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Teaching Independence and Conditional Probability

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Abstract

Understanding independence and conditional probability is essential for a correct application of many probabilistic and statistical concepts and methods. Although an intuitive definition of these two concepts is possible, psychological research shows that its application in some specific circumstances becomes difficult and produces biases and wrong decisions. This paper describes some of these findings with the aim of warning teachers to take them into account when planning the teaching and assessment of this topic. We finally suggest some possible teaching devices that can help students to better understand these two concepts.

Keywords: Conditional probability, independence, learning difficulties.
AMS Subject classifications: 60-01, 97C70.

1. Introduction

In traditional mathematics, the boundary between elementary thinking and advanced thinking is reasonably well-defined, since advanced thinking is usually taken to rest on an understanding of the differential and integral calculus. The differences between advanced and elementary stochastic thinking are much less well defined. Stochastic thinking rests on ideas of randomness and independence which are frequently assumed as obvious in elementary courses, but which are in fact deep ideas requiring advanced thought.

While the learning of technical procedures can help develop stochastic thinking, these techniques need to be applied by a mature, critical stance which is often lacking in many students after a statistics course. This critical stance is essential for making interpretations which go beyond intuitive approaches, and is especially important, because research has shown that even trained statisticians retain and use invalid intuitions in some circumstances (e.g. Nisbett, & Ross, 1980; Kahneman, Slovic & Tversky, 1982).

In this paper we discuss the teaching and learning of two main concepts, the related ideas of Independence and Conditional Probability, whose understanding is closely related to that of Randomness and are therefore at the foundations of advanced stochastic thinking. We start with some considerations about the basic definitions of the two concepts and then discuss research findings related to psychological biases in conditional reasoning. We conclude with some suggestions about good pedagogic practice.

2. Two fundamental stochastic ideas: Conditional probability and independence

Conditional probability and independence were included by Heitele (1975) in his list of fundamental stochastic ideas that have helped probability theory to develop throughout history. Their relevance for statistical applications is due to the fact that conditional probability allows us to change our degree of belief in random events when new information is available. As even the unconditional probabilities are conditioned by the sample-space in which the events are defined and it is always theoretically possible to have some information about a given phenomena, within the subjective view of probability (Bayesian school) all probabilities should be considered as conditional probabilities (Lindley, 1993).

At a theoretical level, understanding conditional probability and independence is required in understanding classical and bayesian inference, as well as in correlation and regression models. Independent sampling is usually assumed as a crucial hypothesis in deriving many sampling distributions, such as the normal and T-tests for comparing two means; F-test for comparing two standards deviations or Chi-square test to assess the homogeneity of several samples. These concepts are also needed to be able to combine experiments themselves and to assign probabilities to these multiple random experiments: "*What matters in probability is almost never one single probability field, but rather interrelatedness of many probability fields*" (Freudenthal 1973, pp. 613).

Heitele hold the view that these fundamental concepts can be studied at various degrees of formalization, which are manifested in more complex cognitive and linguistic levels as one proceeds through school to university using a spiral curriculum. He also suggested that even young children may be helped to build intuitive models for these fundamental ideas that later help them to establish correct analytic knowledge. However, Heitele also pointed out the fact that these fundamental ideas are sometimes accompanied by misconceptions or errors:

"*There are fundamental ideas, as there are fundamental errors, and both are counterparts of each other. Such errors bridge the centuries, the ages and the cultural layers, and may be criteria of what is really fundamental*" (Heitele 1975, p. 191).

These errors also appear in relation to conditional probability. As suggested by Feller (1968, p. 114) "*conditional probability is a basic tool of probability theory, and it is unfortunate that its great simplicity is somewhat obscured by a singularly clumsy terminology*". Let's consider the following simple definition:

Definition 2.1. *Suppose an event B, in a sample space, for which $P(B) > 0$. In this case, for every event A in the same sample space, the conditional probability of A given that B happened is defined by:*

$$P(A/B) = P(A \cap B)/P(B) \tag{2.1}$$

Independence is closely liked to conditional probability. Classically, two events A and B are said to be independent if and only if:

$$P(A \cap B) = P(A) \times P(B),\ or \tag{2.2}$$

$$P(A/B) = P(A) \tag{2.3}$$

These two definitions are equivalent from the mathematical point of view (provided that $P(B)$ is different from 0), and illustrate the close links between independence and conditional probability. However, the intuitive understandings which each generates are different. Consider definition (2.3), which is really an abbreviation for

$$P(A/B) = P(A/E) \tag{2.4}$$

and it requires that A is as likely to be obtained from within the subset B as from the whole universal set E. For example, the probability that a Spanish new-born baby is a boy is the same than the probability that any new-born is a boy. Although, (2.3) is a "pure mathematics" definitions, people might interpret it in a "causal" way as saying that two events are independent if one do not affect the other. On the other hand, statistical data will rarely lead to exact equality for independent events, and perfect independence is not found in "real" applications. Therefore, a variety of modifications to the basic definition (2.3) are required to establish independence in practice. But the underlying principle in this definition is simple, and the classical concept of independence is clearly reflected in the conditional probability form of its definition.

It is the alternative, more common, form (2.2), which causes difficulties in understanding, partly because it is more difficult to visualise. An additional difficulty is that many textbooks argue that two probabilities may be multiplied together because "A has no effect on B". By doing so they are arguing that "effect" is the criterion for establishing independence, in defiance of the well-established view that independence neither confirms nor denies cause and effect.

3. Biases in understanding conditional probability

Apparently, the definitions of conditional probability and independence are easy, although different researchers have shown that its understanding and application are not always correct. There is a substantial body of literature which shows that many people, including statistically trained people, make poor judgements about independence and conditional probability. In fact, conditional probability is, at the same time, enriched and complexified when it is related to several particular situations which are discussed below with the aim of informing teachers and lecturers so that they can use these results in their classrooms.

3.1. Conditioning and causation

Causation is a very complex scientific concept, although it is intuitively perceived by human beings, because most of our knowledge about the world we live in was built by taking into account causes and effects. The concept of causation develops after the period of formal thought (Inhelder & Piaget, 1955), though our perceptions about causation are sometimes biased and other times causation and conditionality are confused (Pozo, 1987).

It is well known that if an event B is cause of another event A whenever B is present, A is also present and therefore $P(A/B) = 1$. On the contrary $P(A/B) = 1$ does not imply that B is a cause for A, though the existence of a conditional relationship indicates a possible causal relationship. For example birth rate is smaller in those countries where the life expectance of the population is higher. This do not imply that raising birth rates lead to a decrease in life expectancy, but that there are other factors (such as the higher proportion of women at work in developed countries) which are contributing at the same time to increase the life expectancy and to decrease the birth rate.

From a psychological point of view, the person who assesses the conditional probability $P(A/B)$ may perceive different type of relationships between A and B depending of the context. If B is perceived as a cause of A, then $P(A/B)$ is viewed as a causal relation, if A is perceived as a possible cause of B, then $P(A/B)$ is viewed like a diagnostic relation (Tversky & Kahneman, 1982). This distinction is immaterial in the mathematical computation or in the assessment of independence of events. However, causal data have greater impact in our perceptions and inferences. In the following problem students usually find relation a) to be more likely than relation b), although in fact the two events are equally probable.

Problem 3.1. *Which of the following events is more probable? a) That a girl has blue eyes if her mother has blue eyes; b) That the mother has blue eyes, if her daughter has blue eyes; c) The two events are equally probable.*

3.2. The Fallacy of the Time Axis

Falk (1979, 1989), suggested that students who confuse conditional and causal reasoning believe that an event cannot condition another event that occurs before it. When given Problem 3.2 students easily answer part (a) but are confused in part (b). Students typically argue that, because the second marble had not been drawn at the time of drawing the first marble, the result of the second draw could not influence the first. Hence the students claim that the probability in Part (b) is 1/2.

Problem 3.2. *An urn contains two white marbles and two red marbles. We pick up two marbles at random, one after the other without replacement. (a) What is the probability that the second marble is red, given that the first marble is also red? (b). What is the probability that the first marble is red, given that the second marble is also red?*

This is false reasoning, because the information in the problem that the second marble is red has reduced the sample space for the first drawing. In essence, there is now just one red marble and two white marbles for the first drawing. Hence, $P(M_1 \text{ } is \text{ } red/M_2 \text{ } is \text{ } red) = 1/3$.

In relation to this type of problems, Gras and Totohasina (1995) identified three different misconceptions or false beliefs in students about conditional probability:

- The chronological conception where students interpret the conditional probability $P(A/B)$ as a temporal relationship; that is, the conditioning event B should always precede the occurrence of event A.
- The causal conception where students interpret the conditional probability $P(A/B)$ as an implicit causal relationship; that is, the conditioning event B is the cause and A is the consequence.
- The cardinal conception where students interpret the conditional probability $P(A/B)$ as the ratio $\frac{Card(A \cap B)}{Card(B)}$. This conception is correct in the case of finite equiprobable sample spaces. However, when we are dealing with a continuous sample space or the probabilities for the simple events are not equal, this conception leads to an error.

Gras and Totohasina suggested that the origin of the chronological and causal misconceptions is cognitive, while the cardinal conception is induced by teaching. All of these misconceptions can hide the reversible character of conditional probability, which is needed if students are to understand the Bayes theorem and statistical inference.

3.3. Synchronical and Diachronical Situations

Another difference involving time in conditional probability problems are synchronical and diachronical situations. Synchronical situations are static and do not incorporate an underlying sequence of experiments. Problem 3.3, adapted from Feller (1968) is an example.

Problem 3.3. *In a population of N people there are N_F fair haired and N_W women. We pick up a person at random. Let F denote the event: "selecting at a fair-haired person". If we just consider the women population in doing the selection, the probability of selecting a fair-haired person is then $N_{F\cap W}/N_W$, that is, the conditional probability $P(F/W)$.*

As shown in this example, computing the conditional probability implies reducing the sample space (in the example to the subset of women). This reduction is, however, not easy to realise in *diachronical situations* where the problem is formulated as a series of sequential experiments, which are carried out over time, as in Problem 3.4.

Problem 3.4. *Two black marbles and two white marbles are put in an urn. We pick a white marble from the urn. Then, without putting again the white marble in the urn, we pick a second marble at random from the urn. Which is the probability that this second marble is white?*

To compute this probability we need to consider the composition of the urn after having taken a white marble out in the first experience (two black marbles and a white marble), so that the probability of obtaining a white marble in the second drawing is 1/3. We observe that this is a conditional probability, even when it sees unnatural apply here the definition (2.1). Diachronical situations can be reduced to "event" situations, if we consider a convenient sample space. We can distinguish the different marbles in the urn and its colour by the notation $W_1,\ W_2,\ B_1,\ B_2$. Let us note by W_1W_2 the event "white in the two drawings". By using a similar notation we can describe the sample space in the compound experiment by: $E = (W_1W_2, W_1B_1, W_1B_2, W_2W_1, W_2B_1, W_2B_2, B_1B_2, B_1W_1, B_1W_2, B_2B_1, B_2W_1, B_2W_2)$. After the first drawing the sample space has been reduced to: $E' = (B_1B_2, B_1W_1, B_1W_2, B_2B_1, B_2W_1, B_2W_2)$ and the probability we look for is 2/6, that is,1/3.

Problem 3.4. is a "without replacement" situation. Tarr and Lannin (2005) suggest that conditional probability problems becomes still more complex in "with replacement" situations since it is harder for students visualize the sample space in these cases. Being able to understand the reduction of sample space in conditional probability problems and correctly compute conditional probability in with and without replacement situations is typical of the highest level of understanding conditional probability, as described by Tarr and Jones (1997).

3.4. Exchanging the events in a conditional probability

The probability $P(A/B)$ is often confused with the probability $P(B/A)$ and even with the joint probability $P(A \cap B)$. For example, Pollatsek et al. (1987), found that a 69 percent of subjects considered c) to be the correct answer to the item shown in the following example.

Problem 3.5. *In which of the following statements do you have the most confidence?*

a) *That a blue cab is correctly identified at night as a blue cab;*

b) *That a cab identified at night as a blue cab is a blue cab;*

c) *The two events are equally probable.*

The authors suggest that part of this problems seem to be caused by formal notation or verbal ambiguity in expressing conditional probabilities. However in a second experiment they also found a big amount of difficulty even when the conditional probabilities were given in percentages. Even when patterns of responses were consistent with the existence of a causal bias in judging conditional probabilities, data also suggested that a major source of error was the confusion between conditional probabilities and join probabilities. Similar results were obtained by Batanero et al. (1996) in students interpretation of contingency tables, where many students confused "the percentage of smokers who get bronchial disease" with "the percentage of people with bronchial disease who smoke".

3.5. Confusing mutually exclusive events with independent events

Because probability has been taught in a formal way to many students, it is easy for them to confuse the concepts of independent events and mutually exclusive events. This belief is false. For example if we throw a die and we flip a coin, the events A "getting 6 on the die" and B "getting a head" are independent. However, they are not mutually exclusive. A less formal teaching where more emphasis is put on examples and investigations might help to avoid the confusions described.

3.6. Other difficulties

Finally, for the specific case of independence, Chapman & Chapman (1967) have shown that in emotionally charged situations people make judgements about the independence of events which disregard the numerical data and rely principally on their subjective beliefs about the truth of a situation. Yet the making of accurate decisions in such real-world situations is at the heart of applied advanced stochastic thinking.

There is also substantial evidence that children hold beliefs about random generators (e.g. a die, a coin) that might lead then to difficulties in perceiving independence of random experiments carried out with them. For example Truran (1992) reported cases of children who think that a die has "a mind of its own" or may be controlled by outsides forces.

Fischbein, Nello and Marino (1991) suggested that even when the simultaneous operation of random generator (e.g., tossing three dice together) is mathematically equivalent to their consecutive operation (e.g., tossing a dice three times) many children do not consider these two experiments to be equivalent.

4. Some teaching resources

In this section we are suggesting ways in which the teaching of these concepts might be improved. Firstly, there are many paradoxes in probability which relate to misunderstandings on conditional probability (Bar-Hillel & Falk, 1982), and could e used as didactic resources in the teaching of the topic. An example is given below:

Problem 4.1. *Three cards are in an urn. One is blue on both sides, one is red on both sides, and one is blue on one side and red on the other. We pick a card at random from the urn and put it on the table as it comes out. If the face we can see is red, which is the probability that the other is also red?*

Falk (1989) discusses this problem, where most people give the answer 1/2, because they condition the probability supposing the card with blue on both sides is out of the sample space. They, therefore, think that each of the two remaning cards have the same probability to be drawn. Even when it is true that we cannot obtain the double blue card, when we can see a red face, "the hidden face is red" is not the event that should condition our calculus, but the event that "one of the faces is red". Therefore we should take into account the card with blue on boths sides in the sample space of this experiment and the answer to the problem is 1/3.

4.1. Experiments and simulations

One simple way to convince the students that their ideas or their solutions to probability problems are wrong is to confront their ideas with experiments. For example, if students are confused with the solution to Problem 4.1, we might organise a classroom experiment, where students working in pairs repeat many times the trial described in problem 4.1 and record the results. They can then compare those trials where the visible side of the card is red and count in how many of them the hidden card is also red. Experimentally they can estimate and compare the two probabilities involved in the problem.

Internet applets may be used to simulate some classical games, for example the Monty Hall problem where independence and conditional probability are needed to solve the related problem. Contreras (2009) analysed the mathematical objects implicit in the use of some games related to conditional probability, part of which are listed in Table 1. These games include variations of the Betrand's box paradox (Monty Hall and several generalizations of the same; the two coins problem, etc.) and the Saint Petersburg paradox.

When working with one of these applets, once the students understand the problem, and after doing some trials of the game with the applet, the lecturer asks the students to solve the problem or to find the strategy that produces the best chance to win over a long series of trials. After a while each student will list his/her solution in the blackboard and the lecturer will give them the opportunity of expressing their ideas and check their conjectures. Results will be compared time to time and, when necessary, the game will be repeated to increase the total number of experiments. Generally, when increasing the number of trials, some solutions are discarded, because results contradict the teachers' initial expectations. Finally, the students have a clear preference for one or several favourite solutions, though some of them might keep to a wrong solution. A discussion will be organised by the lecturer where different teams should give a mathematical proof of their solution. Along this discussion, both correct reasoning and possible misconceptions will be revealed. These activities will serve to reflect on the properties of conditional probability and compound experiments. At the same time students would use other concepts such as event, probability and convergence, combinatorial operations, addition and multiplication rules, independence, random variable, expectation and sampling.

Table 1: Games related to conditional probability

Nombre	Web address
100 puertas	estadisticaparatodos.es/taller/montyhall/montyhall.html
Advanced Monty Hall	www.shodor.org/interactivate/activities/AdvancedMontyHall
Cartas	web.educastur.princast.es/ies/iesreype/Departamentos/ /Mates/paradojas.htm
Coin Toss Applet	www.ibiblio.org/links/applets/appindex/cointoss_a.html
Dice Table	www.shodor.org/interactivate/activities/DiceTable
Gamblers Fallacy Simulation	onlinestatbook.com/simulations/gambler_fallacy/gambler.html
Generalized Monty Hall	www.shodor.org/interactivate/activities/GeneralizedMontyHall
Java Applets: TwoArm	www.dim.uchile.cl/ mkiwi/ma34a/libro/chapter4/ /TwoArm/TwoArm.html
Las dos monedas	www.betweenwaters.com/probab/coingame/coinmainD.html
Let's make a deal	www.stat.sc.edu/˜west/javahtml/LetsMakeaDeal.html
Marbles	www.shodor.org/interactivate/activities/marbles
Monty Knows	math.ucsd.edu/˜crypto/Monty/monty.html
Probabilty by Surprise	www-stat.stanford.edu/˜susan/surprise
Racing Game with One Die	www.shodor.org/interactivate/activities/RacingGameWithOneDie
Random Birthday Applet	www-stat.stanford.edu/˜susan/surprise/Birthday.html
The gambler's ruin problem	math.ucsd.edu/˜anistat/gamblers_ruin.html
The three door dilemma	www.dcity.org/braingames/3doors/index.htm
Three doors simulation	www.decisionhelper.com/montyhall.htm
Two colours	www.shodor.org/interactivate/activities/twocolors

4.2. Adequate notations

Experimental probability can "show" the students that the initial conjecture was false. Convincing them, however, of the reason why their intuitions were incorrect requires a combinatorial or mathematical approach to solve the problem. In Problem 4.1, one way to visualise the mathematics behind the experimental solution is imagining that in each card we put the number "1" on one side and "2" on the other side. Thus, we might represent each card by a pair of letters with sub index, which indicate the sides. We agree with our students in that the first position indicates the face we can observe on the table and the second position refers to the hidden face. The sample space is then represented by:

$$E = \{R_1R_2; R_2R_1; R_1B_2; B_2R_1; B_1B_2; B_2B_1\}$$

and the conditioning event "we see a red side" is:

$$E' = \{R_1R_2; R_2R_1; R_1B_2\}$$

from where the $P(\textit{the second side is red}) = 2/3$.

4.3. Tree diagrams and visualization

Students may use a tree diagram to write down all the elements of the sample space and to help them visualise the nature of the experiment, in particular for diachronic experiments (Sanchez & Hernández, 2003). According to Fischbein (1975), tree diagram belong to "diagramatic models" and present important intuitive characteristics. They offer a global representation of the situation structure and this contributes to the immediacy of understanding and to finding the problem solution. We might use the next tree diagram to visualize the structure of Problem 4.1, and to make clear the solution to the problem, because there are three cases where the face shown is red and in two of them the hidden face is also red.

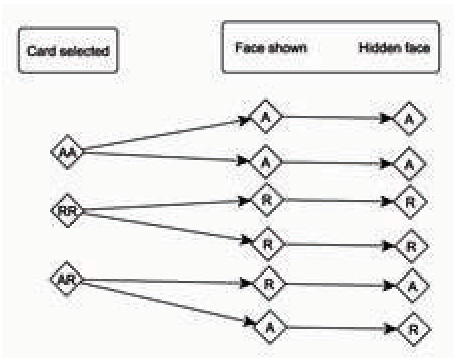

Figure 1

Table 2: Resources for visualizing conditional probability and related objects

Nombre	Web address
Bayes Rule	www.bolderstats.com/gallery/prob/bayes.html
Conditional probability	www.rfbarrow.btinternet.co.uk/htmasa2/Prob2.htm
Conditional Probability and Multiplication Rule	www.spsu.edu/math/deng/m2260/stat/cond/cond.html
Dice and conditional probability	www.math.fau.edu/Richman/Liberal/dice.htm
Probability Tree	http://www-stat.stanford.edu/ susan/surprise/ProbabilityTree.html
Tree Diagram Applet	http://www.stat.tamu.edu/˜west/applets/tree.html
Two Events: Conditioning	www.stat.wvu.edu/SRS/Modules/ProbLaw/GivenProb.html
Venn Diagram Applet	www.teachers.ash.org.au/miKemath/VennDiagramApplet/ /VennGame.html
Venn Diagrams	www.shodor.org/interactivate/activities/VennDiagrams
Venn Diagrams and probability	www.stat.berkeley.edu/˜stark/Java/Html/Venn3.htm

Again, Internet is a source of tools that can help students understand and build tree diagrams and visualize the meaning of conditional probability, independence and other related ideas. These tools are analysed in Contreras (2009). In table 2 we present a list of some of these resources and include calculators (for example calculators for Bayes' formula), Venn's diagrams that visualize the relationships between two events, its complementaries and the effect on conditional probability of the relative size of different events, different visualizations of conditional and compound probability, and tools that help building a tree diagram.

5. Final remarks

As suggested by Tarr and Lannin (2005), the inclusion of conditional probability in the school curriculum is a challenge for teachers, and instruction should be informed by research results in order to help students acquire this concept. Moreover, as stated by Romberg et al. (1991), when assessment is viewed as a continuous and dynamic process that can be used by teachers to help students to attain curricular goals. Garfield (1995) suggests that effective teaching should be based on knowledge of students' preconceptions, because when learning something new, students contruct their own meaning by connecting the new information to what they already believe to be true.

A knowledge of student's difficulties is particularly needed in case of independence and conditional probability where most teachers agree that students have a big deal of difficulty. In this sense we hope the examples along this paper (all of which were used in different research work either in questionnaires or interviews) can help teachers build their own in-classroom exercises and problems for teaching and assessing conditional probability.

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