Momento Angular

Relaciones de conmutación del momento angular

$$\vec{l} = \vec{x} \times \vec{p} \Rightarrow \vec{L} = \vec{X} \times \vec{P}, \qquad \vec{L}(\vec{x}) = -\mathrm{i}\hbar\vec{x} \times \vec{\nabla}, \qquad [X_i, P_j] = \mathrm{i}\hbar\delta_{ij} \;\Rightarrow\; [L_i, L_j] = \mathrm{i}\hbar\epsilon_{ijk}L_k$$

Generador de rotaciones

$$R_{\hat{n}}(\phi) = R(\varphi, \theta, \phi)$$

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{pmatrix}, \quad R_y(\phi) = \begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix}, \quad R_z(\phi) = \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

Rotaciones infinitesimales

$$R_i(\mathrm{d}\phi) = \mathbb{1} - \mathrm{i}K_i\mathrm{d}\phi$$

$$K_x(\phi) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix}, \quad K_y(\phi) = \begin{pmatrix} 0 & 0 & \mathrm{i} \\ 0 & 0 & 0 \\ -\mathrm{i} & 0 & 0 \end{pmatrix}, \quad K_z(\phi) = \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[K_i, K_j] = \mathrm{i}\epsilon_{ijk}K_k \;\Rightarrow\; [J_i, J_j] = \mathrm{i}\hbar J_k \quad (J_i \equiv \hbar K_i)$$

Iterando obtenemos una rotación finita

$$R_z(\phi) = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}J_z\phi} \;\Rightarrow\; R_{\hat{n}}(\phi) = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\vec{J}\cdot\hat{n}} \qquad |\alpha\rangle \to R_{\hat{n}}(\phi)|\alpha\rangle$$

Sistema de momento angular 1

Momento Angular

Sistemas de momento angular ½

$$J_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad J_y = \frac{\hbar}{2}\begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix}, \quad J_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \qquad [J_i = \frac{\hbar}{2}\sigma_i]$$

$$[J_i, J_j] = \mathrm{i}\hbar\epsilon_{ijk}J_k$$

H(j=1/2) esp de Hilbert de dimensión 2 de momento angular ½ $[\{|\frac{1}{2},+\frac{1}{2}\rangle, |\frac{1}{2},-\frac{1}{2}\rangle\}]$

$$R_z(\phi) = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}J_z\phi} = \begin{pmatrix}\mathrm{e}^{-\mathrm{i}\phi/2} & 0\\ 0 & \mathrm{e}^{\mathrm{i}\phi/2}\end{pmatrix} \Rightarrow R_z(2\pi) = \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$$

El signo – es físico (existen sistemas de momento angular “espín” igual a ½)

Rotación general

$$R(\alpha,\beta,\gamma) = R_z(\alpha)R_y(\beta)R_z(\gamma) = \begin{pmatrix}\mathrm{e}^{-\mathrm{i}\frac{\alpha+\gamma}{2}}c_{\beta/2} & -\mathrm{e}^{-\mathrm{i}\frac{\alpha-\gamma}{2}}s_{\beta/2}\\ \mathrm{e}^{\mathrm{i}\frac{\alpha-\gamma}{2}}s_{\beta/2} & \mathrm{e}^{\mathrm{i}\frac{\alpha+\gamma}{2}}c_{\beta/2}\end{pmatrix}$$

Matrix SU(2)

Momento Angular

Representaciones de momento angular

$$J^2 = J_x^2 + J_y^2 + J_z^2, \qquad [J^2, J_i] = 0$$

Elegimos una base de autoestados simultáneos de J^2 y J_z, $\quad J^2|a,b\rangle = a|a,b\rangle \quad J_z|a,b\rangle = b|a,b\rangle$

Operadores "escalera"

$$J_\pm = J_x \pm \mathrm{i} J_y$$

$$J_+^\dagger = J_-$$

$$J^2 = J_z^2 + \hbar J_z + J_- J_+$$

$$[J_+, J_-] = 2\hbar J_z$$

$$[J_z, J_\pm] = \pm\hbar J_\pm$$

$$[J^2, J_\pm] = 0$$

Obtención de la base

$$J^2 J_\pm|a,b\rangle = a J_\pm|a,b\rangle$$

$$J_z J_\pm|a,b\rangle = (b \pm \hbar) J_\pm|a,b\rangle$$

$$J_-|a,b_{\min}\rangle = 0, \quad J_+|a,b_{\max}\rangle = 0$$

$$\{|a,b_{\min}\rangle, |a,b_{\min}+1\rangle, \ldots |a,b_{\max}-1\rangle, |a,b_{\max}\rangle\}$$

$$b \equiv \hbar m, \quad m_{max} \equiv j \;\Rightarrow\; |a,b\rangle \equiv |j,m\rangle$$

$$J^2|j,m\rangle = \hbar^2 j(j+1)|j,m\rangle$$

$$J_z|j,m\rangle = \hbar m|j,m\rangle$$

$$J_\pm|j,m\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}\,|j,m\pm 1\rangle$$

$$\Rightarrow\; j = \frac{n}{2} = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

$$\dim(\mathcal{H}_j) = 2j+1$$

Momento Angular

Momento angular orbital y espín

$$\vec{L} = \vec{X} \times \vec{P} \qquad \langle \vec{x}|L_z|\Psi\rangle = -\mathrm{i}\hbar(x\partial_y\Psi - y\partial_x\Psi), \ldots$$

Una partícula puede tener momento angular orbital l=0 y todavía tener momento angular no nulo: espín (momento angular intrínseco)

$$\vec{J} = \vec{L} + \vec{S} \qquad l = 0, 1, 2, \ldots, \quad s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

Estados propios del momento angular orbital: armónicos esféricos

$$L^2 Y_l^m(\theta,\phi) = -\hbar^2 \left(\frac{1}{s_\theta^2}\partial_\phi^2 + \frac{1}{s_\theta}\partial_\theta(s_\theta\partial_\theta) \right) Y_l^m(\theta,\phi) = \hbar^2 l(l+1) Y_l^m(\theta,\phi)$$

$$L_z Y_l^m(\theta,\phi) = -\mathrm{i}\hbar\partial_\phi Y_l^m(\theta,\phi) = \hbar m Y_l^m(\theta,\phi)$$

Potencial central: $\{|nlm\rangle\}$ bon de autoestados de $\{H, L^2, L_z\}$

$$\Psi(\vec{x}) = R_{nl}(r) Y_l^m(\theta,\phi)$$

Si la parte radial factoriza podemos proyectar sobre estados con momento angular bien definido

$$\phi(r,\theta,\phi) = f(r)\psi(\theta,\phi)$$

$$\psi(\theta,\phi) = \sum_{lm} c_l^m Y_l^m(\theta,\phi) = \sum_{lm} \left[\int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{1} \mathrm{d}\cos\theta' Y_l^{m\,*}(\theta',\phi')\psi(\theta',\phi') \right] Y_l^m(\theta,\phi)$$

Momento Angular

Suma de momentos angulares: sean dos sistemas con momentos angulares cinemáticamente independientes, ¿cómo describimos el momento angular del sistema completo?
El sistema completo es descrito por el producto tensorial de espacios de Hilbert $\mathcal{H}_1 \otimes \mathcal{H}_2$

$$\left.\begin{array}{l}[J_{1i}, J_{1j}] = \mathrm{i}\hbar\epsilon_{ijk}J_{1k} \\ [J_{2i}, J_{2j}] = \mathrm{i}\hbar\epsilon_{ijk}J_{2k} \\ [J_{1i}, J_{2j}] = 0\end{array}\right\} \Rightarrow [J_i, J_j] = \mathrm{i}\hbar\epsilon_{ijk}J_k \qquad \vec{J} = \vec{J}_1 + \vec{J}_2 = \vec{J}_1 \otimes 1 + 1 \otimes \vec{J}_2$$

$$\{J_1^2, J_{1z}, J_2^2, J_{2z}\} \text{ forman parte de un CCOC } \Rightarrow \{|j_1 m_1 j_2 m_2\rangle\} \text{ bon de } \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$[J^2, J_1^2] = [J^2, J_2^2] = [J^2, J_z] = [J_1^2, J_z] = [J_2^2, J_z] = 0, \quad [J^2, J_{1i}] \neq 0, \ldots$$

$$\{J_1^2, J_2^2, J^2, J_z\} \text{ forman parte de un CCOC alternativo } \Rightarrow \{|j_1 j_2 j m\rangle\} \text{ bon de } \mathcal{H}_1 \otimes \mathcal{H}_2$$

Tenemos dos bon, los coefficientes de la transformación unitaria que las relaciona se llaman coefficientes de Clebsch-Gordan

$$|j_1 m_1 j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{j_1+j_2} \langle j_1 j_2 j m | j_1 m_1 j_2 m_2\rangle |j_1 j_2 j m\rangle \quad (m = m_1 + m_2)$$

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			
.	.			
.	.			

1/2×1/2

m_1	m_2	J=1, M=+1
+1/2	+1/2	1

m_1	m_2	J=1, M=0	J=0, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=1, M=−1
−1/2	−1/2	1

1×1/2

m_1	m_2	J=3/2, M=+3/2
+1	+1/2	1

m_1	m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	J=3/2, M=−1/2	J=1/2, M=−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	J=3/2, M=−3/2
−1	−1/2	1

2×1/2

m_1	m_2	J=5/2, M=+5/2
+2	+1/2	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	J=5/2, M=−5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	J=2, M=+2
+3/2	+1/2	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	J=2, M=0	J=1, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=2, M=−1	J=1, M=−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	J=2, M=−2
−3/2	−1/2	1

2×1

m_1	m_2	J=3, M=+3
+2	+1	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	J=3, M=−2	J=2, M=−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	J=3, M=−3
−2	−1	1

1×1

m_1	m_2	J=2, M=+2
+1	+1	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	J=2, M=−1	J=1, M=−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	J=2, M=−2
−1	−1	1

3/2×1

m_1	m_2	J=5/2, M=+5/2
+3/2	+1	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	J=5/2, M=−5/2
−3/2	−1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta \, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m \, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

3/2×3/2

m_1	m_2	J=3, M=+3
+3/2	+3/2	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	J=3, M=−2	J=2, M=−2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	J=3, M=−3
−3/2	−3/2	1

2×3/2

m_1	m_2	J=7/2, M=+7/2
+2	+3/2	1

m_1	m_2	J=7/2, M=+5/2	J=5/2, M=+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	J=7/2, M=+3/2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	J=7/2, M=+1/2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	J=7/2, M=−1/2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	J=7/2, M=−3/2	J=5/2, M=−3/2	J=3/2, M=−3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	J=7/2, M=−5/2	J=5/2, M=−5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	J=7/2, M=−7/2
−2	−3/2	1

2×2

m_1	m_2	J=4, M=+4
+2	+2	1

m_1	m_2	J=4, M=+3	J=3, M=+3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	J=4, M=+2	J=3, M=+2	J=2, M=+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	J=4, M=+1	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	J=4, M=0	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	J=4, M=−1	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	J=4, M=−2	J=3, M=−2	J=2, M=−2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	J=4, M=−3	J=3, M=−3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	J=4, M=−4
−2	−2	1

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{\,3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{\,3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{\,2}_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{\,2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{\,2}_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^{\,2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{\,2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{\,2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{\,2}_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory,* Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

Momento Angular

Tensores y teorema de Wigner-Eckart

En MC los operadores actúan sobre estados, ¿cómo definimos un operador vectorial? (que transforma de manera adecuada bajo rotaciones)

$$\mathcal{D}(R)^\dagger V_i \mathcal{D}(R) = R_{ij} V_j, \quad \text{with } \mathcal{D}(R) = \mathrm{e}^{-\mathrm{i}\hat{n}\cdot\vec{J}\phi/\hbar}$$

$$V_i - \frac{\mathrm{i}\epsilon}{\hbar}[V_i, \hat{n}\cdot\vec{J}] = R_{ij}(\hat{n},\epsilon)V_j \Rightarrow \boxed{[V_i, J_j] = \mathrm{i}\hbar\epsilon_{ijk}V_k}$$

Tensor esférico irreducible de rango j

$$T_m^{(j)} \to \mathcal{D}^\dagger(R) T_m^{(j)} \mathcal{D}(R) = \sum_{m'=-j}^{j} T_{m'}^{(j)} \mathcal{D}^j_{m'm}(R) \Rightarrow \boxed{[\hat{n}\cdot\vec{J}, T_m^{(j)}] = \sum_{m'=-j}^{j} T_{m'}^{(j)} \langle jm'|\hat{n}\cdot\vec{J}|jm\rangle}$$

Producto de tensores:

$$T_m^{(j)} = \sum m_1, m_2 \langle j_1 j_2 m_1 m_2 | j_1 j_2 j m\rangle X_{m_1}^{(j_1)} Z_{m_2}^{(j_2)} \text{ es tensor esférico irreducible de rango } j$$

Momento Angular

Teorema de Wigner-Eckart

$$\langle \alpha' j' m' | T_q^{(k)} | \alpha j m \rangle = \langle j m k q | j k j' m' \rangle \frac{\langle \alpha' j' || T^{(k)} || \alpha j \rangle}{\sqrt{2j+1}}$$

Regla de selección m

$$\langle \alpha' j' m' | T_q^{(k)} | \alpha j m \rangle = 0 \text{ salvo que } m' = m + q$$