One-loop Flavor Change in Little Higgs Models

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in collaboration with:

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1. Little Higgs: the hierarchy and the flavor problems
2. Models and lepton flavor mixing:

   [LHT] *Littlest* Higgs with T-parity

   [SLH] *Simplest* little Higgs
3. One-loop contributions to LFV processes: $\mu \to e\gamma$, $\mu \to ee\bar{e}$, $\mu\,N \to e\,N$
4. Discussion
5. Conclusions

JHEP **01** (2009) 080 [arXiv: 0811.2891 [hep-ph]] and [Work to appear]

Little Higgs

[Arkani-Hamed, Cohen, Georgi '01]

Hierarchy problem: the Higgs mass should be of order v (electroweak scale) but it receives quadratic loop corrections of the order of the theory cutoff (Planck scale?)

Naturalness $\Rightarrow$ New Physics at the TeV scale

[SUSY: cancellations of quadratic Higgs mass corrections provided by superpartners]

In LH models the Higgs is a pseudo-Goldstone boson of an approximate global symmetry broken at f (TeV scale)

(i) Product group

the SM $SU(2)_L$ group from the diagonal breaking of two or more gauge groups

e.g.: *Littlest* Higgs [Arkani-Hamed, Cohen, Katz, Nelson '02]

(ii) Simple group

the SM $SU(2)_L$ group from the breaking of a larger group into an $SU(2)$ subgroup

e.g.: *Simplest* Little Higgs ($SU(3)$ simple group) [Kaplan, Schmaltz '03]

Little Higgs

– The low energy *dof* described by a nonlinear sigma model, an effective theory valid below a cutoff $\Lambda \sim 4\pi f$ (order of 10 TeV) since then the loop corrections are

$$\Delta M_h^2 \sim \left\{ y_t^2, g^2, \lambda^2 \right\} \frac{\Lambda^2}{16\pi^2} \lesssim (1 \text{ TeV})^2$$

Ultraviolet completion (unknown) is required only for physics above Λ

– The global symmetry explicitly broken by gauge and Yukawa interactions, giving the Higgs a mass and non-derivative interactions, preserving the cancellation of one-loop quadratic corrections (collective symmetry breaking)

The sensitivity at two loops to a 10 TeV cutoff is *not unnatural*

LH introduce extra fermions and gauge bosons: new source of flavor mixing

$\Rightarrow$ Obtain and revise predictions for $\boxed{\text{lepton flavor changing processes}}$

Littlest Higgs

[Arkani-Hamed, Cohen, Katz, Nelson '02]

(1) $SU(5) \to SO(5)$ by $\Sigma_0 = \begin{pmatrix} \mathbf{0}_{2\times2} & 0 & \mathbf{1}_{2\times2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2\times2} & 0 & \mathbf{0}_{2\times2} \end{pmatrix}$, $\quad \Sigma(x) = \mathrm{e}^{\mathrm{i}\Pi/f}\Sigma_0\mathrm{e}^{\mathrm{i}\Pi^T/f} = \mathrm{e}^{2\mathrm{i}\Pi/f}\Sigma_0$

where $\Pi(x) = \pi^a(x)X^a$ and X^a are the $24-10=$ 14 broken generators $\Rightarrow$ 14 GB

$$G \equiv SU(5) \supset [SU(2)\times U(1)]_1 \times [SU(2)\times U(1)]_2 \text{ (gauge)} \xrightarrow{\langle\Sigma\rangle=\Sigma_0} SU(2)_L \times U(1)_Y$$

[unbroken]: $Q_1^a + Q_2^a,\ Y_1 + Y_2 \ \Rightarrow$ 4 gauge bosons (γ, Z, W^+, W^-) remain massless

[broken]: $Q_1^a - Q_2^a,\ Y_1 - Y_2 \ \Rightarrow$ 4 gauge bosons (A_H, Z_H, W_H^+, W_H^-) get masses of order f

4 WBGB: $(\eta, \omega^0, \omega^+, \omega^-)$ eaten by (A_H, Z_H, W_H^+, W_H^-)

10 GB: $\underbrace{H \text{(complex } SU(2) \text{ doublet)}}$, Φ (complex $SU(2)$ triplet)

(2) EWSB: $SU(2)_L \times U(1)_Y \xrightarrow{\langle H\rangle} U(1)_{\mathrm{QED}} \Rightarrow H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^+ \\ v + h + \mathrm{i}\phi^0 \end{pmatrix}$

3 WBGB: (ϕ^0, ϕ^+, ϕ^-) eaten by (Z, W^+, W^-)

1 GB: h

Littlest Higgs with T-parity

[Cheng, Low '03]

New particles at TeV scale coupling to SM particles $\Rightarrow$ tension with EW precision tests

$\rightsquigarrow$ T-parity discrete symmetry under which SM (most of new) particles are even (odd)

– Gauge sector: $G_1 \stackrel{\mathrm{T}}{\longleftrightarrow} G_2$ with $G_j = (W_j^a, B_j)$ gauge bosons of $[SU(2)\times U(1)]_{j=1,2}$

and $g \equiv g_1 = g_2$, $g' \equiv g'_1 = g'_2$

T-even: $B, W^3(\gamma, Z), W^+, W^- \leftarrow \frac{1}{\sqrt{2}}(G_1+G_2)$

T-odd: $A_H, Z_H, W_H^+, W_H^- \leftarrow \frac{1}{\sqrt{2}}(G_1-G_2)$

$$\boxed{\mathcal{L}_G = \sum_{j=1}^{2}\left[-\frac{1}{2}\mathrm{Tr}\left(\widetilde{W}_{j\mu\nu}\widetilde{W}_j^{\mu\nu}\right) - \frac{1}{4}B_{j\mu\nu}B_j^{\mu\nu}\right]}$$

– Scalar sector: $\Pi \stackrel{\mathrm{T}}{\longrightarrow} -\Omega\Pi\Omega$, where $\Omega = \mathrm{diag}(-1,-1,1,-1,-1)$

$$\Rightarrow \Sigma \stackrel{\mathrm{T}}{\longrightarrow} \widetilde{\Sigma} = \Omega\Sigma_0\Sigma^\dagger\Sigma_0\Omega \qquad \Sigma \stackrel{G}{\longrightarrow} V\Sigma V^T$$

T-even: SM H doublet $(h, \phi^0, \phi^+, \phi^-)$

T-odd: the others $(\eta, \omega^0, \omega^+, \omega^-, \Phi)$

$$\boxed{\mathcal{L}_S = \frac{f^2}{8}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)\right]} \supset \text{gauge boson masses}$$

with $D_\mu\Sigma = \partial_\mu\Sigma - \sqrt{2}\mathrm{i}\sum_{j=1}^{2}\left[gW_{j\mu}^a(Q_j^a\Sigma + \Sigma Q_j^{aT}) - g'B_{j\mu}(Y_j\Sigma + \Sigma Y_j^T)\right]$

Littlest Higgs with T-parity

– Fermion (lepton) sector: (similarly for quark sector)

(a) Introduce $SU(2)_L$ doublets $\boxed{l_{1L},\, l_{2L},\, l_{HR}}$ in

$$\Psi_1[\overline{\mathbf{5}}] = \begin{pmatrix} -\mathrm{i}\sigma^2 l_{1L} \\ 0 \\ 0 \end{pmatrix} \quad \Psi_2[\mathbf{5}] = \begin{pmatrix} 0 \\ 0 \\ -\mathrm{i}\sigma^2 l_{2L} \end{pmatrix} \quad \Psi_R = \begin{pmatrix} \tilde{\psi}_R \\ \chi_R \\ -\mathrm{i}\sigma^2 l_{HR} \end{pmatrix}$$

$$\Psi_1 \overset{\mathrm{T}}{\longleftrightarrow} \Omega\Sigma_0\Psi_2 \qquad\qquad \Psi_1 \xrightarrow{G} V^*\Psi_1,\ \Psi_2 \xrightarrow{G} V\Psi_2$$

$$\Psi_R \xrightarrow{\mathrm{T}} \Omega\Psi_R \qquad\qquad \Psi_R \xrightarrow{G} U\Psi_R \Longleftrightarrow (\xi\Psi_R) \xrightarrow{G} V(\xi\Psi_R)$$

$\Rightarrow$ T-even: $(\nu_L,\ \ell_L)^T = l_L = \frac{1}{\sqrt{2}}(l_{1L} - l_{2L})$, χ_R

T-odd: $(\nu_{HL},\ \ell_{HL})^T = l_{HL} = \frac{1}{\sqrt{2}}(l_{1L} + l_{2L})$, $(\nu_{HR},\ \ell_{HR})^T = l_{HR}$, $\tilde{\psi}_R$

To obtain heavy masses respecting gauge and T symmetries:

$$\boxed{\mathcal{L}_{Y_H} = -\kappa f \left(\overline{\Psi}_2\xi + \overline{\Psi}_1\Sigma_0\xi^\dagger\right)\Psi_R + \text{h.c.}}$$

$$\xi = \mathrm{e}^{\mathrm{i}\Pi/f} \xrightarrow{\mathrm{T}} \Omega\xi^\dagger\Omega$$

$$\xi \xrightarrow{G} V\xi U^\dagger \equiv U\xi\Sigma_0 V^T\Sigma_0$$

Littlest Higgs with T-parity

(b) Then the light left-handed and heavy fermion gauge interactions are fixed!

$$\boxed{\mathcal{L}_F = \mathrm{i}\overline{\Psi}_1\gamma^\mu D^*_\mu\Psi_1 \ + \ \mathrm{i}\overline{\Psi}_2\gamma^\mu D_\mu\Psi_2 + \mathrm{i}\overline{\Psi}_R\gamma^\mu\left(\partial_\mu + \tfrac{1}{2}\xi^\dagger(D_\mu\xi) + \tfrac{1}{2}\xi(\Sigma_0 D^*_\mu\Sigma_0\xi^\dagger)\right)\Psi_R}$$

with $D_\mu = \partial_\mu - \sqrt{2}\mathrm{i}g(W^a_{1\mu}Q^a_1 + W^a_{2\mu}Q^a_2) + \sqrt{2}\mathrm{i}g'\left(Y_1 B_{1\mu} + Y_2 B_{2\mu}\right)$

introducing so far ignored $\mathcal{O}(v^2/f^2)$ couplings to Goldstones that render the one-loop amplitudes UV finite [Hubisz, Meade '05]

[del Águila, JI, Jenkins '09]

(c) Introduce light right-handed singlets (ν_R, ℓ_R) and their gauge interactions

$\boxed{\mathcal{L}'_F = \mathrm{i}\overline{\ell}_R\gamma^\mu(\partial_\mu + \mathrm{i}g' y_\ell B_\mu)\ell_R}$ $\qquad y_\ell = -1 \qquad$ [requires enlarging $SU(5)$]

[Goto, Okada, Yamamoto '09]

(d) Introduce masses for light (down-type) fermions from: [Chen, Tobe, Yuan '06]

$\boxed{\mathcal{L}_Y = \frac{\mathrm{i}\lambda_\ell}{2\sqrt{2}} f\epsilon_{ij}\epsilon_{xyz}\left[(\overline{\Psi}'_2)_x\Sigma_{iy}\Sigma_{jz}X + \text{T-transformed}\right]\ell_R}$ $\qquad \Psi'_2 = (0,0,l_{2L})^T\,, \quad X = (\Sigma_{33})^{-\frac{1}{4}}$

Littlest Higgs with T-parity

Flavor mixing: (three families)

- In the SM after EWSB, the Yukawa interactions generate masses and mixings (CC):

$$\overline{u}^0_L M_u u^0_R + \overline{d}^0_L M_d d^0_R + \text{h.c.}$$

$$\text{diag}(m_{q_i}) = V_q^\dagger M_q U_q \Rightarrow q^0_L = V_q q_L, \quad q^0_R = U_q q_R$$

$$\Rightarrow \quad \mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \overline{u}^0_L \not{W}^+ d^0_L + \text{h.c.} = \frac{g}{\sqrt{2}} \overline{u}_L \not{W}^+ (V_u^\dagger V_d) d_L + \text{h.c.} \qquad V_{\text{CKM}} \equiv V_u^\dagger V_d$$

- In the LHT, $\mathcal{L}_{Y_H}$ generates heavy masses inducing heavy-light mixings:

$$\sqrt{2} f\, \overline{q}^0_{HL} \kappa q^0_{HR} + \text{h.c.} \qquad \text{(replace } q \text{ with } l \text{ for leptons)}$$

$$\text{diag}(\kappa_i) = V_H^\dagger \kappa U_H \Rightarrow q^0_{HL} = V_H q_{HL}, \quad q^0_{HR} = U_H q_{HR}$$

$$\Rightarrow \quad \mathcal{L}_{\text{LHT}} \supset g\, \overline{q}^0_{HL} \not{G}^\dagger_H q^0_L + \text{h.c.} = g\, \overline{q}_{HL} \not{G}^\dagger_H \begin{pmatrix} V_H^\dagger V_u\, u_L \\ V_H^\dagger V_d\, d_L \end{pmatrix} + \text{h.c.} \qquad \begin{array}{l|l} V_{Hu} \equiv V_H^\dagger V_u & V_{H\nu} \\ V_{Hd} \equiv V_H^\dagger V_d & V_{H\ell} \end{array}$$

$\Rightarrow$ for instance: $\boxed{V^{i\alpha}_{H\ell}\, \overline{\nu}^i_{HL} \not{W}^\dagger_H \ell^\alpha_L}$ CC and $\boxed{V^{i\alpha}_{H\ell}\, \overline{\ell}^i_{HL} \{\not{A}_H, \not{Z}_H\} \ell^\alpha_L}$ NC (tree level!)

Simplest Little Higgs

[Kaplan, Schmaltz '03]

(1) $G \equiv [SU(3) \times U(1)]_1 \times [SU(3) \times U(1)]_2 \to [SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$

by $\Phi_1[(\mathbf{3},\mathbf{1})]$, $\Phi_2[(\mathbf{1},\mathbf{3})]$ acquiring *vevs* $\langle\Phi_1\rangle = (0,0,f\cos\beta)^T$, $\langle\Phi_2\rangle = (0,0,f\sin\beta)^T$

$\Rightarrow 18 - 8 = 10$ broken generators

$G \supset [SU(3) \times U(1)_\chi]$ (gauge) $\xrightarrow{\langle\Phi_1\rangle,\ \langle\Phi_2\rangle} SU(2)_L \times U(1)_Y$

4 unbroken generators $\Rightarrow$ 4 gauge bosons (γ, Z, W^+, W^-) remain massless

5 broken generators $\Rightarrow$ 5 gauge bosons $(X^+, X^-, Y^0, \overline{Y}^0, Z')$ get masses of order f

5 WBGB: $(x^+, x^-, y^0, y^{0\dagger}, z')$ eaten by $(X^+, X^-, Y^0, \overline{Y}^0, Z')$

5 GB: H (complex $SU(2)$ doublet), η (real $SU(2)$ singlet)

(2) EWSB: $SU(2)_L \times U(1)_Y \xrightarrow{\langle H\rangle} U(1)_{\rm QED} \Rightarrow H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^+ \\ v + h + i\phi^0 \end{pmatrix}$

3 WBGB: (ϕ^0, ϕ^+, ϕ^-) eaten by (Z, W^+, W^-)

1 GB: h

Simplest Little Higgs

– Gauge sector: $\boxed{\mathcal{L}_G = -\frac{1}{2}\mathrm{Tr}\left\{\widetilde{A}_{\mu\nu}\widetilde{A}^{\mu\nu}\right\} - \frac{1}{4}B_{x\mu\nu}B_x^{\mu\nu}}$

$$A^a T_a = \frac{A^3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{A^8}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W^+ & Y^0 \\ W^- & 0 & X^- \\ Y^{0\dagger} & X^+ & 0 \end{pmatrix}$$

– Scalar sector: $\boxed{\mathcal{L}_S = |D_\mu\Phi_1|^2 + |D_\mu\Phi_2|^2} \supset$ gauge boson masses $\quad \Phi_{1,2} \sim \mathbf{3}_{-\frac{1}{3}}$

$$D_\mu = \partial_\mu - \mathrm{i}gA^a_\mu T_a + \mathrm{i}g_x Q_x B_{x\mu}\,, \qquad g_x = \frac{g t_W}{\sqrt{1-t_W^2/3}}$$

gauge boson masses diagonalized by:

$$\begin{pmatrix} A^3 \\ A^8 \\ B_x \end{pmatrix} = \begin{pmatrix} 0 & c_W & -s_W \\ \sqrt{1-t_W^2/3} & s_W t_W/\sqrt{3} & s_W/\sqrt{3} \\ -t_W/\sqrt{3} & s_W\sqrt{1-t_W^2/3} & c_W\sqrt{1-t_W^2/3} \end{pmatrix}\begin{pmatrix} Z' \\ Z \\ A \end{pmatrix} + \mathcal{O}(v^2/f^2)$$

Simplest Little Higgs

– Lepton sector:

For each family $m = 1, 2, 3$ introduce the following multiplets:

$$\boxed{\mathbf{3}_{-\frac{1}{3}} \equiv L_m^T = (\nu_L,\ \ell_L,\ \mathrm{i}N_L)_m \qquad \mathbf{1}_0 \equiv \nu_{Rm} \qquad \mathbf{1}_{-1} \equiv \ell_{Rm} \qquad \mathbf{1}_0 \equiv N_{Rm}}$$

Yukawas:

$$\boxed{\mathcal{L}_Y \supset \mathrm{i}\lambda_N^m \overline{N}_{Rm} \Phi_2^\dagger L_m + \frac{\mathrm{i}\lambda_\ell^{mn}}{\Lambda} \overline{\ell}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j L_n^k + \mathrm{h.c.}}$$

Gauge interactions:

$$\boxed{\mathcal{L}_F \supset \overline{\psi}_m \mathrm{i} \not{D} \psi_m} \qquad \psi_m = \{L_m, \ell_{Rm}, N_{Rm}\}$$

Simplest Little Higgs

– Quark sector:

(i) Universal embedding (U):

$$\boxed{\mathbf{3}_{\frac{1}{3}} \equiv Q_m^T = (u_L,\ d_L,\ \mathrm{i}U_L)_m \qquad \mathbf{1}_{\frac{2}{3}} \equiv u_{Rm} \qquad \mathbf{1}_{-\frac{1}{3}} \equiv d_{Rm} \qquad \mathbf{1}_{\frac{2}{3}} \equiv U_{Rm}}$$

Yukawas: $\{u^1_{Rm}, u^2_{Rm}\} \leftrightarrow \{u_{Rm}, U_{Rm}\}$

$$\boxed{\mathcal{L}_Y \supset \mathrm{i}\lambda_1^{um}\overline{u}^1_{Rm}\Phi_1^\dagger Q_m + \mathrm{i}\lambda_2^{um}\overline{u}^2_{Rm}\Phi_2^\dagger Q_m + \frac{\mathrm{i}\lambda_d^{mn}}{\Lambda}\overline{d}_{Rm}\epsilon_{ijk}\Phi_1^i\Phi_2^j Q_n^k + \text{h.c.}}$$

Gauge interactions:

$$\boxed{\mathcal{L}_F \supset \overline{Q}_m \mathrm{i}\not{D}^L Q_m + \overline{u}_{Rm}\mathrm{i}\not{D}^u u_{Rm} + \overline{d}_{Rm}\mathrm{i}\not{D}^d d_{Rm} + \overline{U}_{Rm}\mathrm{i}\not{D}^u U_{Rm}}$$

Simplest Little Higgs

– Quark sector:

(ii) Anomaly-free embedding (AF): [Kong '03]

$$
\begin{array}{llllll}
\bar{\mathbf{3}}_0 \equiv Q_1^T = (d_L,\ -u_L,\ \mathrm{i}D_L) & \mathbf{1}_{-\frac{1}{3}} \equiv d_R & \mathbf{1}_{\frac{2}{3}} \equiv u_R & \mathbf{1}_{-\frac{1}{3}} \equiv D_R \\
\bar{\mathbf{3}}_0 \equiv Q_2^T = (s_L,\ -c_L,\ \mathrm{i}S_L) & \mathbf{1}_{-\frac{1}{3}} \equiv s_R & \mathbf{1}_{\frac{2}{3}} \equiv c_R & \mathbf{1}_{-\frac{1}{3}} \equiv S_R \\
\mathbf{3}_{\frac{1}{3}} \equiv Q_3^T = (t_L,\ b_L,\ \mathrm{i}T_L) & \mathbf{1}_{\frac{2}{3}} \equiv t_R & \mathbf{1}_{-\frac{1}{3}} \equiv b_R & \mathbf{1}_{\frac{2}{3}} \equiv T_R
\end{array}
$$

Yukawas: $\{d^1_{R1}, d^2_{R1}\} \leftrightarrow \{d_R, D_R\}$, $\{d^1_{R2}, d^2_{R2}\} \leftrightarrow \{s_R, S_R\}$, $\{u^1_{R3}, u^2_{R3}\} \leftrightarrow \{t_R, T_R\}$

$$
\begin{aligned}
\mathcal{L}_Y \supset\ & \mathrm{i}\lambda_1^t \bar{u}^1_{R3}\Phi_1^\dagger Q_3 + \mathrm{i}\lambda_2^t \bar{u}^2_{R3}\Phi_2^\dagger Q_3 + \frac{\mathrm{i}\lambda_b^m}{\Lambda}\bar{d}_{Rm}\epsilon_{ijk}\Phi_1^i\Phi_2^j Q_3^k \\
& + \mathrm{i}\lambda_1^{dn}\bar{d}^1_{Rn}Q_n^T\Phi_1 + \mathrm{i}\lambda_2^{dn}\bar{d}^2_{Rn}Q_n^T\Phi_2 + \frac{\mathrm{i}\lambda_u^{mn}}{\Lambda}\bar{u}_{Rm}\epsilon_{ijk}\Phi_1^{*i}\Phi_2^{*j}Q_n^k + \text{h.c.}
\end{aligned}
$$

$d_{Rm} \in \{d_R, s_R, b_R, D_R, S_R\}$ $u_{Rm} \in \{u_R, c_R, t_R, T_R\}$ $n = 1, 2$

Gauge interactions:

$$
\mathcal{L}_F \supset \bar{Q}_m \mathrm{i}\not{D}^L_m Q_m + \bar{u}_{Rm}\mathrm{i}\not{D}^u u_{Rm} + \bar{d}_{Rm}\mathrm{i}\not{D}^d d_{Rm} + \bar{D}_R\mathrm{i}\not{D}^d D_R + \bar{S}_R\mathrm{i}\not{D}^d S_R + \bar{T}_R\mathrm{i}\not{D}^u T_R
$$

Simplest Little Higgs

(Lepton) Flavor mixing: (three families)

– After EWSB the light and the heavy neutrino of the same family mix at $\mathcal{O}(v/f)$

– If λ_N^m and λ_ℓ^{mn} are not aligned there is also family mixing:

$$\ell^0_{Lm} = [V_\ell^\dagger \ell_L]_m$$

$$\begin{pmatrix} \nu_L^0 \\ N_L^0 \end{pmatrix}_m = \left[\begin{pmatrix} \mathbf{1} & -\delta_\nu \\ \delta_\nu & \mathbf{1} \end{pmatrix} \begin{pmatrix} V_\ell^\dagger \nu_L \\ N_L \end{pmatrix} \right]_m + \mathcal{O}(\delta_\nu^2), \qquad \delta_\nu \equiv -\frac{v}{\sqrt{2} f \tan\beta}$$

$\Rightarrow$ Heavy-light mixings in CC only: (mixings in $\overline{N}_{Lm}\{Y^0, Z'\}\nu_{Li}$ are irrelevant)

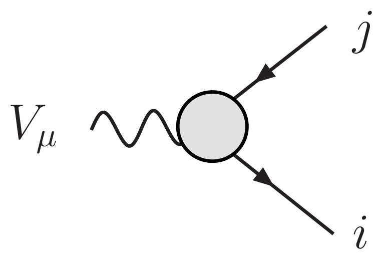
$$\mathcal{L}_{\rm SLH} \supset \underbrace{-\frac{g}{\sqrt{2}}\left(1-\frac{\delta_\nu^2}{2}\right) V_\ell^{im*}\overline{N}_{Lm}\gamma^\mu X_\mu^\dagger \ell_{Li}}_{\mathcal{O}(1)} \underbrace{-\frac{\mathrm{i}g}{\sqrt{2}}\delta_\nu V_\ell^{im*}\overline{N}_{Lm}\gamma^\mu W_\mu^\dagger \ell_{Li}}_{\mathcal{O}(v/f)} \quad + \text{h.c.}$$

[no mixing in NC because there is no heavy charged lepton]

One-loop contributions to Lepton FV processes

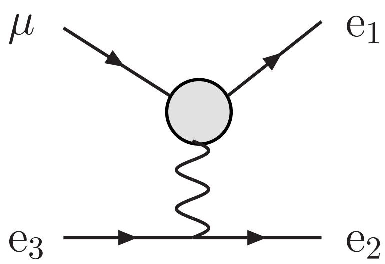
[Buras *et al* ’07 . . .]
[del Águila, JI, Jenkins ’09 . . .]

$\mu \to e\gamma$:



vertex (triangles)

$\mu \to e e \bar{e}$:



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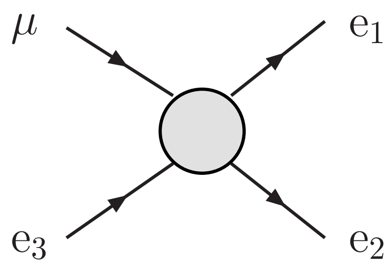
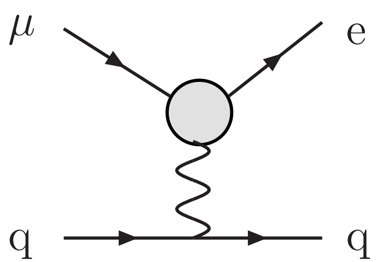


+ crossed ($e_1 \leftrightarrow e_2$)

V-penguins (triangles+SE) e-boxes

$\mu\, N \to e\, N$:



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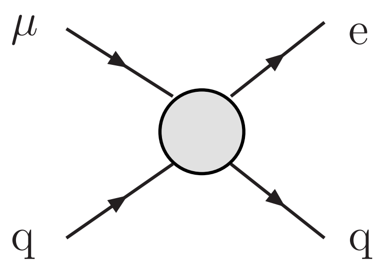


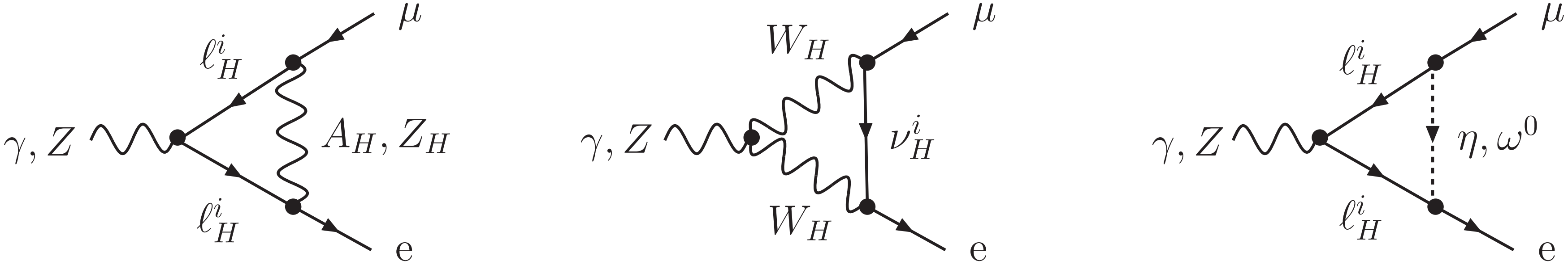
(q = u, d)

V-penguins (triangles+SE) q-boxes

One-loop contributions to Lepton FV processes

LHT

- Triangle diagrams $\Rightarrow$ vertex and $\gamma,\ Z$ penguins



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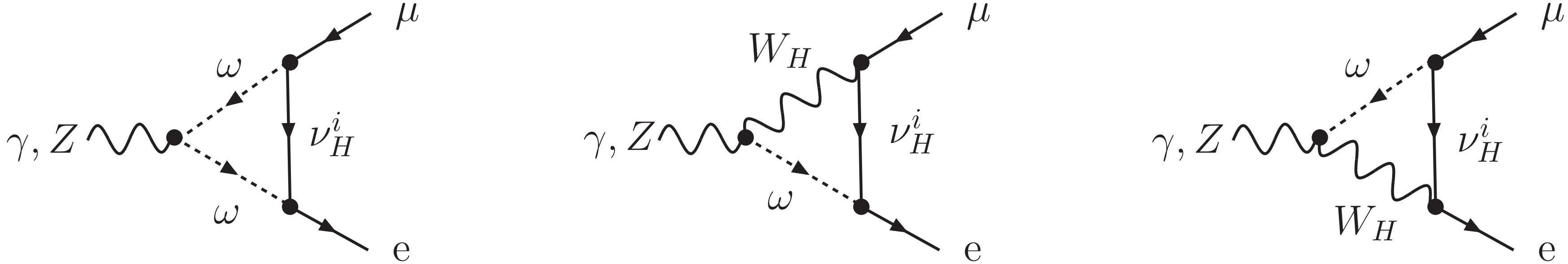

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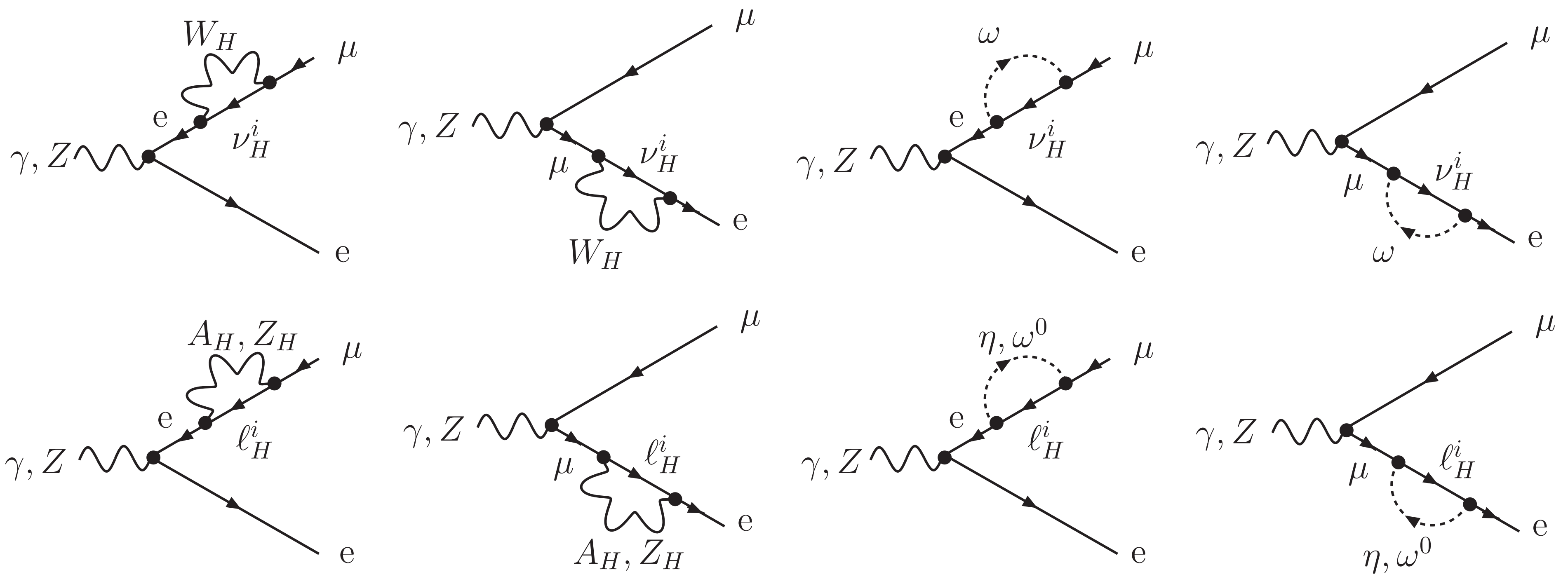
One-loop contributions to Lepton FV processes

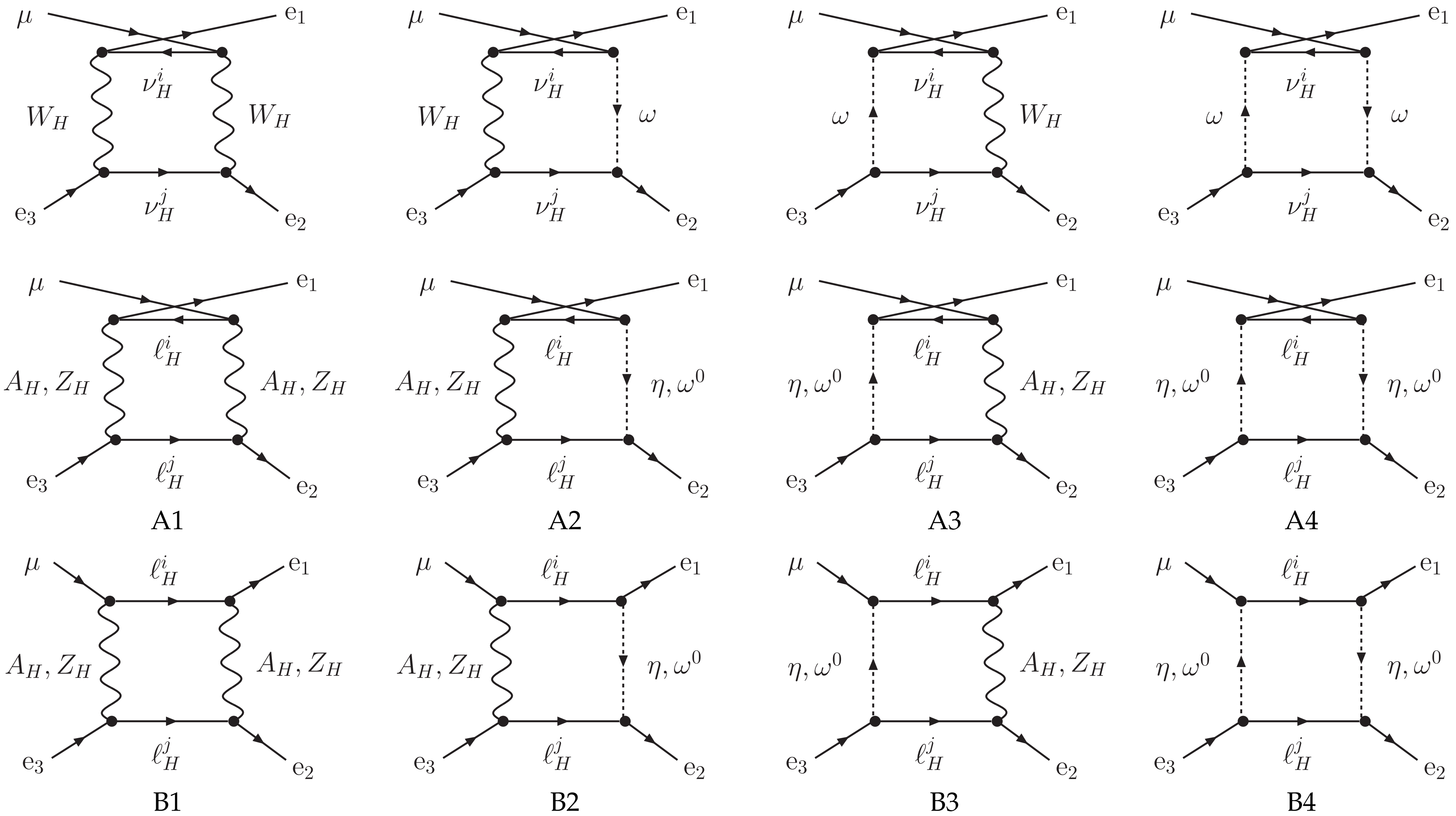
LHT

- Self-energy diagrams $\Rightarrow \gamma,\ Z$ penguins



One-loop contributions to Lepton FV processes

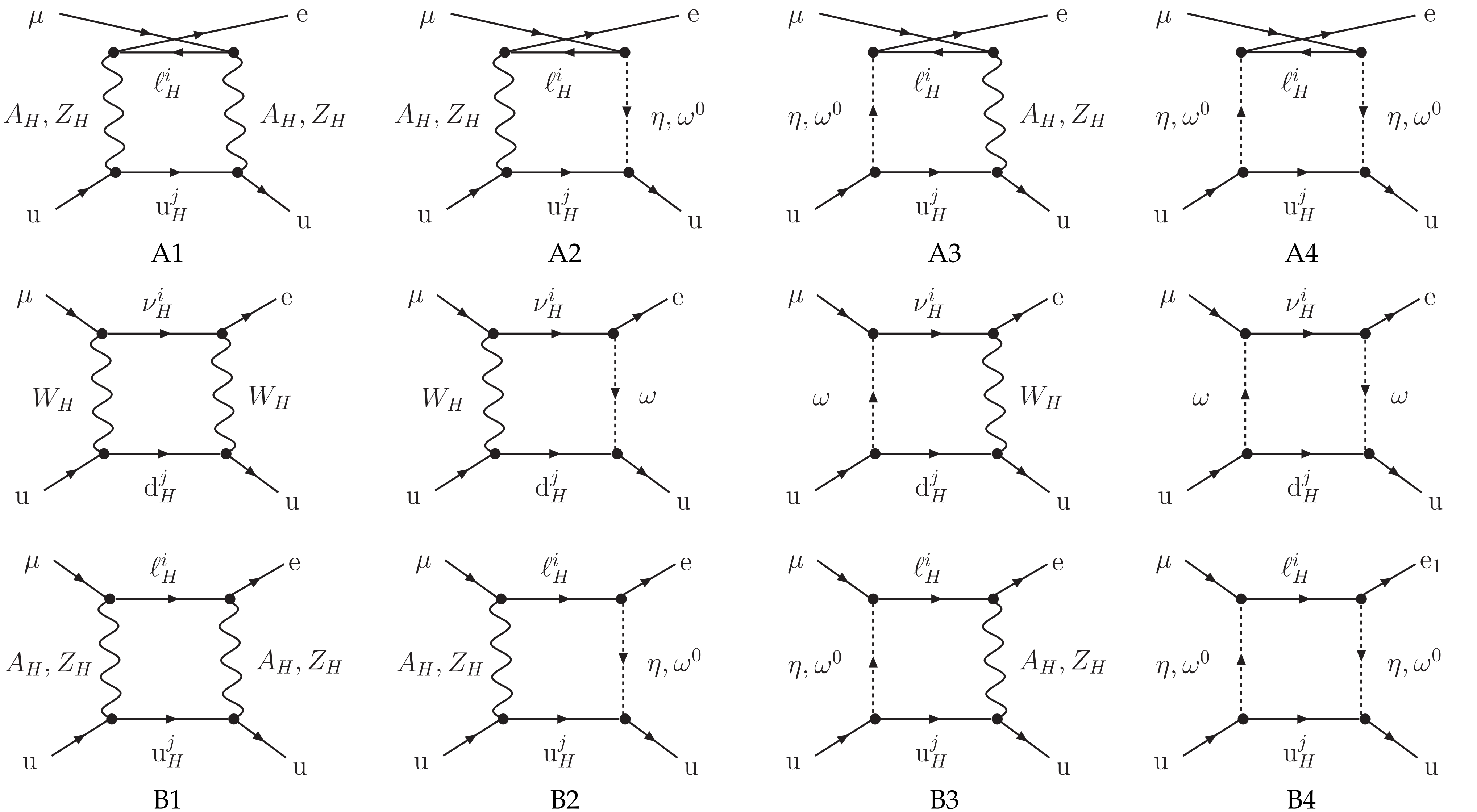
- e-Box diagrams



One-loop contributions to Lepton FV processes

LHT

- q-Box diagrams for quark u (similarly for quark d)

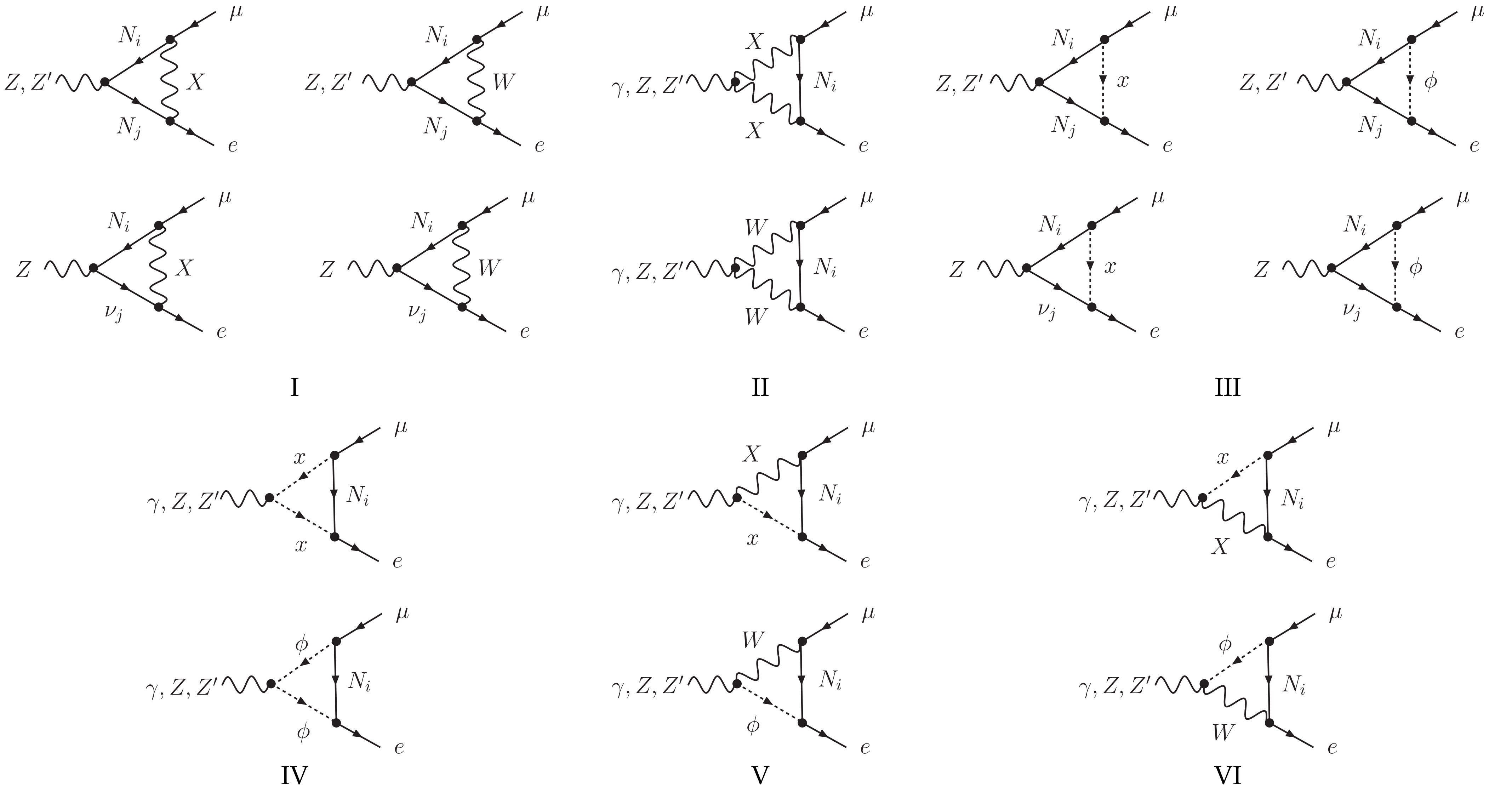


One-loop contributions to Lepton FV processes

SLH

[del Águila, JI, Jenkins '10 . . .]

- Triangle diagrams $\Rightarrow$ vertex and γ, Z, Z' penguins

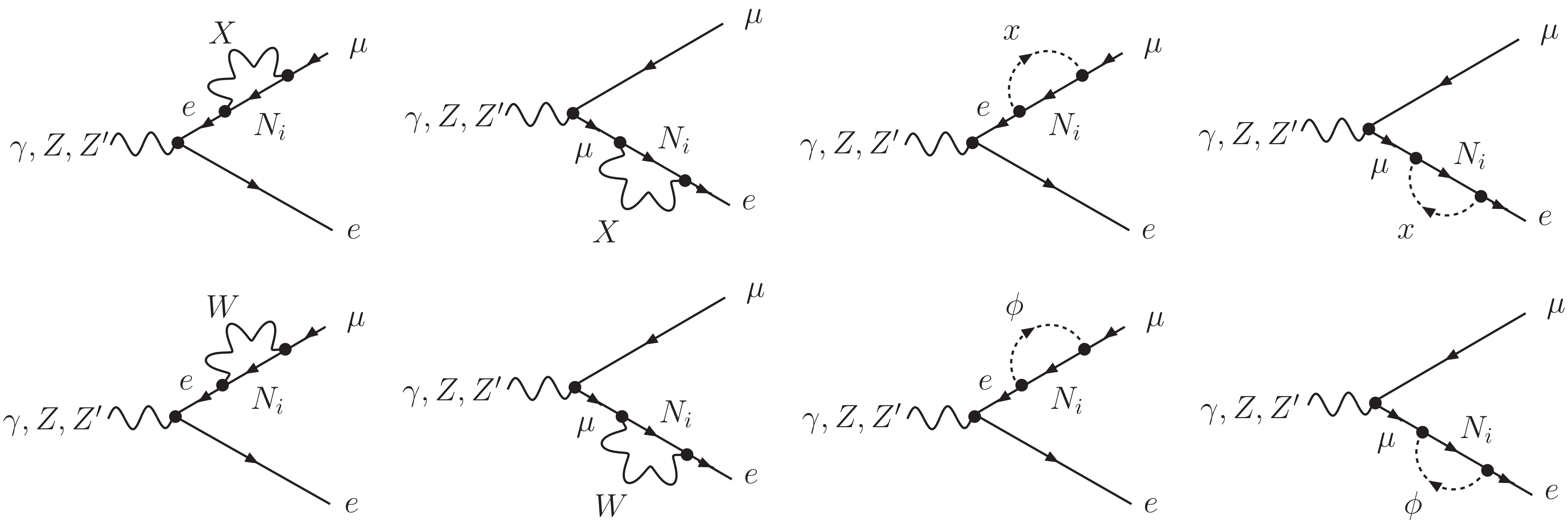


One-loop contributions to Lepton FV processes

SLH

[del Águila, JI, Jenkins '10 ...]

- Self-Energy diagrams ⇒ $\gamma,\ Z,\ Z'$ penguins

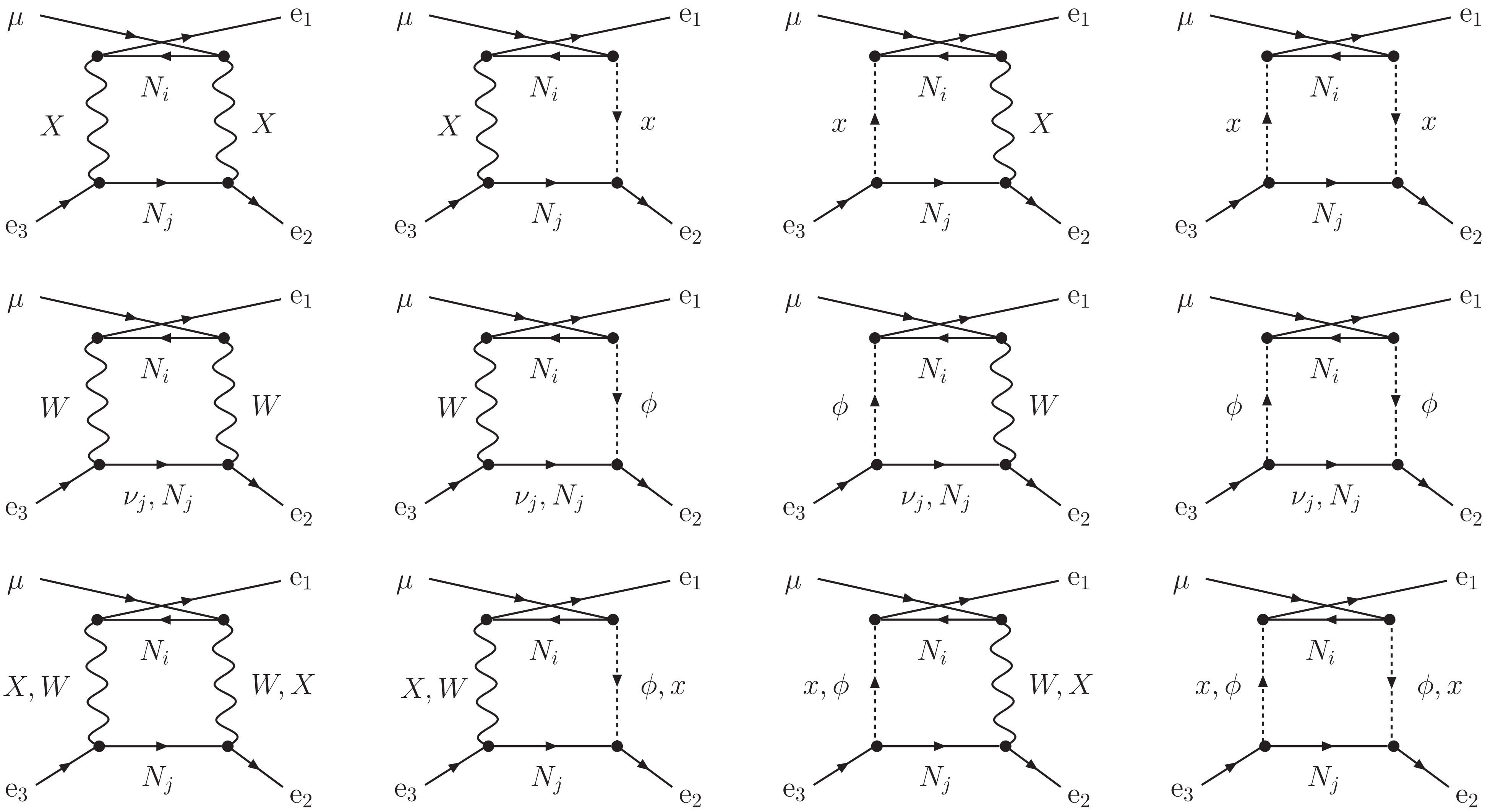


One-loop contributions to Lepton FV processes

SLH

[del Águila, JI, Jenkins '10 . . .]

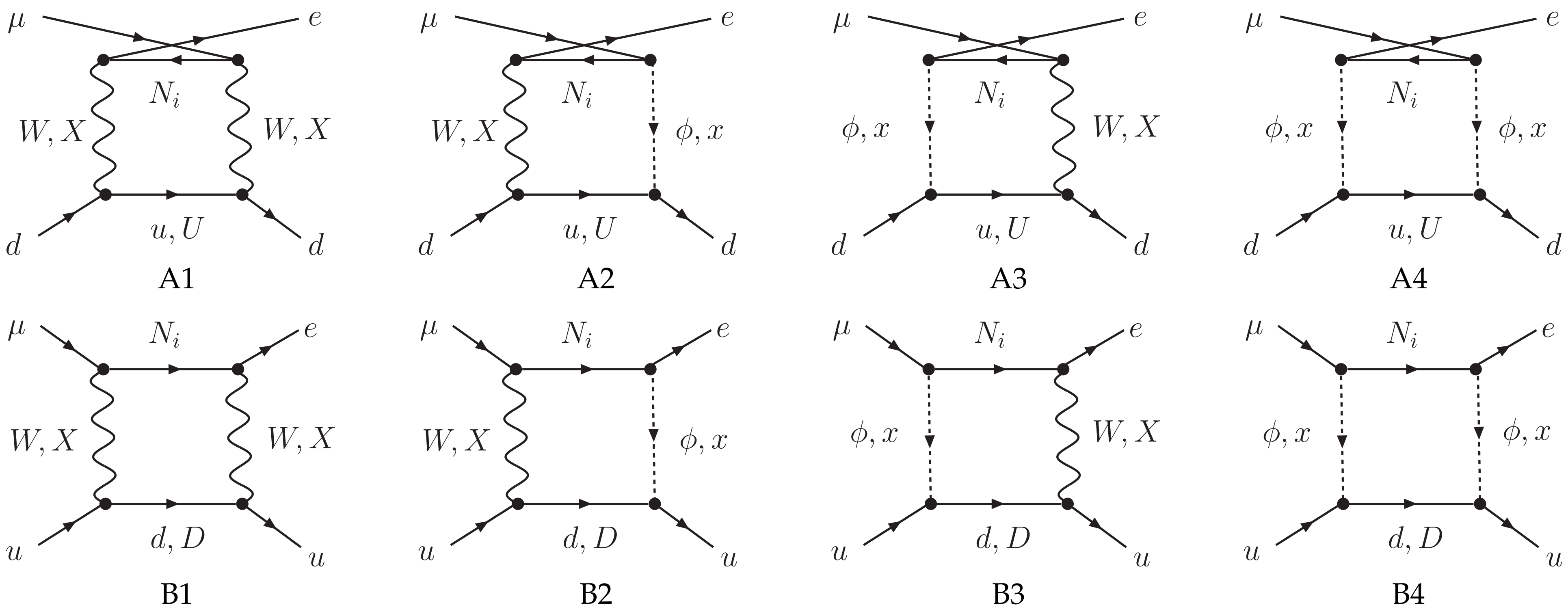
- e-Box diagrams



A1 A2 A3 A4

[del Águila, JI, Jenkins ’10 . . .]

- q-Box diagrams



Discussion

The calculation is to $\boxed{\text{leading order in } v/f}$ (Lagrangian expanded consistently)

- LHT: T-parity forces only particles with masses of $\mathcal{O}(f)$ (T-odd) in the loop

  $\Rightarrow$ The loop functions are at most of $\mathcal{O}(v^2/f^2)$:

$$
\begin{aligned}
B_1,\ C_{00} &\propto \Delta_\epsilon + \mathcal{O}(1) \\
C_1,\ C_2,\ C_{11},\ C_{22},\ C_{12} &\propto \frac{1}{M^2_{W_H}} \\
m_{Hi} m_{Hj} D_0 &\propto \frac{m_{Hi} m_{Hj}}{M^4_{W_H}} = y_i y_j \frac{1}{M^2_{W_H}} \qquad \text{with} \quad \frac{1}{M^2_{W_H}} = \frac{1}{M^2_W} \frac{v^2}{4f^2} \\
D_{00} &\propto \frac{1}{M^2_{W_H}}
\end{aligned}
$$

- SLH: heavy+light neutrinos or heavy+light gauge bosons in the loop

  $\Rightarrow$ Perform a careful v/f expansion of loop functions to $\mathcal{O}(v^2/f^2)$

Discussion

All form factors are $\boxed{\text{UV finite}}$ in the LHT ...

- Box diagrams $\Leftarrow$ power counting
- γ penguins: – dipole form factors $\Leftarrow$ finite for generic vertices
  – $F_L^\gamma [\propto Q^2/M^2] \Leftarrow$ triangle and SE divergences cancel (unitary mix.)
- Z penguins: – dipole form factors suppressed by $(m_\mu/M)^2 \Leftarrow$ finite generically
  – F_L^Z finite when full $Z\nu_{HR}\nu_{HR}$ interaction is included:

$$F_L^Z|_{A_H} = F_L^Z|_{Z_H} = 0$$

$$F_L^Z|_{W_H} = \frac{\alpha_W}{16\pi}\frac{1}{s_W c_W}\sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu}\left[-4c_W^2\left(\Delta_\epsilon - \log\frac{M_{W_H}^2}{\mu^2}\right) + \frac{v^2}{4f^2} y_i H_W(y_i)\right], \quad y_i = \frac{m_{Hi}^2}{M_{W_H}^2}$$

that agrees with $Z\mu e$ in the SM extended with massive neutrinos ($Q^2 = 0$): [JI, Riemann '01]

$$F_L^Z|_{\nu\mathrm{SM}} = \frac{\alpha_W}{16\pi}\frac{1}{s_W c_W}\sum_i V_{\mathrm{PMNS}}^{ei} V_{\mathrm{PMNS}}^{\mu i*}\left[-4c_W^2\left(\Delta_\epsilon - \log\frac{M_W^2}{\mu^2}\right) + x_i H_W(x_i)\right], \quad x_i = \frac{m_{\nu i}^2}{M_W^2}$$

... and also in the SLH: divergences of γ, Z and Z' penguins vanish (unitarity mix.)

Discussion

✓ FRs including WBGBs ('t Hooft-Feynman gauge) obtained to $\mathcal{O}(v^2/f^2)$

✓ All form factors in terms of standard loop integrals computed analytically

✓ Amplitudes reduced to exact and simple expressions

✓ Ultraviolet finite (independent of the cutoff Λ)

– Simplification: just 2-gen lepton mixing with $\{\nu_{Hi}, \ell_{Hi} | N_{Hi}\}$ of $m^2_{Hi} \equiv y_i M^2_{\{W_H|X\}}$

$$V = \begin{pmatrix} V^{1e} & V^{1\mu} \\ V^{2e} & V^{2\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad \delta = \frac{m^2_{H2} - m^2_{H1}}{m_{H1} m_{H2}}, \quad \tilde{y} = \sqrt{y_1 y_2}$$

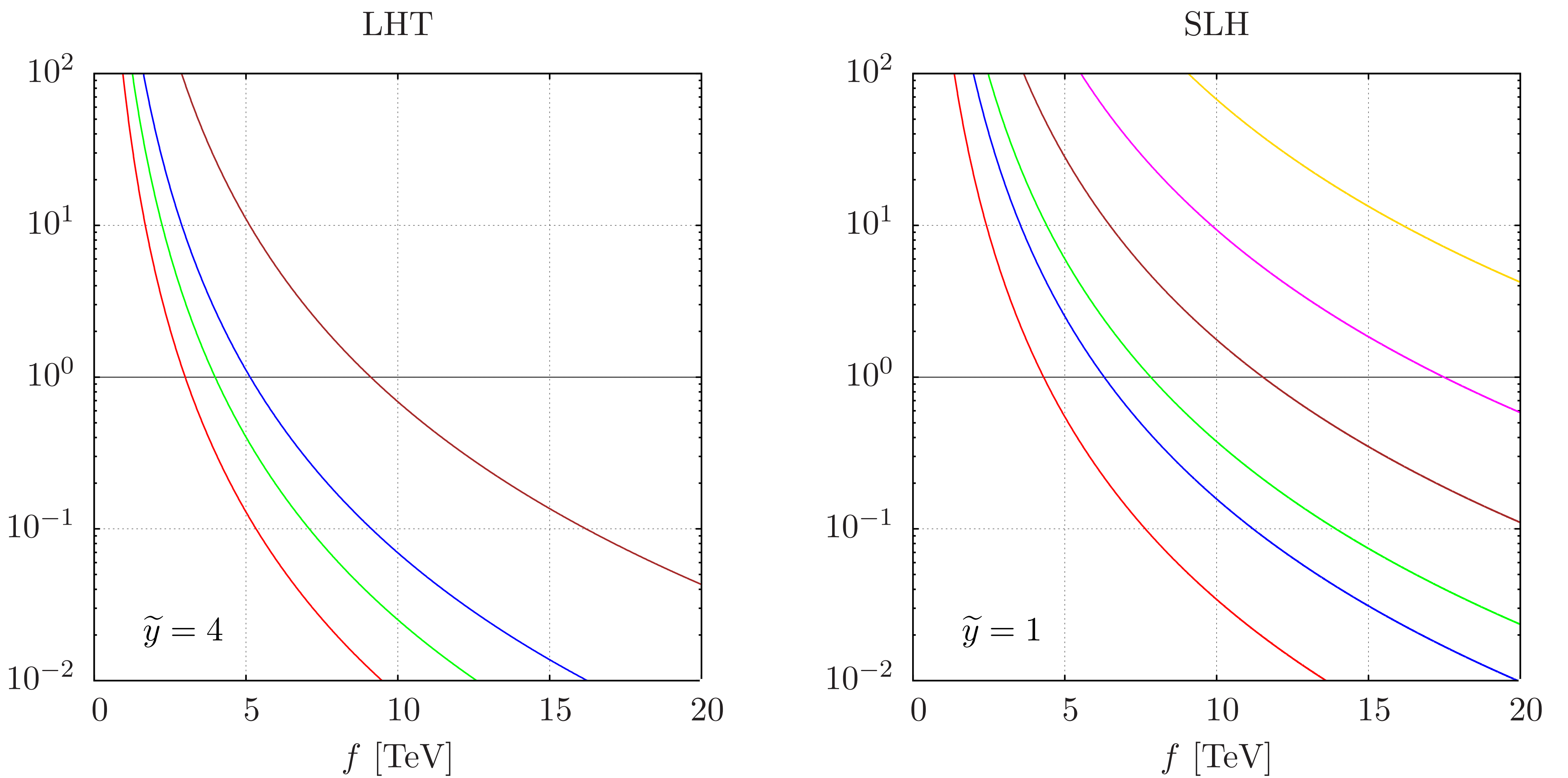
$\rightsquigarrow$ amplitudes approximately scale like $\boxed{\dfrac{v^2}{f^2} \sin 2\theta \; \delta}$ and vary with $\boxed{\tilde{y}}$

– *Natural* reference values: $f \sim 1$ TeV, $\sin 2\theta \sim 1$, $\delta \sim 1$, $\tilde{y} \sim 1$, degenerate quarks

[for LHT: $m_{q_{Hi}} = M_{W_H}$] [for SLH: $\tan\beta = 1$, $m_U = m_D = M_X$]

Ratio of expectations to current limits

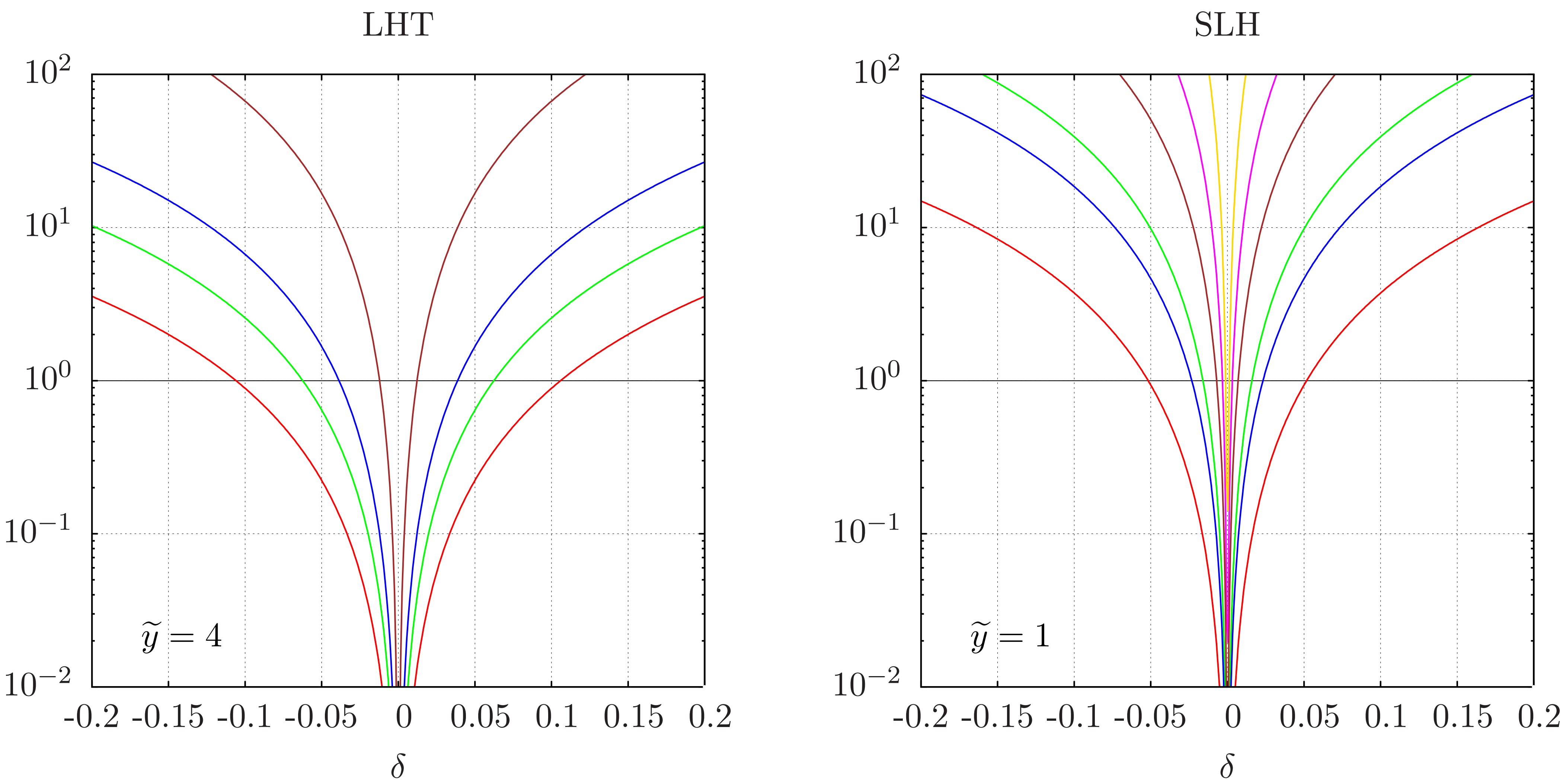
$\delta = 1 \quad \sin 2\theta = 1 \quad \widetilde{y} \sim 1$



$\mu \to e\gamma$	$\mu - \mathrm{e}$ in Ti	$\mu \to e\gamma$	$\mu - \mathrm{e}$ in Ti (AF) ; Ti (U)
$\mu \to ee\bar{e}$	$\mu - \mathrm{e}$ in Au	$\mu \to ee\bar{e}$	$\mu - \mathrm{e}$ in Au (AF) ; Au (U)

Ratio of expectations to current limits

$f = 1$ TeV $\quad \sin 2\theta = 1 \;\; \widetilde{y} \sim 1$



$\mu \to e\gamma \qquad \mu - e$ in Ti

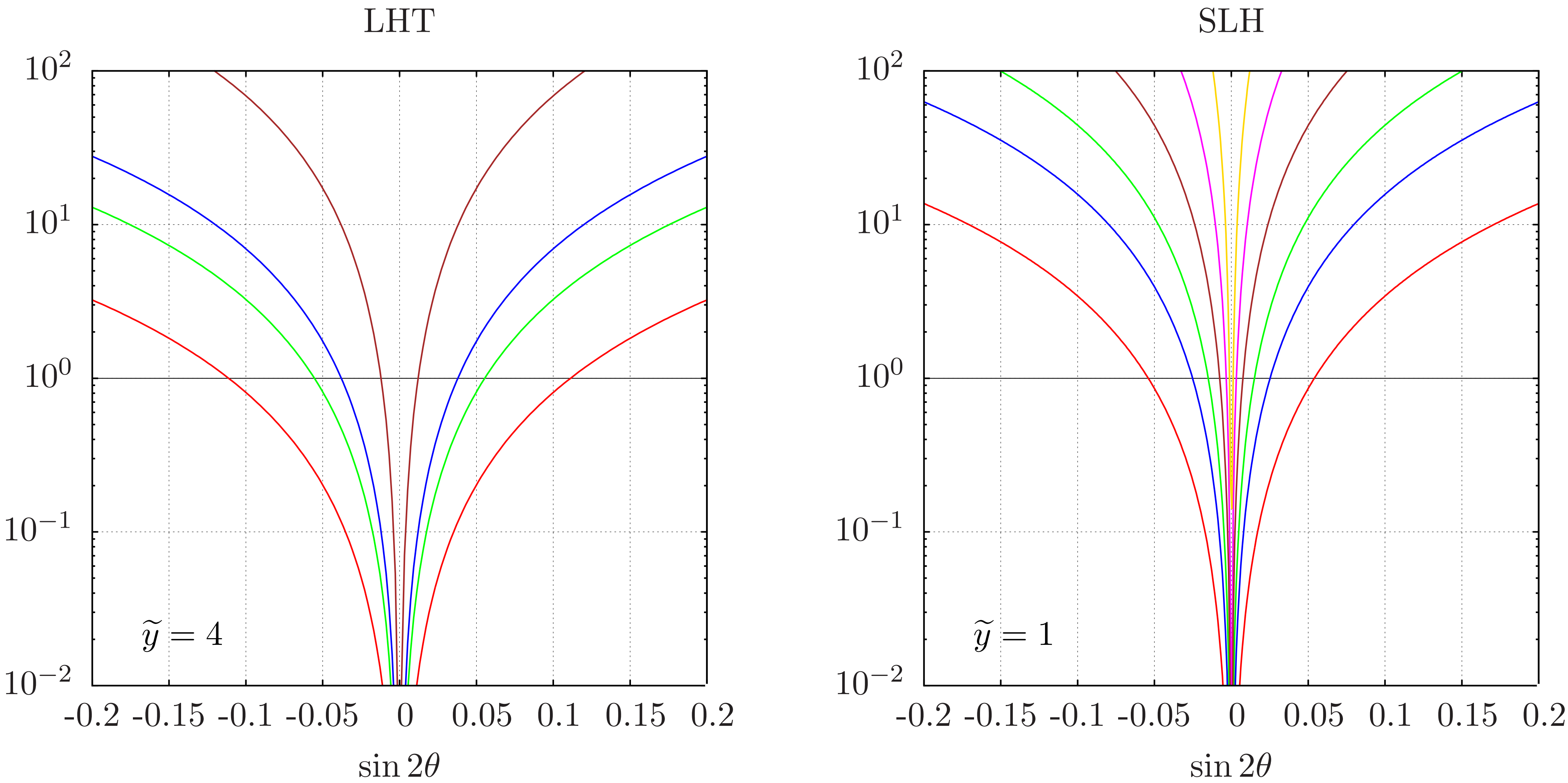
$\mu \to ee\bar{e} \qquad \mu - e$ in Au

$\mu \to e\gamma \qquad \mu - e$ in Ti (AF) ; Ti (U)

$\mu \to ee\bar{e} \qquad \mu - e$ in Au (AF) ; Au (U)

Ratio of expectations to current limits

$f = 1$ TeV $\delta = 1$ $\widetilde{y} \sim 1$



$\mu \to \mathrm{e}\gamma$ $\mu - \mathrm{e}$ in Ti

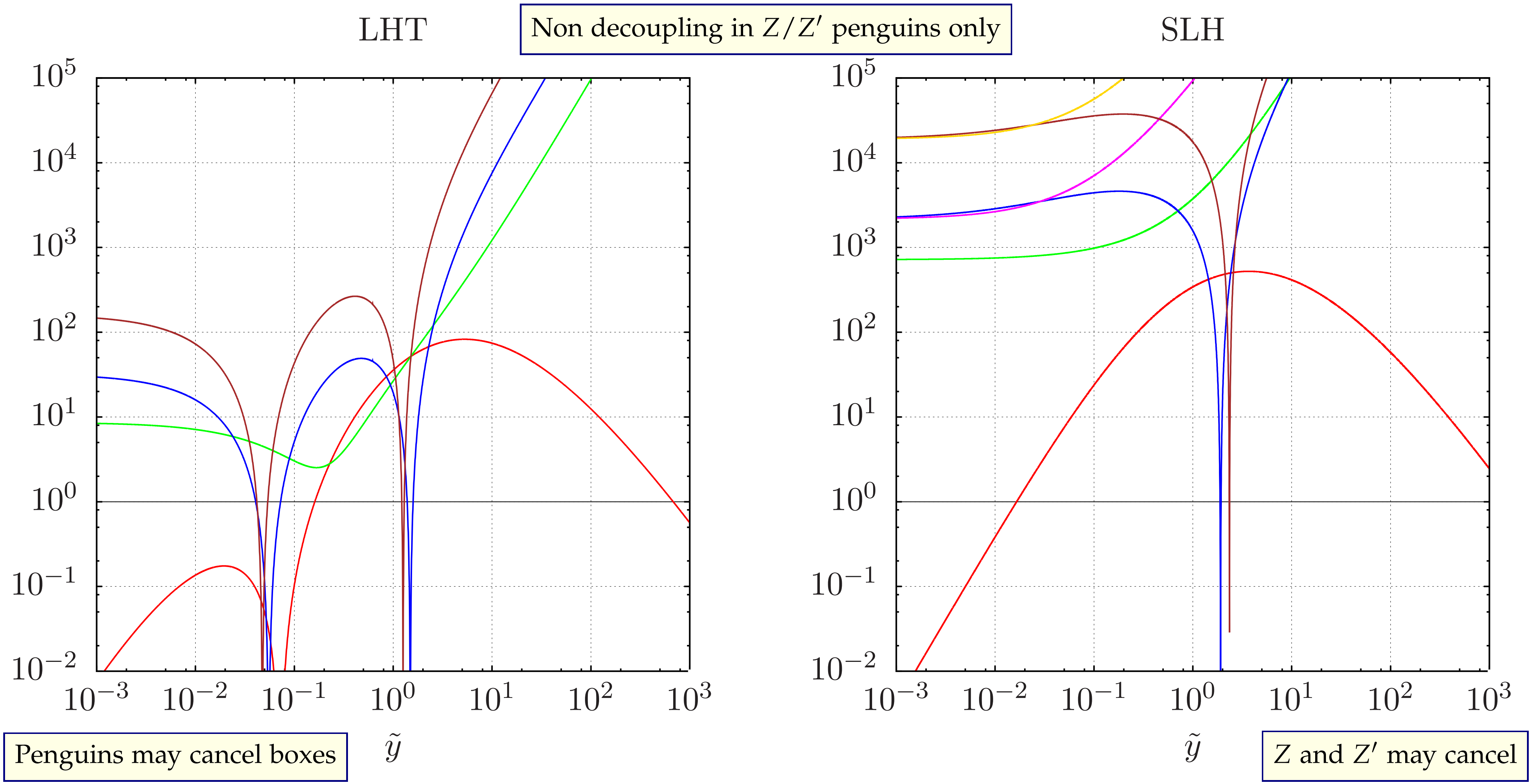
$\mu \to \mathrm{ee}\bar{\mathrm{e}}$ $\mu - \mathrm{e}$ in Au

$\mu \to \mathrm{e}\gamma$ $\mu - \mathrm{e}$ in Ti (AF) ; Ti (U)

$\mu \to \mathrm{ee}\bar{\mathrm{e}}$ $\mu - \mathrm{e}$ in Au (AF) ; Au (U)

Ratio of expectations to current limits

$f = 1$ TeV $\delta = 1$ $\sin 2\theta = 1$



$\mu \to e\gamma$ $\mu - \mathrm{e}$ in Ti

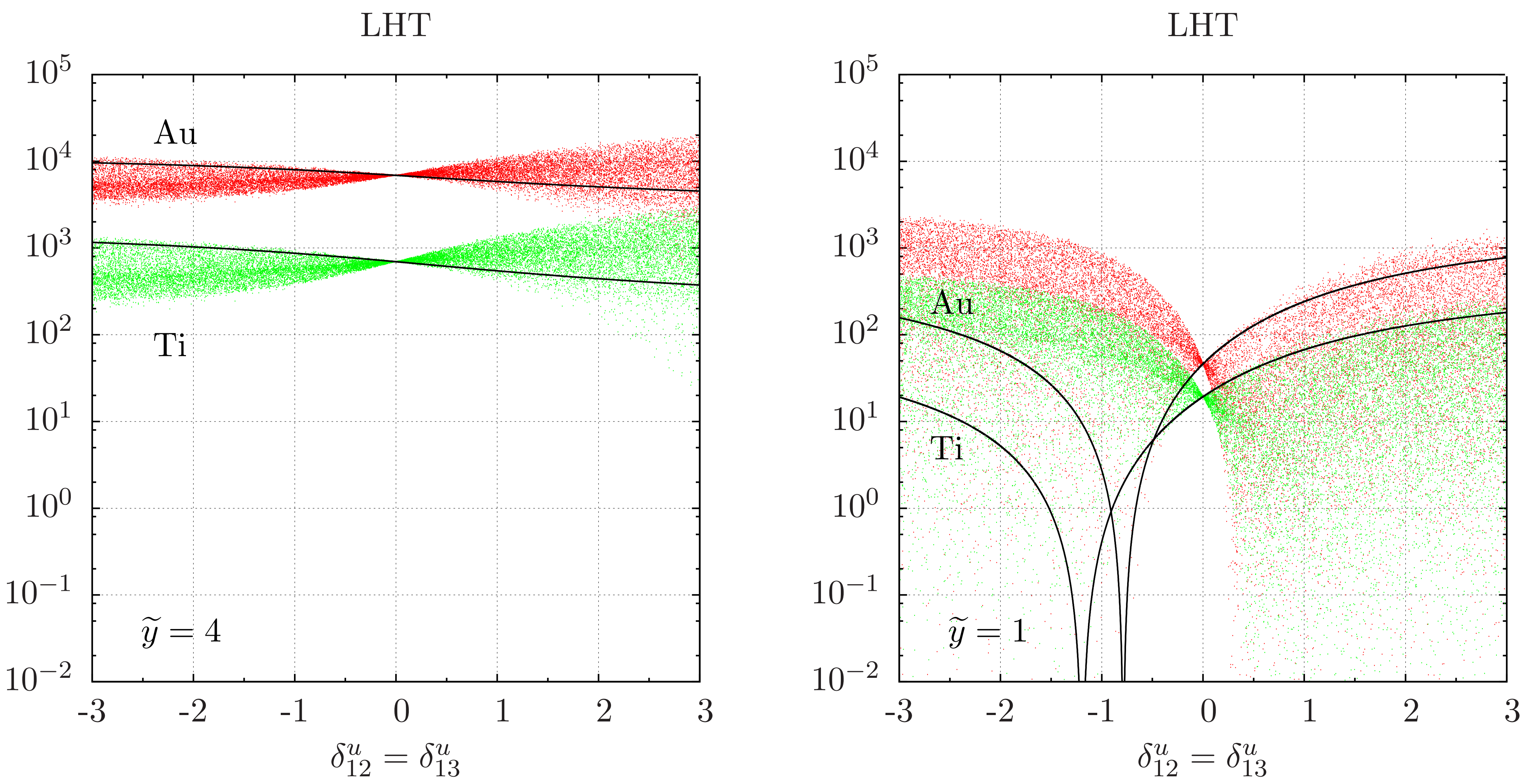
$\mu \to ee\bar{e}$ $\mu - \mathrm{e}$ in Au

$\mu \to e\gamma$ $\mu - \mathrm{e}$ in Ti (AF) ; Ti (U)

$\mu \to ee\bar{e}$ $\mu - \mathrm{e}$ in Au (AF) ; Au (U)

Ratio of expectations to current limits

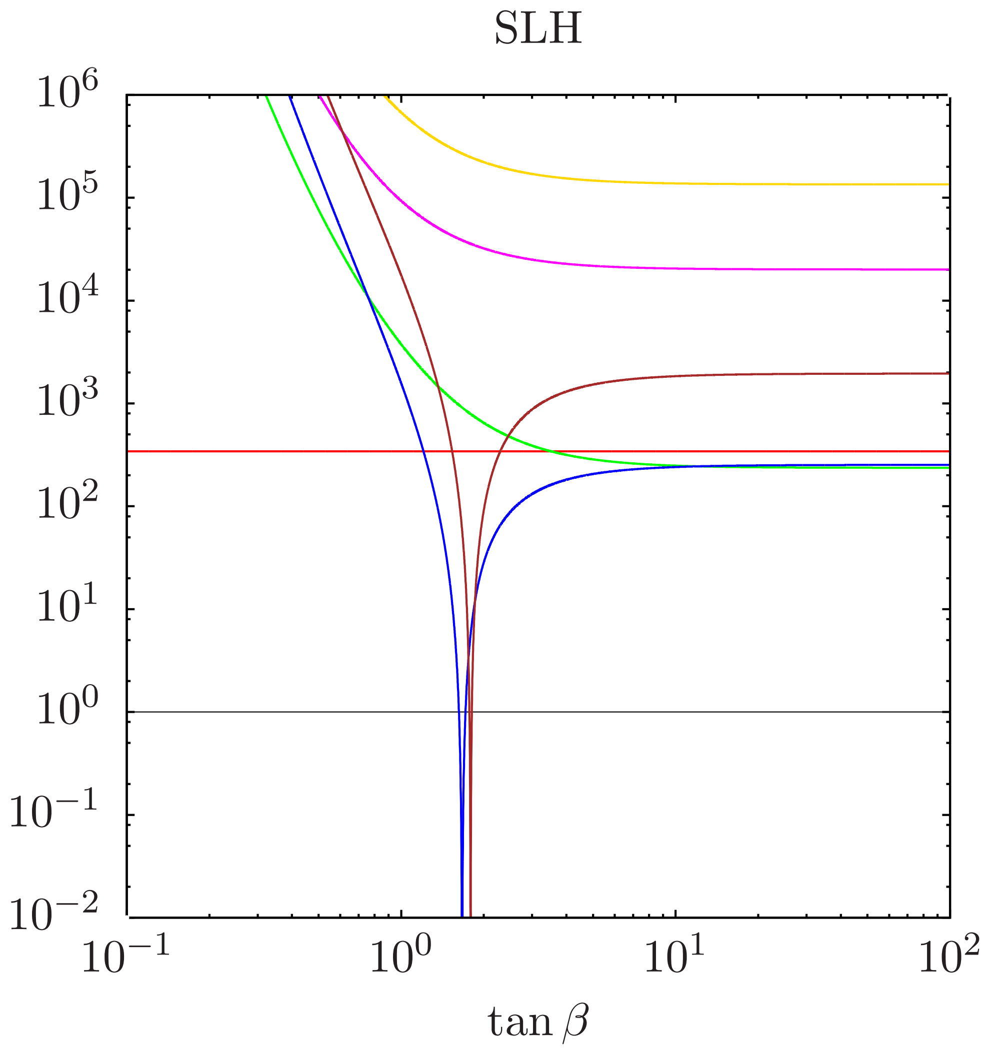
quark mixing in $\mu\,\mathrm{N} \to \mathrm{e}\,\mathrm{N}$



points: random mixing

lines: $V_{Hd} = V_{Hu} V_{\mathrm{CKM}} = V_{\mathrm{CKM}}$

Ratio of expectations to current limits



$\mu \to e\gamma$	$\mu - \mathrm{e}$ in Ti	$\mu \to e\gamma$	$\mu - \mathrm{e}$ in Ti (AF) ; Ti (U)
$\mu \to ee\bar{e}$	$\mu - \mathrm{e}$ in Au	$\mu \to ee\bar{e}$	$\mu - \mathrm{e}$ in Au (AF) ; Au (U)

Constraints based on naturalness

(with degenerate heavy quark masses)

Present | Future

LHT	$\mu \to e\gamma$		$\mu \to ee\bar{e}$	
Limit	1.2×10^{-11}	10^{-13}	10^{-12}	10^{-14}
$f/\mathrm{TeV} >$	3.00	9.93	3.98	12.6
$\sin 2\theta <$	0.111	0.010	0.055	0.006
$\lvert\delta\rvert <$	0.106	0.010	0.062	0.006

LHT	$\mu\,\mathrm{Au} \to \mathrm{e}\,\mathrm{Au}$	$\mu\,\mathrm{Ti} \to \mathrm{e}\,\mathrm{Ti}$	
Limit	7×10^{-13}	4.3×10^{-12}	10^{-18}
$f/\mathrm{TeV} >$	9.11	5.13	234
$\sin 2\theta <$	0.012	0.038	$< 10^{-4}$
$\lvert\delta\rvert <$	0.012	0.039	$< 10^{-4}$

SLH (AF)	$\mu\,\mathrm{Au} \to \mathrm{e}\,\mathrm{Au}$	$\mu\,\mathrm{Ti} \to \mathrm{e}\,\mathrm{Ti}$	
Limit	7×10^{-13}	4.3×10^{-12}	10^{-18}
$f/\mathrm{TeV} >$	11.5	6.3	287
$\sin 2\theta <$	0.0074	0.0252	$< 10^{-4}$
$\lvert\delta\rvert <$	0.0070	0.0232	$< 10^{-4}$

SLH	$\mu \to e\gamma$		$\mu \to ee\bar{e}$	
Limit	1.2×10^{-11}	10^{-13}	10^{-12}	10^{-14}
$f/\mathrm{TeV} >$	4.3	14.2	7.8	24.8
$\sin 2\theta <$	0.0540	0.0048	0.0150	0.0014
$\lvert\delta\rvert <$	0.0517	0.0047	0.0159	0.0015

SLH (U)	$\mu\,\mathrm{Au} \to \mathrm{e}\,\mathrm{Au}$	$\mu\,\mathrm{Ti} \to \mathrm{e}\,\mathrm{Ti}$	
Limit	7×10^{-13}	4.3×10^{-12}	10^{-18}
$f/\mathrm{TeV} >$	28.7	17.5	796
$\sin 2\theta <$	0.0012	0.0032	$< 10^{-4}$
$\lvert\delta\rvert <$	0.0011	0.0032	$< 10^{-4}$

Conclusions

- The one-loop predictions for flavor violating processes in the $\boxed{\text{LHT}}$ are finite when *all* Goldstone interactions compatible with gauge and T symmetry included
- EWPT allow f as low as 500 GeV in the LHT model [Hubisz, Meade, Noble, Perelstein '06]
  and dark matter limits on the lightest T-particle set $f \gtrsim 1.8$ TeV [Hubisz, Meade '05]
  but present experimental limits on LFV processes (μ Au $\to$ e Au) require:
  - somewhat heavier scale ($f \gtrsim 9$ TeV), or
  - flavor alignment of light and heavy leptons ($\sin 2\theta \lesssim 0.01$), or
  - small splitting of heavy lepton masses ($\delta \lesssim 1\%$)
- The Feynman rules for the $\boxed{\text{SLH}}$ in the 't Hooft-Feynman gauge obtained and predictions for LFV processes computed for the first time (also finite)
- The constraints on the SLH from LFV are even more demanding:
  $f \gtrsim 12\ (29)$ TeV, $\quad \sin 2\theta \lesssim 0.007\ (0.001)$, $\quad \delta \lesssim 0.7\%\ (0.1\%)$ assuming AF (U)

$$\boxed{\text{LH models are severely constrained by flavor !!}}$$

BACKUP

Littlest Higgs

The matrix of the 14 Goldstone Bosons:

$$\Pi = \begin{pmatrix} -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^+}{\sqrt{2}} & -\mathrm{i}\frac{\phi^+}{\sqrt{2}} & -\mathrm{i}\Phi^{++} & -\mathrm{i}\frac{\Phi^+}{\sqrt{2}} \\ -\frac{\omega^-}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & \frac{v+h+\mathrm{i}\phi^0}{2} & -\mathrm{i}\frac{\Phi^+}{\sqrt{2}} & \frac{-\mathrm{i}\Phi^0 + \Phi^P}{\sqrt{2}} \\ \mathrm{i}\frac{\phi^-}{\sqrt{2}} & \frac{v+h-\mathrm{i}\phi^0}{2} & \sqrt{\frac{4}{5}}\eta & -\mathrm{i}\frac{\phi^+}{\sqrt{2}} & \frac{v+h+\mathrm{i}\phi^0}{2} \\ \mathrm{i}\Phi^{--} & \mathrm{i}\frac{\Phi^-}{\sqrt{2}} & \mathrm{i}\frac{\phi^-}{\sqrt{2}} & -\frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} & -\frac{\omega^-}{\sqrt{2}} \\ \mathrm{i}\frac{\Phi^-}{\sqrt{2}} & \frac{\mathrm{i}\Phi^0 + \Phi^P}{\sqrt{2}} & \frac{v+h-\mathrm{i}\phi^0}{2} & -\frac{\omega^+}{\sqrt{2}} & \frac{\omega^0}{2} - \frac{\eta}{\sqrt{20}} \end{pmatrix}$$

Generators of the gauge subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2 \subset SU(5)$:

$$Q_1^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{0}_{2\times 2} \end{pmatrix} \qquad Q_2^a = \frac{1}{2}\begin{pmatrix} \mathbf{0}_{2\times 2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*} \end{pmatrix}$$

$$Y_1 = \frac{1}{10}\mathrm{diag}(3,3,-2,-2,-2) \qquad Y_2 = \frac{1}{10}\mathrm{diag}(2,2,2,-3,-3)$$

Littlest Higgs with T-parity

Enlarging $SU(5)$ to assign $\boxed{\text{proper hypercharges}}$ ($y = y_1 + y_2$) to *all* SM fermions:

$$SU(5) \times U(1)_1'' \times U(1)_2'' \supset [SU(2)_1 \times \underbrace{U(1)_1' \times U(1)_1''}_{\supset U(1)_1}] \times [SU(2)_2 \times \underbrace{U(1)_2' \times U(1)_2''}_{\supset U(1)_2}]$$

Leptons	$SU(2)_1$	$SU(2)_2$	$y_1 = y_1' + y_1''$	$y_2 = y_2' + y_2''$	y_1'	y_2'	y_1''	y_2''
$l_1 = \begin{pmatrix} \nu_{1L} \\ \ell_{1L} \end{pmatrix}$	$\mathbf{2}$	$\mathbf{1}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$-\frac{3}{10}$	$-\frac{1}{5}$	0	0
$l_2 = \begin{pmatrix} \nu_{2L} \\ \ell_{2L} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{5}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$-\frac{3}{10}$	0	0
ν_R	$\mathbf{1}$	$\mathbf{1}$	0	0	0	0	0	0
ℓ_R	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
$l_{HR} = \begin{pmatrix} \nu_{HR} \\ \ell_{HR} \end{pmatrix}$	—	—	—	—	—	—	0	0

Littlest Higgs with T-parity

Enlarging $SU(5)$ to assign proper hypercharges ($y = y_1 + y_2$) to *all* SM fermions:

$$SU(5) \times U(1)''_1 \times U(1)''_2 \supset [SU(2)_1 \times \underbrace{U(1)'_1 \times U(1)''_1}_{\supset U(1)_1}] \times [SU(2)_2 \times \underbrace{U(1)'_2 \times U(1)''_2}_{\supset U(1)_2}]$$

Quarks	$SU(2)_1$	$SU(2)_2$	$y_1 = y'_1 + y''_1$	$y_2 = y'_2 + y''_2$	y'_1	y'_2	y''_1	y''_2
$q_1 = \begin{pmatrix} u_{1L} \\ d_{1L} \end{pmatrix}$	**2**	**1**	$\frac{1}{30}$	$\frac{2}{15}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{3}$
$q_2 = \begin{pmatrix} u_{2L} \\ d_{2L} \end{pmatrix}$	**1**	**2**	$\frac{2}{15}$	$\frac{1}{30}$	$-\frac{1}{5}$	$-\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{3}$
u_R	**1**	**1**	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$
d_R	**1**	**1**	$-\frac{1}{6}$	$-\frac{1}{6}$	0	0	$-\frac{1}{6}$	$-\frac{1}{6}$
$q_{HR} = \begin{pmatrix} u_{HR} \\ d_{HR} \end{pmatrix}$	—	—	—	—	—	—	$\frac{1}{3}$	$\frac{1}{3}$