_{j,h,e,a,b,p}$ be a probability measure defined on $\Re = J \times H \times \mathcal{E} \times \mathcal{A} \times B \times P$.[10] Then, given a set of exogenous demographic parameters $\{\mu_{jh}, \psi_j\}$, a stationary competitive equilibrium for this economy is a government policy, $\{G, P, Z, T_k, T_s, T_y, T_c, E\}$, a household policy, $\{c(j,h,e,a,b,p),\ l(j,h,e,a,b,p),\ a^{'}(j,h,e,a,b,p)\}$, factor prices, $\{r, w\}$, macroeconomic aggregates, $\{C, Y, K, L\}$, and a function, Q, such that:

(i) The government policy satisfies the consolidated government and pension system budget constraint described in Expression (6).

(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r = f_1(K, L) - \delta$ and $w = f_2(K, L)$.

(iii) Given the government policy and factor prices, the household policy solves the households' decision problem defined in Expressions (15), through (21).

(iv) Gross savings, consumption, factor inputs, pension payments, tax revenues, and accidental bequests are obtained aggregating over the model economy households as follows:

$$K = \int a_{jh} d\mu \tag{22}$$

$$C = \int c_{jh} d\mu \tag{23}$$

$$L = \int \epsilon_{jh} s l_{jh} d\mu \tag{24}$$

$$P = \int p_{jh} d\mu \tag{25}$$

$$T_c = \int \tau_c(c_{jh}) d\mu \tag{26}$$

$$T_a = \int \tau_a(y^a_{jh}) d\mu \tag{27}$$

$$T_s = \int \tau_s(y^l_{jh}) d\mu \tag{28}$$

$$T_y = \int \tau_y(y^b_{jh}) d\mu \tag{29}$$

$$E = \int (1 - \psi_j) a^{'}_{jh} d\mu \tag{30}$$

where $y^a_{jh} = a_{jh} r$, $y^l_{jh} = w \epsilon_{jh} s l_{jh}$, and $y^b_{jh} = y^a_{jh} + y^l_{jh} + p - \tau_k(y^k) - \tau_s(y^l)$, and all the integrals are defined over the state space $\Re$.

(vi) The goods market clears:

$$C + \int [a_{j+1h} - (1-\delta) a_{jh}] d\mu + G + Z = F(K, L). \tag{31}$$

The last term of the left-hand side of this expression is not standard, and would show up as net exports in the standard national income and product accounts. This is because transfers other than pensions, Z, show up in this expression are assumed that the government throws them to the sea.

[10] Recall that, for convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

(vii) The law of motion for μ is:

$$\mu^{'} = \int_{\Re} Q(j,h,e,a,b,p,A)d\mu. \tag{32}$$

where set $A \subset \Re$ describes the type tomorrow, for a household who has the type (j,h,e,a,b,p) today.

2 Appendix 1: Calibration of The Benchmark Model Economy

To calibrate the benchmark model economy we choose 2010 as the calibration target year. Then we choose the parameter values that allow the benchmark model economy to replicate as closely as possible selected macroeconomic aggregates and ratios, distributional statistics, and the institutional details of Spain in the target year.

2.1 Parameters

We take the distribution by age and education, μ_{jh}, directly from the Spanish economy published by the *National Institute of Statistics* (INE)[11]. We also take from this source the conditional probabilities of surviving, ψ_j. Then, and to characterize the model economy fully, we must choose the values of a total of 48 parameters. Of these 48 parameters, 3 describe the household preferences, 21 the process on the endowment of efficiency labor units, 4 the disability risk, 2 the production technology, 11 the pension system rules, and 7 the remaining components of the government policy. To choose the values of these 48 parameters we need 48 equations or calibration targets which we describe below.

2.2 Equations

To determine the values of the 48 parameters that identify the model economy, we do the following. First, we determine the values of a group of 29 parameters directly using equations that involve one parameter only. To determine the values of the remaining 19 parameters we construct a system of 19 non-linear equations. Most of these equations require that various descriptive statistics of the benchmark model economy replicate the values of the corresponding Spanish statistics in 2010. We describe the determination of both sets of parameters in the subsections below.

2.2.1 Parameters determined using single equations

The life-cycle profile of earnings. We estimate the values of parameters of the three quadratic functions that we describe in Expression (1), using the age and educational distributions of hourly wages reported by the INE in the *Encuesta de Estructura Salarial* (2010) for Spain. This procedure allows us to identify the values of 9 parameters.

The disability risk. We use the *Boletín de Estadísticas Laborales* (2007) dataset to estimate the values of parameters a_4 and a_5 of Expression (3) using an ordinary least squares regression of q_j on j. And according to the *Instituto de Mayores y Servicios Sociales*, in 2008 in Spain 62.6 percent of the total number of disabled people aged 25 to 44 years old had not completed high school, 26.9 percent had completed high school, and the remaining 10.5 percent had completed college. We use these shares to determine the values of parameters

[11] The data is available at *www.ine.es*.

Table 1: The values of 36 of the model economy parameters

		Parameter	Value
Parameters obtained directly			
Earnings Life-Cycle			
		$a_{1,1}$	0.9189
		$a_{1,2}$	0.8826
		$a_{1,3}$	0.5064
		$a_{2,1}$	0.0419
		$a_{2,2}$	0.0674
		$a_{2,3}$	0.1648
		$a_{3,1}$	0.0006
		$a_{3,2}$	0.0008
		$a_{3,3}$	0.0021
Disability Risk			
		a_4	0.000449
		a_5	0.0924
		a_6	0.4291
		a_7	0.1677
Preferences			
	Curvature	σ	4.0000
Technology			
	Capital share	θ	0.3669
Public Pension System			
	Maximum early retirement penalty	a_{16}	0.4000
	Early retirement penalty per year	a_{17}	0.0800
	Number of years of contributions	N_b	15
	First retirement age	R_0	60
	Normal retirement age	R_1	65
Government Policy			
	Household Income Tax function		
		a_9	0.4500
		a_{11}	1.0710
Parameters determined by guesses for (K, L)			
Public Pension System			
	Payroll tax cap	a_{13}	0.4553
	Maximum covered earnings	a_{15}	1.6089
	Minimum retirement pension	b_0	0.6639
	Maximum retirement pension	b_m	4.6021
Government Policy			
	Government consumption	G	0.7562
	Capital income tax rate	a_8	0.1907
	Consumption tax rate	a_{12}	0.2113
Parameters determined solving the system of equations			
Preferences			
	Leisure share	α	0.2979
	Time discount factor	β	1.0460
Technology			
	Capital depreciation rate	δ	0.0724
Public Pension System			
	Payroll tax rate	a_{14}	0.2385
	Pension replacement rate	ϕ	0.8279
Government Policy			
	Household Income tax function	a_{10}	0.0672
	Government transfers	Z	-0.0807

a_6 and a_7 of Equation (4). Specifically, we choose $a_6 = 0.269/0.626 = 0.4297$ and $a_7 = 0.105/0.626 = 0.1677$. This procedure allows us determine the values of 4 parameters.

The pension system. In 2010 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 44,772 euros of annual gross labor income. Hence, the maximum contribution was 12,670 euros which correspond to 45.53 percent of the Spanish GDP per person who was 20 or older. To replicate this feature of the Spanish pension system we choose the value of parameter a_{13} of the payroll tax function to be $a_{13} = 0.4553$.

Following the the Spanish *Régimen General de la Seguridad Social* we choose $N_b = 15$. We assume that the minimum pension, the maximum pension, and the maximum covered earnings are directly proportional to per capita income, and that the targets for the proportionality coefficients are $b_0 = 0.1731$, $b_m = 1.2567$, and $a_{15} = 1.6089$, as these numbers correspond to their values in 2010 in Spain[12]. We also choose the first and normal retirement ages to be $R_0 = 60$ and $R_1 = 65$. Finally, to identify the early retirement penalty function, we choose $a_{16} = 0.4$, and $a_{17} = 0.08$. This is because we have chosen $R_0 = 60$, and because in Spain the penalties for early retirement are 8 percent for every year before age 65. These choices allow us to determine directly the values of 9 parameters.

Government policy. We choose directly the values of government consumption, G, of the tax rate on capital income, a_8, of parameters a_9 and a_{11} of the household income tax function, and of the tax rate on consumption, a_{12}. We describe the procedure to choose the value of these five parameters in Appendix 2.

Preferences. Of the four parameters in the utility function, we choose the value of $\sigma = 4.0$ directly[13].

Technology. According to the OECD data, the capital income share in Spanish GDP was 0.3669 in 2008. Consequently, we choose $\theta = 0.3669$ directly.

Adding up. So far we have determined the values of 29 parameters directly. We report their values in the first two blocks of Table 1.

2.2.2 Parameters determined using a system of equations

We still have to determine the values of 19 parameters. To find the values of those 19 parameters we need 19 equations. Of those equations, 14 require that model economy statistics replicate the value of the corresponding statistics for the Spanish economy in 2010. The government budget constraint allows us to determine the value of Z residually. And the 4 remaining equations are normalization conditions. In the third block of Table 1 and in the first two blocks of Table 4, we report the values of the 19 unknowns.

Aggregate Targets. We report the values of the 6 Spanish macroeconomic aggregates and ratios that we target in Table 2. According to the Spanish *Encuesta de Empleo del Tiempo (2010)*, the average number of hours worked per worker was 36.79 per week, so our target for the share of disposable time allocated to working in the market is 37.5 percent[14]. Consequently, the *average* Frisch elasticity of labour supply implied

[12] Specifically, in 2010 the minimum retirement pension in Spain was 4,817 euros, the maximum pension was 34,970 euros, the maximum covered earnings were 44,772 euros, and GDP per person who was 20 or older was 27,827 euros.

[13] This choice and the value of the share of consumption in the utility function, imply that the relative risk aversion in consumption is 1.8937, which falls within the 1.5-3 range which is standard in the literature.

[14] If we consider the endowment of disposable time to be 14 hours per day, the total amount of disposable time is 96 hours per week. Dividing 36.79 by 96 we obtain 37.5 percent.

Table 2: Macroeconomic Aggregates and Ratios in 2010 (%)

	C/Y^{*a}	K/Y^{*b}	H^c	T_y/Y^*	T_s/Y^*	P/Y^*
Spain	51.5	3.28	37.5	7.4	10.1	10.3

[a]Variable Y^* denotes GDP at market prices.
[b]The target for K/Y^* is in model units and not in percentage terms.
[c]Variable h denotes the average share of disposable time allocated to the market.

in the model is 0.77, which is in the middle of the range of recent econometric estimates (see, for instance, Fuster et al. (2007)). We describe how we obtain the remaining targets in Appendix 2[15].

Distributional Targets. We target the 3 Gini indexes and 5 points of the Lorenz curves of the Spanish distributions of earnings, income and wealth for 2004. We have taken these statistics from Budría and Díaz-Giménez (2006), and we report them in bold face in Table 3[16]. These targets give us a total of 8 additional equations.

Table 3: The Distributions of Earnings, Income, and Wealth*

		Bottom Tail			Quintiles					Top Tail		
	Gini	1	1–5	5–10	1st	2nd	3rd	4th	5th	10–5	5–1	1
The Earnings Distributions (%)												
Spain	**0.49**	0.0	0.7	1.2	**5.3**	10.9	**16.2**	23.3	**44.3**	10.9	11.5	5.6
Model	0.48	0.1	0.8	1.3	5.2	9.4	13.5	16.0	55.7	17.5	18.1	6.6
The Income Distributions (%)												
Spain	**0.42**	0.0	0.7	1.1	5.1	10.1	15.2	22.5	47.1	11.1	12.8	6.7
Model	0.44	0.1	0.9	1.5	6.3	9.6	13.9	17.3	52.8	14.8	18.3	6.9
The Wealth Distributions (%)												
Spain	**0.57**	–0.1	0.0	0.0	0.9	**6.6**	12.5	**20.6**	59.5	12.5	16.4	13.6
Model	0.57	0.0	0.0	0.0	0.9	6.6	13.2	20.5	58.7	15.7	22.8	6.2

*The source for the Spanish data of earnings, income, and wealth is the 2004 *Encuesta Financiera de las Familias Españolas* as reported in Budría and Díaz-Giménez (2006). The statistics in bold face have been targeted in the calibration procedure.

The Government Budget. The government budget is an additional equation that allows us to obtain residually the government transfers Z.

Normalization conditions. Finally, in the model economy there are 4 normalization conditions. The transition probability matrix on the stochastic component of the endowment of efficiency labor units process is a Markov matrix and therefore its rows must add up to one. This gives us three normalization conditions. We also normalize the first realization of this process to be $s(1)=1$.

[15]The Frisch elasticity in this case varies over the life-cycle and depends on the ratio of leisure to work hours over the life-cycle. Consequently, we compute the *average* Frisch elasticity as $\frac{1-\overline{h}}{\overline{h}}\frac{1-\alpha(1-\sigma)}{\sigma}$, where $\overline{h}$ is the average time devoted to market activities over the life cycle.

[16]Castañeda Díaz-Giménez and Rios-Rull (2003) argue in favor of this calibration procedure to replicate the inequality reported in the data.

Table 4: The Stochastic Component of the Endowment Process

		Transition Probabilities			
	Values	$s' = s_1$	$s' = s_2$	$s' = s_3$	$\pi^*(s)^a$
$s = s_1$	1.0000	0.9417	0.0582	0.0000	31.41
$s = s_2$	2.0856	0.0319	0.9680	0.0000	57.25
$s = s_3$	11.2892	0.0000	0.0002	0.9997	11.32

$^a\pi^*(s)$% denotes the invariant distribution of s.

Appendix 2: Calibration of the Model Economy Ratios

A2.1 Calibration of the Macroeconomic Ratios

In Table 5 we report the Spanish GDP and its components for 2010. We adjust the amounts reported in this table according to Cooley and Prescott (1995), and we obtain that the adjusted private consumption is 552,784 ($= 596,322 - 54,127 + 10,589$) million euros[17], the adjusted public consumption is 221,715 million euros, and the adjusted investment is 299,114 ($= 244,987 + 54,127$) million euros.

Table 5: Spanish GDP and its Components for 2010 at Current Market Prices

	Millon Euros	Shares of GDP (%)
Private Consumption	596,322	56.72
Public Consumption	221,715	21.08
Consumption of Non-Profits	10,589	1.00
Gross Capital Formation	244,987	23.30
Exports	283,936	27.00
Imports	306,207	29.12
Total (GDP)	1,051,342	100.00

Source: Spanish National Institute of Statistics.

The next adjustment is to allocate Net Exports to the measures of C, I, and G. To that purpose, we compute the shares of each of those three variables in the sum of the three and we allocate Net Exports according to those shares. The sum of the three variables is 1,073,613 million euros and the shares of C, I, and G are 51.49, 27.86, and 20.65 percent. Next, we redefine the model economy's output and consumption from factor cost to market prices as follows: $Y^* = Y + T_c$, where Y^* is the model economy's output at market prices and T_c is the consumption tax collections, and $C^* = C + T_c$, where C^* is the model economy's consumption at market prices. Finally we use $C^*/Y^* = 51.49$ and $G/Y^* = 20.65$ as targets.

A2.2 Calibration of the Government Policy Ratios

In Table 6 we report the 2010 revenue and expenditure items of the consolidated Spanish public sector. Notice that the GDP share of Government consumption differs from the one that we have computed in Section A2.1 because here we use its unadjusted value. If we ignore the public pension system, the government budget in

[17] Note that 54,127 million euros is the private consumption of durables.

Table 6: Spanish Public Sector Expenditures and Revenues in 2010*

Expenditures	Millions of euros	Percentage of GDP	Revenues	Millions of euros	Percentage of GDP
Consumption	221,715	21.08	Sales and gross receipts taxes[a]	94,234	8.96
Investment	40,091	3.81	Payroll taxes[b]	106,599	10.13
Pensions[c]	109,000	10.36	Individual income taxes	77,542	7.37
Other	108,839	10.35	Corporate profit taxes	19,425	1.84
			Other revenues	83,626	9,96
			Deficit	98,218	9.33
Total	479,645	45.62	Total	479,645	45.62

Source: Spanish National Institute of Statistics, Spanish Social Security, and Eurostat.
*Shares of nominal GDP at market prices.
[a]It includes the tax collections from the Value Added Tax and other taxes on products.
[b]Total revenues from the Spanish Social Security.
[c]Total expenditure from the Spanish Social Security.

the model economy in 2010 is

$$G + Z = T_c + T_a + T_y + E \tag{33}$$

In this expression, unitentional bequests, E, are exogenous, and we target the output shares of T_c, T_a, and T_y, so that they replicate the GDP shares of Sales and Gross Receipt Taxes, Corportate Profit Taxes, and Individual Income taxes. Note that we have already targeted the output ratio of government consumption and we have already accounted for government investment. Consequently, we define the output share of transfers other than pensions, Z, residually to satisfy the budget. We report the model economy government budget items in Table 7 below.

Table 7: Model Economy Public Sector Expenditures and Revenues in 2010 (%Y^*Shares)

Expenditures		Revenues	
Consumption and Investment (G)	20.65	Consumption taxes (T_c)	8.96
Pensions (P)	10.35	Payroll taxes (T_s)	10.12
Other Transfers (Z)	0.83	Household income taxes (T_y)	7.66
		Capital Income Taxes (T_k)	1.84
		Unitentional Bequests (E)	3.25
Total	31.83	Total	31.83