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Understanding Statistical Graphs: A Research Survey

Pedro Arteaga, Carmen Batanero, José Miguel Contreras and Gustavo Raúl Cañadas

Departamento de Didáctica de las Matemáticas
University of Granada

✉ parteaga@ugr.es, batanero@ugr.es, jmcontreras@ugr.es, jmcontreras@ugr.es

Abstract

Reading and building statistical graphs are part of the statistics literacy that well informed citizens need to critically face the information society. In this paper we first analyse the graph elements and the semiotic activity involved in graphical work. Then, we discuss the definition of graphical understanding, and the graph understanding levels. Next, we focus on errors in reading and building graphs, the use of graphs in exploratory data analysis, and teachers' understanding of graphs. The conclusion of our analysis is that graphical competence is neither reached by students nor by prospective primary school teachers. Consequently, this survey provides information that can be used in improving the graphical competence of students and teachers as well as the didactical education of teachers.

Keywords: statistical graphs, graphical understanding, research survey
AMS Subject classifications: 97K30

1. Introduction

Graphs are pervasive in our society and can be used to communicate information efficiently, as a tool for data analysis (Cazorla, 2002). Moreover, in many school topics graphs are used to visualize abstract concepts and relationships (Postigo and Pozo, 2000). New curricula in many countries (e.g., NCTM, 2000; MEC, 2006) include statistical graphs at all levels of primary education. The construction and interpretation of statistical graphs is also an important part of statistical literacy according Watson (2006).
Despite this relevance, educational research alerts us that the competence related to statistical graphic language is neither achieved in the students (Friel, Curcio and Bright, 2001) nor in the preparation of future primary

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school teachers (González, Espinel and Ainley, 2011). The reason is that statistics have only recently become an important part of the elementary school mathematics curriculum, so that these teachers may not have had adequate opportunities to learn about graphs.
In this paper we summarize research on this subject, to provide teachers with information about the main difficulties in their students. Below we first analyse the graph elements, and the semiotic activity involved in graphical work. The following sections discuss the definition of graphical understanding, and the graph understanding levels. Next, we focus on errors in reading and building graphs, and the use of graphs in exploratory data analysis. We finish with some didactic reflections.

2. Graph Structural Components

Knowledge of the following structural elements is needed to understand the information expressed in statistical graphs (Kosslyn, 1985):

- Background plane: This supports the graph and is white in most graphs, but could be a photograph or a drawing, depending on the graph.
- Graph structure that consists of Cartesian axes in many graphs, but not in all of them, for example, in pie charts. It provides information about the entities being represented and interrelated.
- Pictorial content: which is the way data is represented and transmitted through the graph, for example, by rectangles in histograms, or circular sectors in pie charts.
- Labels: which provide the information needed to interpret the graph and include words, and numbers in the title and axes.

Curcio (1987) considered the following elements in a graph:

- Words that appear in the title, axis labels and scales, and provide the necessary keys to understand the context, variables and relationships expressed in the graph.
- Underlying mathematical content embedded in the graph, which student should know to interpret the graph. For example, integer numbers; area in a pie chart, length in a line graph or Cartesian coordinate in a scatter plot.
- The specific conventions used in each graph that need to be known to make a correct reading or construction. For example, the student must know that in a pie chart, the circular sector is proportional to frequency or that in scatter plots, each point represents one case and the coordinates the values of the two variables represented.

Starting from the above analysis, Friel, Curcio and Bright (2001) identified the graph background, which includes the colours, grid and images which could have been imposed on the graph and the title and labels, which indicate the graph content and the contextual variables represented in the same. They also consider two more elements:

- Framework of the graph, including axes, scales, grids and reference markings on each axis, which provides information about the kinds of measurement and units being represented. There are different types of frames and coordinate systems (linear, Cartesian two or multidimensional, polar).
- Graph specifiers, or elements used to represent data, such as rectangles (in histograms) or points (in scatter plots). The authors suggest that not all specifiers are equally simple to understand, and suggest the following order of difficulty: (a) Position in a homogeneous scale (line graphs, bar charts, some pictograms), (b) position on a not homogeneous scale (polar graphs or bivariate graphs), (c) length (star graphs without reference axes, trees), (d) angle or slope (pie chart), (e) area (circles, pictograms, histograms), (f) volume (cubes, some statistical maps), and (g) colours (colour-coded statistical maps).

3. Graph as semiotic object

Bertin (1967) suggested that a graph is a complex semiotic object. The graph itself and every component of the same are made up of signs that require a semiotic activity by those who interpret them. A reader has to perform three successive operations to read the graph:

- External identification, to find the conceptual and real-world referents that relate the information contained in the graph, through the analysis of the graph alphanumeric labels.
- Internal identification, to identify the relevant dimensions of variation in the graph pictorial content and the correspondence between the visual and conceptual dimensions and scales.
- Perception of the correspondence, by which the reader uses the levels of each visual dimension to draw conclusions about the levels of each conceptual dimension.

For each of these steps one or more semiotic functions can be identified. Eco (1977) called semiotic function to a correspondence established by an individual between an antecedent (expression) and a consequent (content). In reading the graphs, several translation activities between the graph as a whole, and each part of the graph should be performed. Each piece of

information obtained from a graph requires establishing a correspondence between elements, subsets or sets of this graph. Bertin argues that in general, two possible types of questions are possible as regards a graph representing a function $Y = f(X)$: (a) Finding the value Y from an input X, or direct reading; and (b) Finding a value X, from an input Y or inverse reading.
Bertin defined the image as a significant visual-perception carried out in a moment of vision. An image is formed through the perception of the correspondences originated by a question: (a) first, we define an entry in X; then (b) we observe the correspondence, for example a point; and finally (c) we identify an output or response (the value Y). The author termed visual selection the process of concentrating on isolate information contained in the graph in a minimum moment of vision. Bertin (1981, p. 15) suggested that, "the efficacy of a graphic construction is revealed by the level of question that receives an immediate response" and, therefore, considers a graph to be more effective when any question at any level, can be answered through a single image.
Cleveland and McGill (1984) studied graphic perception. The information in a graph is encoded (e. g. using position or size) and the person that reads the graph should decode that information, through a process of graphical perception, consisting in "visually decoding the information encoded in a graph" (p. 531). The authors identified the basic graphic perception tasks that are carried out during the visual information decoding process and ordered these tasks by the degree of precision they allow in the conclusions reached in reading the graphs, as follows: (a) determining the position of a point or element along a common scale; (b) determining the position when using two scales are not aligned, for example, in the scatter plot; (c) determining length, direction and angle, (d) estimating an area; and (e) estimating a volume or curvature. The authors considered a graph to be more useful than another that represents the same information when the reading of the first graph pr ovides more precise conclusions. The best graph is that allowing the reader to extract quantitative information, that lead to organizing and perceiving patterns and structures that are not revealed by other alternative representation of data.

4. Defining graphical understanding

Several authors analysed graph understanding and its components. For example, Kosslyn (1985) studied the cognitive processes involved in the generation, interpretation and processing of graphs using computer networks models and identified three levels of graph comprehension:

- *Syntactic level*: In this level the properties of graphical elements are considered; for example, in deciding whether there are visual distortions.

- *Semantic leve*l, where the goal is performing quantitative and qualitative interpretations and evaluation of the meaning of the graph.
- *Pragmatic level*, that seeks to recognise the aim of the graph and examine the purpose of the information transmitted.

According to Pinker (1990), the following processes related to the reader's cognitive abilities are activated when reading a graph: (a) the recognition process, to classify the graph as belonging to a particular type; (b) the creation of a conceptual message process, to select the data available to be extracted; (c) the questioning process, by which new information (required information) is recovered or coded base on the conceptual message; and (d) the inferential process, where new information, which is not explicitly represented is deduced from the graph through inferential, logical and mathematical rules. Pinker suggests that reading efficiency depends both on the reader's graphical competence and information processing capacity, and on the graph effectiveness to convey information. This last efficiency depends on the type of graph, the concepts involved, and their mathematical complexity.
Friel, Curcio and Bright (2001) defined graphical understanding as the "graph readers' abilities to derive meaning from graphs created by others or by themselves" (p. 132). For these authors, graph understanding includes the following competences:

- *Recognizing the graph structural elements* (axes, scales, labels, specific elements) and their relationships. This competence is achived when it is possible to distinguish each of these elements and evaluate whether each of them is or not appropriate in the particular graph.
- *Assessing the impact of each structural element* on the presentation of information in a graph (for example, being able to predict the change of the graph when changing the scale in an axis).
- *Translating the relationships reflected in the graph to the data represented* in the same and vice versa. For example, translating the tendency in scatter plot to a correlation between two variables.
- *Recognizing when a graph is more useful than another*, depending on the problem and data represented, that is, being able to choose the appropriate graph for each type of variable and problem.

5. Levels in Graphical Understanding

In addition to the above competences, different levels of understanding in the critical reading of data have been defined. Bertin (1967) described the following levels:

- *Extracting data* or direct reading of the data represented on the graph.

For example, in a bar graph, reading the frequency associated with a value of the variable.

- *Extracting trends*; being able to perceive a relationship between two subsets of data that can be defined a priori or visually in the graph. For example, visually determining the mode of a distribution in a bar graph.
- *Analyzing the data structure*; comparing trends or clusters and making predictions. For example, in an attached bar graph, analyzing the differences in mean and range of two distributions.

A related classification was due to Curcio (1989), who termed the three levels defined by Bertin as: reading between the data (literal reading of the graph without interpreting the information contained in it), reading within the data (integrating the data in the graph) and reading beyond the data (making predictions and inferences from the data to information that is not directly reflected in the graph). Friel, Curcio and Bright (2001) expanded the above classification by defining a new level of reading behind the data, which consists on judging the method of data collection, and assessing the data validity and reliability, as well as the possible generalization of findings. Wainer (1992) in turn classified the type of questions that may be posed from a graph in three levels:

- *Elementary level.* Questions related solely to the extraction of data directly from the graph.
- *Intermediate level.* Questions regarding the assessment of trends based on a portion of the data.
- *Upper level.* Questions about the deep structure of the whole data set, usually comparing trends and looking for groups.

The basis of his theory is the distinction between relations of first, second and third class. First class relations consider a single variable such as weight. Second class relates two objects, such as the weight of a book. The third class connects three variables, such as the weight of a textbook. In Wainer's levels, elementary level questions correspond to first class relations, intermediate to second-class and upper level to the third class.
A more complex model is due to Gerber, Boulton-Lewis and Bruce (1995), who distinguished seven levels of graph understanding, depending on the students' skills:

- *Level 1.* Students do not focus on the data, but associate some characteristics of the same with their knowledge of the world, which is often inaccurate. For example, when asking about the children ages represented on a graph, they may respond by giving their own age.

- *Levels 2 and 3.* Students focus on the data represented, in an incomplete way. At level 2 they do not appreciate the purpose of the graph, and interpret only partial aspects of the data; for example, they only read one of the bars in the bar chart. At level 3 students appreciate the purpose of the graph and analyse all the elements, but fail to reach a global synthesis, since they do not understand a specific element that is critical in the graph. For example a student could interpret each group age in a population pyramid (that refers to a group of people) as ages of individual subjects.
- *Levels 4, 5 and 6.* Students achieve a global synthesis when reading a graph, but they still have a static vision of the same. At level 4 students analyse the variables represented in the same graph in isolation; for example, in a graph representing the life expectancy of men and women in various countries, students interpret separately the life expectancy of men and the life expectancy of women, without making comparisons. At level 5 students are able to compare several variables represented on the same graph; in the previous example they could conclude that life expectancy of women is higher than men in the majority of countries displayed. At level 6 students do not only compare different variables en the same graph, but also get general conclusions about a hypothesis; for example, they could use the graph described before to refute the idea that women are weaker than men.
- *Level 7.* In the last level students are able to extrapolate and make predictions for data that is not represented on the graph; in the previous example they could predict the life expectancy of men for a country not shown in the graph, when they are given the life expectancy of women in that country.

When taking into account the critical assessment of the graph and not only its interpretation, the highest reading levels described above can be slightly modified (Aoyama and Stephen, 2003; Aoyama, 2007). Suppose, for example, we give some students a graph displaying data about the number of hours that a group of teenagers spend playing video games and the number of violence episodes they were involved at school. The graph clearly shows a growing number of violence episodes at school when the time spent on these games by the adolescents increases. Students are asked whether they think that school violence would decrease if video games were banned. Once students reach the highest stage in the previous classifications, Aoyama and Stephen distinguish three reasoning levels according to the students' critical ability to judge the information reflected in the graph:

- *Rational / Literal Level.* Students read the graph correctly, and are able to interpolate, identify trends and predict. The students answer the

question posed, using the graph properties, but they neither question the information shown in the graph, nor they give alternative explanations to draw a conclusion. A typical response at this level would be "*Yes, because the group of guys who played games for a long time also had many episodes of violence*".

- *Critical Level*. Students read graphs, understand the context and assess the reliability of information; they sometimes question this information, but are unable to find other possibilities to respond the question: "I do not think so, because although the guys who play videogames more often are the most violent, there could be other reasons to explained this results, although I cannot imagine which".
- *Hypothetical Level*. Students read graphs, interpret and evaluate information, and are also able to form their own hypotheses and models: "I do not agree that the reason of violence is playing videogames, perhaps their parents do not give much attention to these guys, and this can explain both, the violence episodes and the time spent with the game".

6. Errors in Reading or Producing Statistical Graphs

Other researchers discuss the common errors in producing or reading statistical graphs. The first step in building a graph is selecting a graph that is adequate to the type of variable and the problem, but students often fail in this selection. Li and Shen (1992) analyzed the statistical graphs produced by their students when working with a statistical project. They found some students who built frequency polygons with qualitative variables, used bar charts to represent data that should be displayed in a scatter plot or represented unrelated variables in the same graph. They also found the following problems in the scales: (a) Choosing an inappropriate scale for the intended purpose (e.g., not covering the entire range of variation of the variable represented); (b) Ignoring the graph scales in one or both axes; (c) Not specifying the origin of coordinates, and (d) Not providing enough divisions for the graph scales.

Other authors focussed on the reading and understanding of specific graph. For example, Pereira-Mendoza and Mellor (1990) found that students make simple reading mistake, when changing the arrangement of data in a bar chart the (e.g. using a horizontal bar chart instead of vertical bar chart). Lee and Meletiou (2003) described four main errors related to constructing and interpreting histograms:

- Perceiving histograms as a representation of isolated data, in assuming that each rectangle refers to a particular observation and not a range of values.

- Tendency to observe the vertical axis and compare the differences in the heights of the bars when comparing the variation of two histograms.
- Tendency to interpret histograms as a two variables graphs (e.g., as scatter plots).

Wu (2004) categorized the errors of high school students when working with different types of statistical graphs and found the following categories: (1) comprehension errors, (2) incorrect reading, (3) calculus errors, (4) errors related to the graph scales, (5) errors in the specifiers, title or labels of the graph, (6) errors in pie charts, (7) errors relating to the sizes of the elements in a pictogram, (8) confusion between similar graphs (for example, between histogram and bar graph), (9) confusion between variable and variable value, (10) errors in handling information provided by a graph, (11) problems in misunderstanding the context. The most common errors were those related to the graph scales and specifiers, as well as misunderstanding the information represented. Wu concluded that, in general, students have better graph reading skills than interpretation and evaluation skills, because these last skills require making inferences from information that is not shown di rectly in the graph.

These errors are also common in prospective primary-school teacher, as shown by Bruno and Espinel (2005). Although few participants in their study had difficulty in grouping data into intervals, they failed to represent these intervals on the number line. For example, some participants represented the whole interval with just a point (different from the class centre); others omitted the intervals with zero frequency or built a histogram with non-attached rectangles, even when being aware that they were working with continuous variables.

Carrión and Espinel (2005a and b, 2006) analysed students' difficulties when working with histograms, scatter plots, box plots, and stem and leaf graphs and the students' ability to translate between different ways of presenting information, i.e., moving from one graph to another. Their questionnaire was completed by primary school students and prospective primary school teachers in two different groups: the first group (A) completed the questionnaire before statistics instruction, and a second group (B) after a statistics training process. The most difficult task for all the participants was traducing verbal information to a graph, with bar graph having the higher success rate and box plot the lowest success rate. Moreover, similar errors were produced by both the prospective teachers and the primary school students. When comparing the two groups of prospective teachers, the authors found a noticeable improvement in responses related to stem and leaf plots and box plots in g roup B, i.e. after statistics instruction.

Arteaga and Batanero (2010) analysed the errors made by 201 prospective

teachers in the construction of statistical graphs when working with the same statistical project. About 50 % of prospective teachers built correct graphs or graphs with not serious flaws. The remaining teachers produced many errors described in previous research, such as, for example, incorrect representation of frequency intervals on the number line, confusing the variable frequency and value or representing unrelated variables on the same graph. The authors also analysed the way in which 201 the prospective primary school teachers in their sample interpreted and extracted useful information from the graphs they constructed. Their results showed that about 30 % of prospective teachers did not use their graphs as a data analysis tool, because they neither read nor interpreted these graphs and an additional 11.6 % of teachers made an incorrect reading. Only 13.3 % of these prospective teachers were able to extract information from the graphs within the highest reading level defined by Curcio (1989), reading beyond the data; these teachers were able to analyze and compare measures of central tendency and dispersion.

7. Statistical Graphs and Exploratory Data Analysis

Graphs play an important role in Exploratory Data Analysis (EDA) as an analysis tool and not only as a way to communicate information (Ben-Zvi, 2000). Until the early 90's, data analysis was primarily based on statistical calculation and procedures; in that approach the main purpose was to test hypotheses, without attempting to explore any other information that could be derived from the data used.

A fundamental idea in EDA is that using different representations facilitates the production of new knowledge; for example, moving from tables to graphs, the data structure is visualized. Although any statistical graph can be used in EDA, this paradigm has developed some new data representations, such as, for example, the steam-and-leaf diagram and the box-plot, which that facilitate the comparison of several samples (Biehler, 1997). Learning these graphs can also help improve critical understanding because they help graph readers to pay attention to particular aspects of the distribution.

Several authors analysed the use of graphics in EDA by students. For example, Bakker, Derry, and Konold (2006) designed experiments using the software TinkerPlots, to improve 11 years old students' EDA abilities. The authors concluded that the work with statistical projects and software can help students to develop statistical language, to become familiar with graphs, to extract information from them, and to start thinking intuitively with statistical concepts, as some measures of central tendency and spread.

Ben-Zvi and Friedlander (1997) analysed the graphs produced by students when working with statistical projects and computers, identifying four levels in the construction of graphs using new technologies:

- *Uncritical use*: Students construct graphs accepting the software default options, even if these options are inappropriate. They also have difficulty in assessing the relationships suggested in their graphical representations, identifying only obvious information, such as the maximum value of the variable.
- *Significant use of a representation*: Students construct a graph correctly if they are told what graphs have to build. They are able to modify and transform the graph (e.g., changing a software option) and interpret the results, but are unable to select the best representation when they have several options to choose from.
- *Significant handling of multiple representations*: In this case, the students can select the most appropriate graphs and take into account what representation is more appropriate to the particular task they are solving.
- *Creative use*: Students construct a correct but unusual graph to present and justify their ideas.

In a later research, Ben-Zvi (2002) evaluated 13 years old students' abilities to appreciate the importance of statistical data and graphs. The author described students' difficulties to move from a local vision of data (e.g. looking only at one bar in a histogram) to a global vision for which the skills of finding, describing and explaining patterns and trends in a data set are needed. Although at the beginning of this course, students tended to work with a local vision of the data in exploratory data analysis, this vision evolved into a global vision of the graphs as the teaching progressed.
Ridgway, Nicholson, and McCusker (2006) suggested that statistical data and graphs presents in the media and on the Internet are often multivariate and show complex interactions between different variables. The authors pointed out that the school curriculum does not prepare students to deal with this type of data, so they highlighted the potential and possibilities offered by new technologies to understand these representations and they designed interactive didactical material to work with dynamic and multivariate representations (available at: www.dur.ac.uk/smart.centre/).

8. Reading Media Graphs

Monteiro and Ainley (2006, 2007) suggested that the interpretation of statistical graph, needs a "critical sense", since the reader needs a variety of different knowledge and experiences to perform this task. In the school context, graphical interpretation is focused primarily on statistical knowledge and processes, without paying enough attention to the social context from which the graph data were taken. Therefore, the authors proposed to use

statistical graphs taken from the media as a pedagogical tool to bridge the gap between using graphs in school contexts and daily life contexts.
To obtain their data, the authors prepared a questionnaire with statistical graphs taken from the daily press. These graphs were chosen by taking into account that: (a) the complexity of the graph were accessible to students; (b) the graph topic was familiar to participants; and (c) the graphical representations did not have errors or misleading information. The authors found that many prospective teachers in their sample did not have enough mathematical knowledge to perform a reading of the graph taken from the daily press. Few participants had had a specific training in reading statistical graphs during their studies at university and most of them recognized their need of specific formal training. The authors also remarked that the interpretation of graphs mobilized knowledge and feelings that influence the teachers' understanding; for example, a graph showing information about cancer incidence in women got much better interpretations than another graph mathematically equiv alent about gestational age for different animal species. They conclude that the teaching of statistical graphs should take into account a wider range of elements and skills than just statistical knowledge.

9. Final Reflections

Research summarised in this paper suggest that reading and interpreting statistical graphs is a highly complex skill that is neither achieved in compulsory education nor in the preparation of prospective primary school teachers. Several factors may explain these difficulties, such as deficits in the cognitive development of students (Berg and Smith, 1994) and the passive use of these representations in the classroom, beyond construction and interpretation by students (Ainley, Nardi and Pratt, 2000).
Another conclusion from our analysis is that the preparation of teachers to teach statistical graphs is an important topic and sometimes overlooked in research and teacher training. The evaluation of the teachers' knowledge remains an important research issue because of the scarce research and the demands that students be taught by qualified teachers. The need to demonstrate the results of teacher training programs and the discussions about what is the mathematical content for teaching that the teacher should possess (Hill, Sleep, Lewis and Ball, 2007) is another reason that makes this topic relevant. Due to the scarce prior research on teachers' content and pedagogical knowledge about statistical graphs, this survey may provide useful information to researchers interested in this topic, as well as help teachers' educators improve for the preparation of teachers in providing them information about prospective teachers' difficulties in the area.

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About the authors

Pedro Arteaga is bachelor in Mathematics and carried out a Master thesis and a doctoral dissertation in Mathematics Education. He is lecturer at the Department of Mathematics Education at the University of Granada. His research focus on Statistical Education.

Carmen Batanero is Doctor in Statistics and professor at the Department of Mathematics Education at the University of Granada. Her research focus on Statistical Education. She has been President at the International Association for Statistical Education and member of the executive comitee for the International Comission on Mathematical Instruction.

José Miguel Contreras is bachelor in Mathematics and Statistics, Master in applied statistics and carried out a doctoral dissertation in Mathematics Education. He is lecturer at the Department of Mathematics Education at the University of Granada. His research focus on Statistical Education.

Gustavo Raúl Cañadas is bachelor in Statistics, Master in applied statistics and Mathematics Education and is carrying out a doctoral dissertation in Mathematics Education. His research focus on Statistical Education. He got a grant of the "Programa de Formación del Profesorado Investigador" at the Department of Mathematics Education, University of Granada.