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Switching dynamics of neural systems in the presence of multiplicative colored noise

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Abstract. We study the dynamics of a simple bistable system driven by multiplicative correlated noise. Such system mimics the dynamics of classical attractor neural networks with an additional source of noise associated, for instance, with the stochasticity of synaptic transmission. We found that the multiplicative noise, which performs as a fluctuating barrier separating the stable solutions, strongly influences the behaviour of the system, giving rise to complex time series and scale-free distributions for the escape times of the system. This finding may be of interest to understand nonlinear phenomena observed in real neural systems and to design bio-inspired artificial neural networks with convenient complex characteristics.

Key words: Bistable systems, switching dynamics, multiplicative colored noise, neural up and down states

1 Introduction

Attractor neural networks (ANN) have been a topic of high interest in the last decades. Most of the paradigmatical approaches in this field consider a number of bioinspired elements from biological neural systems and study the computational properties of the resulting model, resulting in hints and developments in neuroscience and computer algorithm design. One of the most notable models of ANN is the one proposed by Amari and Hopfield [1, 2]. This model assumes a network of N binary neurons linked by connections of certain strength, also called synapses. By considering a simple learning rule based on neurophysiological data [3], this network is able to store a certain number P of patterns of activity. After this learning, the network is able to recover one of these activity pattern from an initial configuration correlated with this pattern, a property which is called *associative memory*. While the behaviour of such models is highly nonlinear, one can derive analytical solutions [4] which help to reach a better understanding for simple and hypothetical situations. For instance, if one assumes that the number of stored patterns does not scale with the size of the network, the previous model is simplified to a bistable system. This is a common approach employed when one is interested in the dynamics of the network activity instead on its storing capacities [5]. It has been employed, for instance, to study the influence of the network topology [6], or the switching between different patterns of activity due to short-term synaptic mechanisms [7, 8].

In this work, we employ a simplified bistable model, which mimics the dynamics of attractor neural networks in the limit of $P \ll N$, to study the effect of multiplicative colored noise in the dynamics of the system. Such noise resembles the stochastic nature of synaptic transmission [9] (which may be relevant to transmit information through dynamic synapses [10, 11]), or other sources of colored noise which could affect the dynamics in a multiplicative way, such as threshold fluctuations. We found that this multiplicative colored noise strongly affects the dynamics of the system, giving rise to complex time series and scale-free distributions for the escape times of the dynamics. Our results could be of interest to understand nonlinear phenomena observed in real neural systems and to design new paradigms in bio-inspired artificial neural networks.

2 Model and results

We consider a bistable system characterized by the variable $y(t)$, which represents the level of activity of the neural network. This variable evolves according to the following discrete dynamics

$$y(t+1) = \tanh[z(t)y(t)] + \xi(t), \tag{1}$$

Here, the variable $z(t)$ is a Gaussian colored noise with mean one, standard deviation σ and correlation time τ. It represents a source of correlated noise associated with the stochasticity of the synaptic transmission or fast threshold variations, for instance. The term $\xi(t)$ is an additive Gaussian white noise of zero mean and standard deviation δ. This term takes into account other possible sources of non-multiplicative noise, and is also employed to prevent the system to be locked in the solution $y = 0$, since the fluctuations due to the multiplicative noise cannot influence the system in this point. In the following, we focus on the role of the correlation time in the dynamics, and therefore we fix the level of the fluctuations $\sigma = 50$ and $\delta = 0.1$. A more complete study of the influence of these parameters will be published elsewhere [12].

From a mathematical point of view, we can see in Eq. (1) that the variable $z(t)$ represents the barrier height of our bistable system. For fixed $z < 1$, our system can be viewed as a particle in a single well potential in the presence of fluctuations (given by $\xi(t)$). Thus, $y(t)$ will be fluctuating around the only stable solution of the dynamics, $y = 0$. On the other hand, for fixed $z > 1$ we have a particle in a double well potential in the presence of fluctuations (given by

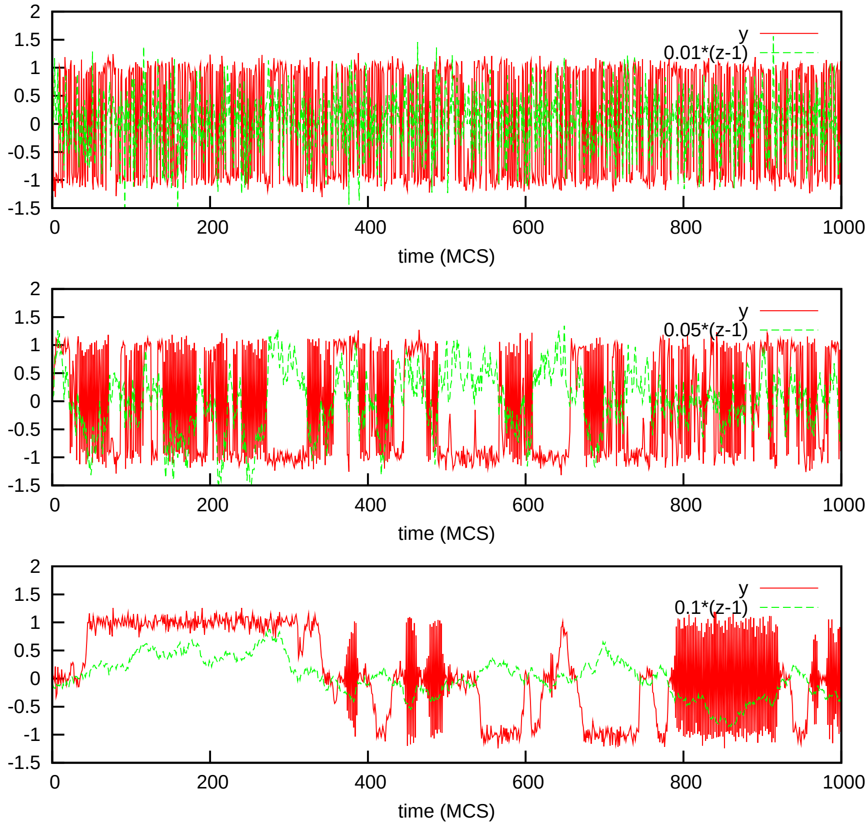

Fig. 1. (Color online) Several realizations of the time evolution of $y(t)$ and $z(t)$, for different values of the correlation time τ. The series correspond, from top to bottom, to $\tau = 0$, $\tau = 10$ and $\tau = 100$, respectively. The mean and variance of $z(t)$ have been conveniently rescaled in each case for a better visualization (see labels in each plot).

$\xi(t)$). In this situation, the particle will be jumping, or switching, from one stable solution to the other in a stochastic manner. The fact that z has not a fixed value but it is also a fluctuating magnitude introduces a high level of complexity which is the aim of this study. For instance, we can control the characteristics of the barrier height by varying the correlation time τ. This variation has a dramatic effect in the dynamics of $y(t)$, as one can see in figure 1. The three plots in the figure shows the relation between the dynamics of $y(t)$ and $z(t)$: for $z(t) < 1$, the variable $y(t)$ rapidly fluctuates around the solution $y = 0$, and for $z(t) > 1$ we enter in the double well regime and $y(t)$ starts to swith between the symmetric stable solutions $y_+ \simeq +1$ and $y_- \simeq -1$. The switching dynamics has a strong dependence with τ, as the figure also shows. For low values of τ the switching is random and has a high frequency, but when τ is increased, some intervals of prolongued permanence in a particular solution appear. Concretely, high values

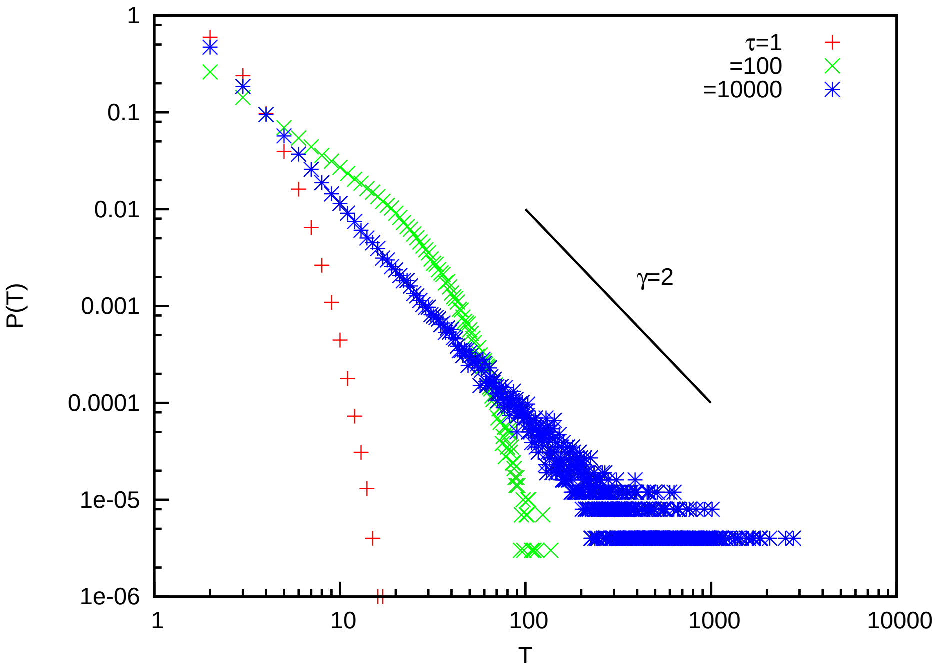

Fig. 2. Probability distribution of escape times for different values of τ. We can see that increasing of the correlation time yields the appearance of power-law distributions in escape times, which reflects the change in the statistics of the dynamics of $y(t)$. The obtained power-law distribution is defined as $P(T) \sim T^{-\gamma}$, with the exponent taking a value of $\gamma \simeq 2$.

of the correlation time induces drastical modifications in the probability of the system to jump between stable solutions after a given interval T, that is, in the probability distribution of escape times.

A more detailed study of the influence of τ in the escape time probability is shown in figure 2. One can observe that the probability distribution of the escape times is a Gaussian distribution for low values of τ, as it is well known. In this situation, the variable $z(t)$ behaves approximately as a white noise, and therefore $z(t)$ is continuously crossing the value $z = 1$. As a consequence of that, the double well configuration is only maintained by a very short time, and long periods of permanence in the solutions y_+ or y_- are unlikely to occur. However, when the value of τ is increased, the excursions of the variable $z(t)$ in the region of $z > 1$ become longer in time and $z(t)$ can eventually take values which are far from $z = 1$. These two factors combined allow the system to eventually stay in one of the stable solutions $y_+,\ y_-$ for long periods of time, as it is shown in bottom panel of figure 1. This eventuals long intervarls of permanence in a double well solutions are reflected in the probability distribution of escapes times as a

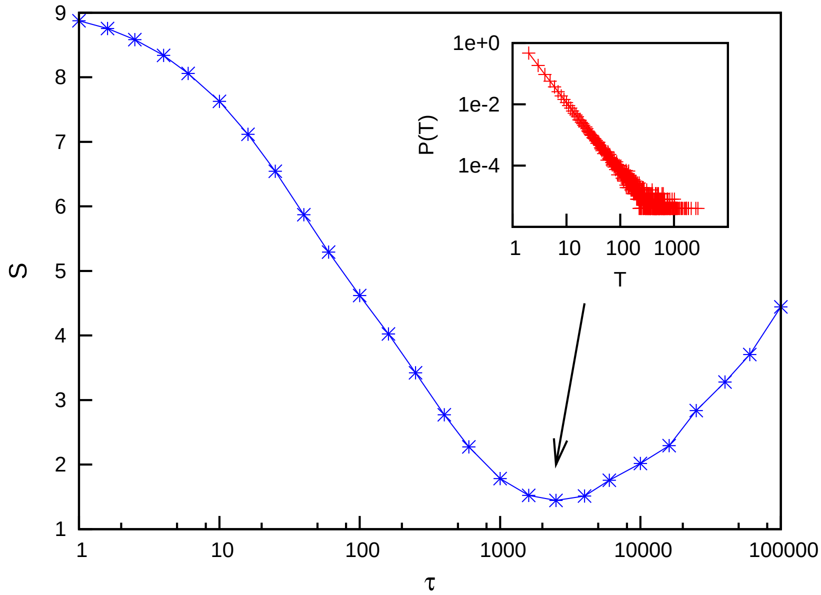

Fig. 3. The entropy function, as defined in the main text, for different values of the correlation time τ. We can observe a minimum in the entropy which approximately corresponds to the value of τ for which scale-free escape times distribution appears.

scale-free relation, which mathematically corresponds to a power-law function $P(T) \sim T^{-\gamma}$. We found an approximate value of $\gamma \simeq 2$ for the exponent.

Finally, to complete the study of the dynamics of the system, we have computed a measure of the irregularity of the time series of $y(t)$, for different values of the correlation time τ. This measure is defined as and entropy of the form

$$S \equiv -\sum_x p(x)\ \log_2\ p(x) \tag{2}$$

Here, $p(x)$ is the normalized power spectrum of the time serie of $y(t)$ for a given value of τ. It is worthy to note that this entropy depends strongly on the dynamics of the system (via the power spectrum), and therefore it is influenced by the value of τ. In figure 3, we can see that the entropy reaches a minimum around the value of τ for which the scale-free distributions for the escape times appear. This is due to the fact that, in this situation, long permanence times in stable double well solutions are allowed eventually, and such permanence decreases significatively the complexity of the time series of the system. Thus, the influence of τ in the dynamics of $y(t)$ is also notorious when studying the complexity of the dynamics with multiple methods.

3 Conclusions

We have study the dynamics of a simple bistable system under the influence of multiplicative colored noise. The results show a high impact of considering high values of the correlation time in the dynamics of the system, and some nonlinear characteristics such as scale-free distributions and minima of entropy are found. One can think that the consequences of such complex behaviour in real neural systems, such as populations of cortical neurons connected by highly fluctuating synapses, could be relevant for different neural protocols of processing and coding of information. Further study of theses characteristics could also reveal strategies to implement the benefits obtained with these class of dynamics in bio-inspired computer algorithms.

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