# 8. El Modelo Estándar de las interacciones electrodébiles (QFD) y fuertes (QCD). El bosón de Higgs

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# 1. Gauge Theories

# The symmetry principle | free Lagrangian

- Lagrangian of a free fermion field $\psi(x)$:

$$\text{(Dirac)} \qquad \boxed{\mathcal{L}_0 = \overline{\psi}(\mathrm{i}\not{\partial} - m)\psi} \quad \not{\partial} \equiv \gamma^\mu \partial_\mu \,, \quad \overline{\psi} = \psi^\dagger \gamma^0$$

$\Rightarrow$ Invariant under global U(1) phase transformations:

$$\psi(x) \mapsto \psi'(x) = \mathrm{e}^{-\mathrm{i}q\theta}\psi(x) \,, \quad q,\ \theta \text{ (constants)} \in \mathbb{R}$$

$\Rightarrow$ By Noether's theorem there is a conserved current:

$$j^\mu = q\,\overline{\psi}\gamma^\mu\psi \,, \quad \partial_\mu j^\mu = 0$$

and a Noether charge:

$$Q = \int \mathrm{d}^3x\, j^0, \quad \partial_t Q = 0$$

## The symmetry principle | free Lagrangian

- A quantized free fermion field:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_{s=1,2} \left( a_{\vec{p},s} u^{(s)}(\vec{p}) e^{-ipx} + b^{\dagger}_{\vec{p},s} v^{(s)}(\vec{p}) e^{ipx} \right)$$

– is a solution of the Dirac equation (Euler-Lagrange):

$$(i\partial\!\!\!/ - m)\psi(x) = 0\,, \quad (p\!\!\!/ - m)u(\vec{p}) = 0\,, \quad (p\!\!\!/ + m)v(\vec{p}) = 0\,,$$

– is an operator from the canonical quantization rules (anticommutation):

$$\{a_{\vec{p},r}, a^{\dagger}_{\vec{k},s}\} = \{b_{\vec{p},r}, b^{\dagger}_{\vec{k},s}\} = (2\pi)^3 \delta^3(\vec{p} - \vec{k})\delta_{rs}\,, \quad \{a_{\vec{p},r}, a_{\vec{k},s}\} = \cdots = 0\,,$$

that annihilates/creates particles/antiparticles on the Fock space of fermions

# The symmetry principle | free Lagrangian

- For a quantized free fermion field:

⇒ Normal ordering for fermionic operators ($H$ spectrum bounded from below):

$$: a_{\vec{p},r} a^{\dagger}_{\vec{q},s} : \equiv -a^{\dagger}_{\vec{q},s} a_{\vec{p},r} \; , \qquad : b_{\vec{p},r} b^{\dagger}_{\vec{q},s} : \equiv -b^{\dagger}_{\vec{q},s} b_{\vec{p},r}$$

⇒ The Noether charge is an operator:

$$: Q : = q \int \mathrm{d}^3x \; : \overline{\psi} \gamma^0 \psi : = q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_{s=1,2} \left( a^{\dagger}_{\vec{p},s} a_{\vec{p},s} - b^{\dagger}_{\vec{p},s} b_{\vec{p},s} \right)$$

$$Q \; a^{\dagger}_{\vec{k},s} \, |0\rangle = +q \; a^{\dagger}_{\vec{k},s} \, |0\rangle \;\; (\text{particle}) \, , \qquad Q \; b^{\dagger}_{\vec{k},s} \, |0\rangle = -q \; b^{\dagger}_{\vec{k},s} \, |0\rangle \;\; (\text{antiparticle})$$

## The symmetry principle | gauge invariance dictates interactions

- To make $\mathcal{L}_0$ invariant under local $\equiv$ gauge transformations of U(1):

$$\psi(x) \mapsto \psi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\psi(x)\,, \quad \theta = \theta(x) \in \mathbb{R}$$

perform the minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu + \mathrm{i}eqA_\mu \quad \text{(covariant derivative)}$$

where a gauge field $A_\mu(x)$ is introduced transforming as:

$$A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x) \quad \Leftarrow \quad \boxed{D_\mu\psi \mapsto \mathrm{e}^{-\mathrm{i}q\theta(x)}D_\mu\psi} \quad \overline{\psi}\not{D}\psi \text{ inv.}$$

$\Rightarrow$ The new Lagrangian contains **interactions** between $\psi$ and $A_\mu$:

$$\boxed{\mathcal{L}_{\text{int}} = -eq\,\overline{\psi}\gamma^\mu\psi A_\mu} \quad \propto \begin{cases} \text{coupling} & e \\ \text{charge} & q \end{cases}$$

$$(= -e\,j^\mu A_\mu)$$

## The symmetry principle | gauge invariance dictates interactions

- Dynamics for the gauge field ⇒ add gauge invariant kinetic term:

$$\text{(Maxwell)} \qquad \boxed{\mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}} \qquad \Leftarrow \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \mapsto F_{\mu\nu}$$

- The full U(1) gauge invariant Lagrangian for a fermion field $\psi(x)$ reads:

$$\mathcal{L}_{\text{sym}} = \overline{\psi}(\mathrm{i}\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (= \mathcal{L}_0 + \mathcal{L}_{\text{int}} + \mathcal{L}_1)$$

- The same applies to a complex scalar field $\phi(x)$:

$$\mathcal{L}_{\text{sym}} = (D_\mu \phi)^\dagger D^\mu \phi - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

# The symmetry principle | non-Abelian gauge theories

- A general gauge symmetry group $G$ is an $N$-dimensional compact Lie group

$$\mathrm{g} \in G\,, \quad \mathrm{g}(\vec{\theta}) = \mathrm{e}^{-\mathrm{i}T^a\theta^a}\,, \quad a = 1,\ldots,N$$

$$\theta^a = \theta^a(x) \in \mathbb{R}\,, \quad T^a = \text{Hermitian generators}\,, \quad [T^a, T^b] = \mathrm{i}f^{abc}T^c \quad \text{(Lie algebra)}$$

$$\mathrm{Tr}\{T^aT^b\} \equiv \tfrac{1}{2}\delta_{ab}\,, \quad \text{structure constants:} \quad \begin{array}{ll} f^{abc} = 0 & \text{Abelian} \\ f^{abc} \neq 0 & \text{non-Abelian} \end{array}$$

$\Rightarrow$ Finite-dimensional irreducible representations are unitary:

$$d\text{-multiplet}: \quad \Psi(x) \mapsto \Psi'(x) = U(\vec{\theta})\Psi(x)\,, \quad \Psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_d \end{pmatrix}$$

$d \times d$ matrices : $\quad U(\vec{\theta})$ [given by $\{T^a\}$ algebra representation]

- Examples:

| $G$ | $N$ | Abelian | |
|---|---|---|---|
| U(1) | 1 | Yes | |
| SU($n$) | $n^2-1$ | No | ($n \times n$ unitary matrices with det $= 1$) |

- U(1): 1 generator ($q$), one-dimensional irreps only

- SU(2): 3 generators
  $f^{abc} = \epsilon^{abc}$ (Levi-Civita symbol)
  - Fundamental irrep ($d = 2$): $T^a = \frac{1}{2}\sigma^a$ (3 Pauli matrices)
  - Adjoint irrep ($d = N = 3$): $(T^a_{\text{adj}})^{bc} = -\mathrm{i} f^{abc}$

- SU(3): 8 generators
  $f^{123} = 1,\ f^{458} = f^{678} = \frac{\sqrt{3}}{2},\ f^{147} = f^{156} = f^{246} = f^{247} = f^{345} = -f^{367} = \frac{1}{2}$
  - Fundamental irrep ($d = 3$): $T^a = \frac{1}{2}\lambda^a$ (8 Gell-Mann matrices)
  - Adjoint irrep ($d = N = 8$): $(T^a_{\text{adj}})^{bc} = -\mathrm{i} f^{abc}$

(for SU($n$): $f^{abc}$ totally antisymmetric)

The symmetry principle | non-Abelian gauge theories

- To make $\mathcal{L}_0$ invariant under local $\equiv$ gauge transformations of $G$:

$$\Psi(x) \mapsto \Psi'(x) = U(\vec{\theta})\Psi(x) \;, \quad \vec{\theta} = \vec{\theta}(x) \in \mathbb{R}$$

substitute the covariant derivative:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g\widetilde{W}_\mu \;, \quad \widetilde{W}_\mu \equiv T^a W_\mu^a$$

where a gauge field $W_\mu^a(x)$ per generator is introduced, transforming as:

$$\widetilde{W}_\mu(x) \mapsto \widetilde{W}'_\mu(x) = U\widetilde{W}_\mu(x)U^\dagger - \frac{\mathrm{i}}{g}(\partial_\mu U)U^\dagger \quad \Leftarrow \quad \boxed{D_\mu\Psi \mapsto UD_\mu\Psi} \quad \overline{\Psi}\not{D}\Psi \text{ inv.}$$

$\Rightarrow$ The new Lagrangian contains interactions between $\Psi$ and $W_\mu^a$:

$$\boxed{\mathcal{L}_{\text{int}} = g\,\overline{\Psi}\gamma^\mu T^a \Psi W_\mu^a} \quad \propto \begin{cases} \text{coupling} & g \\ \text{charge} & T^a \end{cases}$$

$$(= g\, j_a^\mu W_\mu^a)$$

## The symmetry principle | non-Abelian gauge theories

- Dynamics for the gauge fields $\Rightarrow$ add gauge invariant kinetic terms:

$$\text{(Yang-Mills)} \quad \boxed{\mathcal{L}_{\rm YM} = -\frac{1}{2}\mathrm{Tr}\left\{\widetilde{W}_{\mu\nu}\widetilde{W}^{\mu\nu}\right\} = -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}} \quad \Leftarrow \quad \widetilde{W}_{\mu\nu} \mapsto U\widetilde{W}_{\mu\nu}U^\dagger$$

$$\widetilde{W}_{\mu\nu} \equiv \partial_\mu \widetilde{W}_\nu - \partial_\nu \widetilde{W}_\mu - \mathrm{i}g[\widetilde{W}_\mu, \widetilde{W}_\nu]$$

$$\Rightarrow \quad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu$$

$\Rightarrow \mathcal{L}_{\rm YM}$ contains cubic and quartic **self-interactions** of the gauge fields $W^a_\mu$:

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{a,\nu} - \partial^\nu W^{a,\mu})$$

$$\mathcal{L}_{\rm cubic} = -\frac{1}{2} g f^{abc} (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) W^{b,\mu} W^{c,\nu}$$

$$\mathcal{L}_{\rm quartic} = -\frac{1}{4} g^2 f^{abe} f^{cde} W^a_\mu W^b_\nu W^{c,\mu} W^{d,\nu}$$

- The (Feynman) propagator of a scalar field:

$$D(x-y) = \langle 0|\, T\{\phi(x)\phi^\dagger(y)\}\,|0\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\mathrm{i}}{p^2-m^2+\mathrm{i}\epsilon}\mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

is a Green's function of the Klein-Gordon operator:

$$(\Box_x + m^2)D(x-y) = -\mathrm{i}\delta^4(x-y) \quad \Leftrightarrow \quad \widetilde{D}(p) = \frac{\mathrm{i}}{p^2-m^2+\mathrm{i}\epsilon}$$

- The propagator of a fermion field:

$$S(x-y) = \langle 0|\, T\{\psi(x)\overline{\psi}(y)\}\,|0\rangle = (\mathrm{i}\partial\!\!\!/_x + m)\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\mathrm{i}}{p^2-m^2+\mathrm{i}\epsilon}\mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

is a Green's function of the Dirac operator:

$$(\mathrm{i}\partial\!\!\!/_x - m)S(x-y) = \mathrm{i}\delta^4(x-y) \quad \Leftrightarrow \quad \widetilde{S}(p) = \frac{\mathrm{i}}{p\!\!\!/ - m + \mathrm{i}\epsilon}$$

- BUT the propagator of a gauge field cannot be defined unless $\mathcal{L}$ is modified:

(e.g. modified Maxwell) $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

Euler-Lagrange: $$\frac{\partial \mathcal{L}}{\partial A_\nu} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0 \quad \Rightarrow \quad \left[g^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right] A_\mu = 0$$

– In momentum space the propagator is the inverse of:

$$-k^2 g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right)k^\mu k^\nu \quad \Rightarrow \quad \widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]$$

$\Rightarrow$ Note that $(-k^2 g^{\mu\nu} + k^\mu k^\nu)$ is singular!

$\Rightarrow$ One may argue that $\mathcal{L}$ above will not lead to Maxwell equations ...
unless we fix a (Lorenz) gauge where:

$$\partial^\mu A_\mu = 0 \quad \Leftarrow \quad A_\mu \mapsto A'_\mu = A_\mu + \partial_\mu \Lambda \;\text{ with }\; \partial^\mu\partial_\mu\Lambda \equiv -\partial^\mu A_\mu$$

# Quantization of gauge theories | gauge fixing (Abelian case)

- The extra term is called Gauge Fixing:

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

$\Rightarrow$ modified $\mathcal{L}$ equivalent to Maxwell Lagrangian just in the gauge $\partial^\mu A_\mu = 0$

$\Rightarrow$ the $\xi$-dependence always cancels out in physical amplitudes

- Several choices for the gauge fixing term (simplify calculations): $R_\xi$ gauges

$$\text{('t Hooft-Feynman gauge)} \quad \xi = 1: \quad \widetilde{D}_{\mu\nu}(k) = -\frac{\mathrm{i}g_{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$$

$$\text{(Landau gauge)} \quad \xi = 0: \quad \widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right]$$

- For a non-Abelian gauge theory, the gauge fixing terms:

$$\mathcal{L}_{\text{GF}} = -\sum_a \frac{1}{2\xi_a}(\partial^\mu W^a_\mu)^2$$

allow to define the propagators:

$$\widetilde{D}^{ab}_{\mu\nu}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1 - \xi_a)\frac{k_\mu k_\nu}{k^2}\right]$$

BUT, unlike the Abelian case, this is not the end of the story ...

# Quantization of gauge theories | Faddeev-Popov ghosts $(\star)$

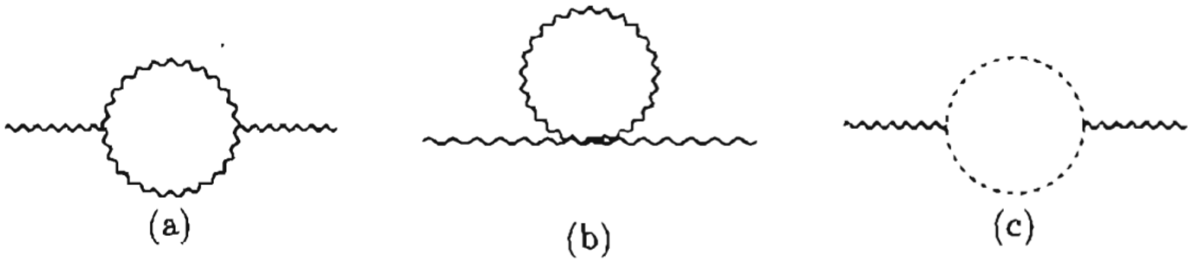
- Add Faddeev-Popov ghost fields $c_a(x)$, $a = 1, \ldots, N$:

$$\boxed{\mathcal{L}_{\text{FP}} = (\partial^\mu \bar{c}^a)(D_\mu^{\text{adj}})^{ab} c^b = (\partial^\mu \bar{c}^a)(\partial_\mu c^a - g f^{abc} c^b W_\mu^c)} \quad \Leftarrow \quad D_\mu^{\text{adj}} = \partial_\mu - \mathrm{i} g T_{\text{adj}}^c W_\mu^c$$

Computational trick: anticommuting scalar fields, just in loops as virtual particles

$$\widetilde{D}_{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 + \mathrm{i}\epsilon} \qquad [(-1) \text{ sign for closed loops! (like fermions)}]$$

$\Rightarrow$ Faddeev-Popov ghosts needed to preserve gauge symmetry:




$$= \mathrm{i}(g_{\mu\nu} k^2 - k_\mu k_\nu)\Pi(k^2)$$

$\Rightarrow$ Faddeev-Popov ghosts needed to preserve unitarity at the loop level:

$$q\bar{q} \to q\bar{q}$$

- Then the complete quantum Lagrangian is

$$\boxed{\mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}}$$

⇒ Note that in the case of a massive vector field

$$\text{(Proca)} \qquad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 A_\mu A^\mu$$

is not gauge invariant

– The propagator is:

$$\widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M^2 + \mathrm{i}\epsilon}\left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{M^2}\right)$$

# 2. Spontaneous Symmetry Breaking

# Spontaneous Symmetry Breaking — discrete symmetry

- Consider a real scalar field $\phi(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4 \quad \text{invariant under} \quad \phi \mapsto -\phi$$

$$\Rightarrow \quad \mathcal{H} = \frac{1}{2}(\dot{\phi}^2 + (\nabla\phi)^2) + V(\phi)$$

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$


$\mu^2,\ \lambda \in \mathbb{R}$ (Real/Hermitian Hamiltonian) and $\lambda > 0$ (existence of a ground state)

(a) $\mu^2 > 0$: min of $V(\phi)$ at $\phi_{\text{cl}} = 0$

(b) $\mu^2 < 0$: min of $V(\phi)$ at $\phi_{\text{cl}} = v \equiv \pm\sqrt{\frac{-\mu^2}{\lambda}}$, in QFT $\langle 0|\, \phi \,|0\rangle = v \neq 0$ (VEV)

– A quantum field must have $v = 0$

$$a\,|0\rangle = 0$$

$$\Rightarrow \qquad \phi(x) \equiv v + \eta(x)\,, \quad \langle 0|\, \eta \,|0\rangle = 0$$

## Spontaneous Symmetry Breaking | discrete symmetry

- At the quantum level, the same system is described by $\eta(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4 \quad \text{not invariant under} \quad \eta \mapsto -\eta$$

$$(m_\eta = \sqrt{2\lambda}\, v)$$

⇒ Lesson:

$\mathcal{L}(\phi)$ had the symmetry but the parameters can be such that the ground state of the Hamiltonian is not symmetric (Spontaneous Symmetry Breaking)

⇒ Note:

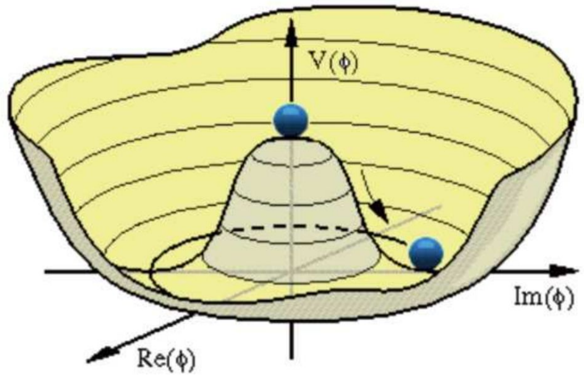
One may argue that $\mathcal{L}(\eta)$ exhibits an explicit breaking of the symmetry. However this is not the case since the coefficients of terms $\eta^2$, $\eta^3$ and $\eta^4$ are determined by just two parameters, $\lambda$ and $v$ (remnant of the original symmetry)

# Spontaneous Symmetry Breaking | continuous symmetry

- Consider a complex scalar field $\phi(x)$ with Lagrangian:

$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 \quad \text{invariant under U(1):} \quad \phi \mapsto e^{-iq\theta}\phi$$

$$\lambda > 0,\ \mu^2 < 0 : \quad \langle 0|\,\phi\,|0\rangle \equiv \frac{v}{\sqrt{2}}, \quad |v| = \sqrt{\frac{-\mu^2}{\lambda}}$$




Take $v \in \mathbb{R}^+$. In terms of quantum fields:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + i\chi(x)], \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\chi\,|0\rangle = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - \lambda v^2 \eta^2 - \lambda v \eta(\eta^2 + \chi^2) - \frac{\lambda}{4}(\eta^2 + \chi^2)^2 + \frac{1}{4}\lambda v^4$$

Note: if $ve^{i\alpha}$ (complex) replace $\eta$ by $(\eta \cos\alpha - \chi \sin\alpha)$ and $\chi$ by $(\eta \sin\alpha + \chi \cos\alpha)$

⇒ The actual quantum Lagrangian $\mathcal{L}(\eta, \chi)$ is not invariant under U(1)

U(1) broken ⇒ one scalar field remains massless: $m_\eta = \sqrt{2\lambda}\, v$, $m_\chi = 0$

# Spontaneous Symmetry Breaking | continuous symmetry

- Another example: consider a real scalar SU(2) triplet $\Phi(x)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^\mathsf{T})(\partial^\mu \Phi) - \frac{1}{2}\mu^2 \Phi^\mathsf{T}\Phi - \frac{\lambda}{4}(\Phi^\mathsf{T}\Phi)^2 \quad \text{inv. under SU(2):} \quad \Phi \mapsto \mathrm{e}^{-\mathrm{i}T^a\theta^a}\Phi$$

that for $\lambda > 0,\ \mu^2 < 0$ acquires a VEV $\langle 0|\,\Phi^\mathsf{T}\Phi\,|0\rangle = v^2 \qquad (\mu^2 = -\lambda v^2)$

Assume $\Phi(x) = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ v + \varphi_3(x) \end{pmatrix}$ and define $\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_1 + \mathrm{i}\varphi_2)$

$$\mathcal{L} = (\partial_\mu \varphi^\dagger)(\partial^\mu \varphi) + \frac{1}{2}(\partial_\mu \varphi_3)(\partial^\mu \varphi_3) - \lambda v^2 \varphi_3^2 - \lambda v(2\varphi^\dagger\varphi + \varphi_3^2)\varphi_3 - \frac{\lambda}{4}(2\varphi^\dagger\varphi + \varphi_3^2)^2 + \frac{1}{4}\lambda v^4$$

$\Rightarrow$ Not symmetric under SU(2) but invariant under U(1):

$$\varphi \mapsto \mathrm{e}^{-\mathrm{i}q\theta}\varphi \quad (q = \text{arbitrary}) \qquad\qquad \varphi_3 \mapsto \varphi_3 \quad (q = 0)$$

SU(2) broken to U(1) $\Rightarrow 3 - 1 = 2$ broken generators

$\Rightarrow$ 2 (real) scalar fields (= 1 complex) remain massless: $m_\varphi = 0,\ m_{\varphi_3} = \sqrt{2\lambda}\,v$

# Spontaneous Symmetry Breaking | continuous symmetry

⇒ Goldstone's theorem: [Nambu '60; Goldstone '61]

*The number of massless particles (Nambu-Goldstone bosons) is equal to the number of spontaneously broken generators of the symmetry*

$$\text{Hamiltonian symmetric under group } G \quad \Rightarrow \quad [T^a, H] = 0\,, \quad a = 1, \ldots, N$$

$$\text{By definition: } H\,|0\rangle = 0 \quad \Rightarrow \quad H(T^a\,|0\rangle) = T^a H\,|0\rangle = 0$$

– If $|0\rangle$ is such that $T^a\,|0\rangle = 0$ for all generators

⇒ non-degenerate minimum: *the* vacuum

– If $|0\rangle$ is such that $T^{a'}\,|0\rangle \neq 0$ for some (broken) generators $a'$

⇒ degenerate minimum: chose one (*true* vacuum) and $\mathrm{e}^{-\mathrm{i}T^{a'}\theta^{a'}}\,|0\rangle \neq |0\rangle$

⇒ excitations (particles) from $|0\rangle$ to $\mathrm{e}^{-\mathrm{i}T^{a'}\theta^{a'}}\,|0\rangle$ cost no energy: massless!

## Spontaneous Symmetry Breaking | gauge symmetry

- Consider a U(1) gauge invariant Lagrangian for a complex scalar field $\phi(x)$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \,, \quad D_\mu = \partial_\mu + \mathrm{i}eqA_\mu$$

inv. under $\quad \phi(x) \mapsto \phi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\phi(x) \,, \quad A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x)$

If $\lambda > 0$, $\mu^2 < 0$, the $\mathcal{L}$ in terms of quantum fields $\eta$ and $\chi$ with null VEVs:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + \mathrm{i}\chi(x)] \,, \quad \mu^2 = -\lambda v^2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi)$$
$$- \lambda v^2\eta^2 - \lambda v\eta(\eta^2 + \chi^2) - \frac{\lambda}{4}(\eta^2 + \chi^2)^2 + \frac{1}{4}\lambda v^4$$
$$\boxed{+ eqvA_\mu\partial^\mu\chi} + eqA_\mu(\eta\partial^\mu\chi - \chi\partial^\mu\eta)$$
$$\boxed{+ \frac{1}{2}(eqv)^2A_\mu A^\mu} + \frac{1}{2}(eq)^2A_\mu A^\mu(\eta^2 + 2v\eta + \chi^2)$$

<u>Comments:</u>

(i) $m_\eta = \sqrt{2\lambda}\, v$

$m_\chi = 0$

(ii) $M_A = |eqv|$ (!)

(iii) Term $A_\mu\partial^\mu\chi$ (?)

(iv) Add $\mathcal{L}_{\mathrm{GF}}$

- Removing the cross term (no mixing in propagators) and new gauge fixing:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu - \xi M_A \chi)^2$$

$$\Rightarrow \quad \mathcal{L} + \mathcal{L}_{\rm GF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2 A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \overbrace{M_A \partial_\mu (A^\mu \chi)}^{\text{total deriv.}}$$
$$+ \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - \frac{1}{2}\xi M_A^2 \chi^2 + \ldots$$

and the propagators of $A_\mu$ and $\chi$ are:

$$\widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_A^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi M_A^2}\right]$$
$$\widetilde{D}(k) = \frac{\mathrm{i}}{k^2 - \xi M_A^2 + \mathrm{i}\epsilon}$$

$\Rightarrow$ $\chi$ has a gauge-dependent mass: actually it is not a physical field!

## Spontaneous Symmetry Breaking | gauge symmetry

- A more transparent parameterization of the quantum field $\phi$ is

$$\phi(x) \equiv \mathrm{e}^{\mathrm{i}q\zeta(x)/v}\frac{1}{\sqrt{2}}[v+\eta(x)]\,, \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\zeta\,|0\rangle = 0$$

$$\phi(x) \mapsto \mathrm{e}^{-\mathrm{i}q\zeta(x)/v}\phi(x) = \frac{1}{\sqrt{2}}[v+\eta(x)] \quad \Rightarrow \quad \zeta \text{ gauged away!}$$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \\ &-\lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{1}{4}\lambda v^4 \\ &+\frac{1}{2}(eqv)^2A_\mu A^\mu + \frac{1}{2}(eq)^2A_\mu A^\mu(2v\eta+\eta^2)\end{aligned}$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\,v$

(ii) $M_A = |eqv|$

(iii) No need for $\mathcal{L}_{\mathrm{GF}}$

$\Rightarrow$ This is the unitary gauge ($\xi \to \infty$): just physical fields

## Spontaneous Symmetry Breaking | gauge symmetry

⇒ Brout-Englert-Higgs mechanism: [Anderson '62]

[Higgs '64; Englert, Brout '64; Guralnik, Hagen, Kibble '64]

*The gauge bosons associated with the spontaneously broken generators become massive, the corresponding would-be Goldstone bosons are unphysical and can be absorbed, the remaining massive scalars (Higgs bosons) are physical (the smoking gun!)*

- The would-be Goldstone bosons are 'eaten up' by the gauge bosons ('get fat') and disappear (gauge away) in the unitary gauge ($\xi \to \infty$)

  ⇒ Degrees of freedom are preserved

| | | |
|---|---|---|
| Before SSB: | 2 (massless gauge boson) | + 1 (Goldstone boson) |
| After SSB: | 3 (massive gauge boson) | + 0 (absorbed would-be Goldstone) |

- For loops calculations, 't Hooft-Feynman gauge ($\xi = 1$) is more convenient:

  ⇒ Gauge boson propagators are simpler, but

  ⇒ Goldstone bosons must be included in internal lines

## Spontaneous Symmetry Breaking | gauge symmetry

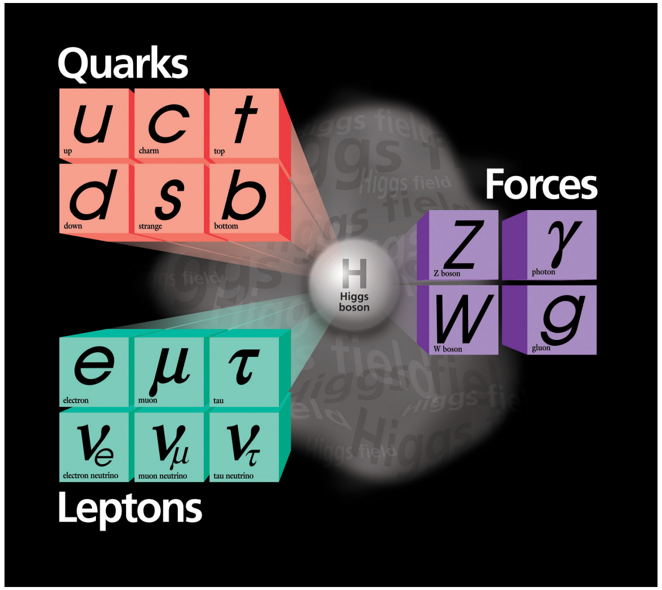
- Comments:

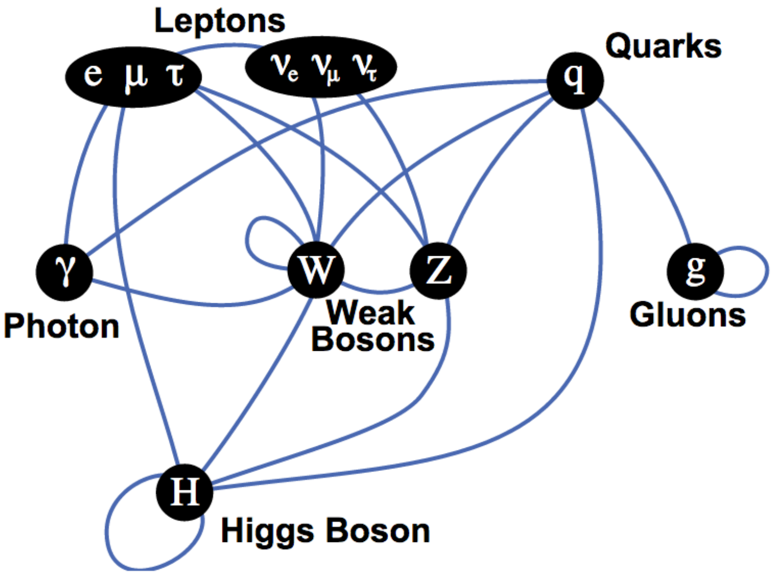
  - After SSB the FP ghost fields (unphysical) acquire a gauge-dependent mass, due to interactions with the scalar field(s):

$$\widetilde{D}_{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 - \xi M_A^2 + \mathrm{i}\epsilon}$$

  - Gauge theories with SSB are renormalizable ['t Hooft, Veltman '72]

    UV divergences appearing at loop level can be removed by renormalization of parameters and fields of the classical Lagrangian ⇒ predictive!




# 3. The Standard Model

## SM: Gauge group and particle reps

[Glashow '61; Weinberg '67; Salam '68]
[D. Gross, F. Wilczek; D. Politzer '73]

- The Standard Model is a gauge theory based on the local symmetry group:

$$\underbrace{SU(3)_c}_{\text{strong}} \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{electroweak}} \to SU(3)_c \otimes \underbrace{U(1)_Q}_{\text{em}}$$

with the electroweak symmetry spontaneously broken to the electromagnetic $U(1)_Q$ symmetry by the Brout-Englert-Higgs mechanism

- The particle (field) content: (ingredients: 12 *flavors* + 12 gauge bosons + H)

| **Fermions** | | | I | II | III | $Q$ |
|---|---|---|---|---|---|---|
| spin $\frac{1}{2}$ | Quarks | $f$ | uuu | ccc | ttt | $\frac{2}{3}$ |
| | | $f'$ | ddd | sss | bbb | $-\frac{1}{3}$ |
| | Leptons | $f$ | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | 0 |
| | | $f'$ | e | $\mu$ | $\tau$ | $-1$ |

$Q_f = Q_{f'} + 1$

| **Bosons** | | |
|---|---|---|
| spin 1 | 8 gluons | strong interaction |
| | $W^\pm$, Z | weak interaction |
| | $\gamma$ | em interaction |
| spin 0 | Higgs | origin of mass |

# SM: Gauge group and particle representations

- The fields lay in the following representations (color, weak isospin, hypercharge):

| Multiplets | $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ | I | II | III | $Q = T^3 + Y$ |
|---|---|---|---|---|---|
| Quarks | $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ | $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ | $\begin{pmatrix} c_L \\ s_L \end{pmatrix}$ | $\begin{pmatrix} t_L \\ b_L \end{pmatrix}$ | $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ <br> $-\frac{1}{3} = -\frac{1}{2} + \frac{1}{6}$ |
| | $(\mathbf{3}, \mathbf{1}, \frac{2}{3})$ | $u_R$ | $c_R$ | $t_R$ | $\frac{2}{3} = 0 + \frac{2}{3}$ |
| | $(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$ | $d_R$ | $s_R$ | $b_R$ | $-\frac{1}{3} = 0 - \frac{1}{3}$ |
| Leptons | $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ | $\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$ | $\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$ | $\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$ | $0 = \frac{1}{2} - \frac{1}{2}$ <br> $-1 = -\frac{1}{2} - \frac{1}{2}$ |
| | $(\mathbf{1}, \mathbf{1}, -1)$ | $e_R$ | $\mu_R$ | $\tau_R$ | $-1 = 0 - 1$ |
| | $(\mathbf{1}, \mathbf{1}, 0)$ | $\nu_{e_R}$ | $\nu_{\mu_R}$ | $\nu_{\tau_R}$ | $0 = 0 + 0$ |
| Higgs | $(\mathbf{1}, \mathbf{2}, \frac{1}{2})$ | (3 families of quarks & leptons) | | | |

$\Rightarrow$ Electroweak (QFD): $SU(2)_L \otimes U(1)_Y$ Strong (QCD): $SU(3)_c$

# 4. Electroweak interactions

## The EWSM with one family (of quarks or leptons)

- Consider two massless fermion fields $f(x)$ and $f'(x)$ with electric charges $Q_f = Q_{f'} + 1$ in three irreps of SU(2)$_L \otimes$U(1)$_Y$:

$$\mathcal{L}_F^0 = \mathrm{i}\overline{f}\partial\!\!\!/ \mathrm{f} + \mathrm{i}\overline{f}'\partial\!\!\!/ f' \qquad f_{R,L} = \frac{1}{2}(1 \pm \gamma_5)f \,, \quad f'_{R,L} = \frac{1}{2}(1 \pm \gamma_5)f'$$

$$= \mathrm{i}\overline{\Psi}_1\partial\!\!\!/\Psi_1 + \mathrm{i}\overline{\psi}_2\partial\!\!\!/\psi_2 + \mathrm{i}\overline{\psi}_3\partial\!\!\!/\psi_3 \quad ; \quad \Psi_1 = \underbrace{\begin{pmatrix} f_L \\ f'_L \end{pmatrix}}_{(\mathbf{2}, y_1)} , \quad \psi_2 = \underbrace{f_R}_{(\mathbf{1}, y_2)} , \quad \psi_3 = \underbrace{f'_R}_{(\mathbf{1}, y_3)}$$

- To get a Langrangian invariant under gauge transformations:

$$\Psi_1(x) \mapsto U_L(x)\mathrm{e}^{-\mathrm{i}y_1\beta(x)}\Psi_1(x), \quad U_L(x) = \mathrm{e}^{-\mathrm{i}T^i\alpha^i(x)}, \quad T^i = \frac{\sigma^i}{2} \quad \text{(weak isospin gen.)}$$

$$\psi_2(x) \mapsto \mathrm{e}^{-\mathrm{i}y_2\beta(x)}\psi_2(x)$$

$$\psi_3(x) \mapsto \mathrm{e}^{-\mathrm{i}y_3\beta(x)}\psi_3(x)$$

**The EWSM with one family** | **covariant derivatives**

⇒ Introduce gauge fields $W_\mu^i(x)$ $(i=1,2,3)$ and $B_\mu(x)$ through covariant derivatives:

$$\left.\begin{aligned} D_\mu \Psi_1 &= (\partial_\mu - \mathrm{i}g\widetilde{W}_\mu + \mathrm{i}g' y_1 B_\mu)\Psi_1\,, \quad \widetilde{W}_\mu \equiv \frac{\sigma^i}{2} W_\mu^i \\ D_\mu \psi_2 &= (\partial_\mu + \mathrm{i}g' y_2 B_\mu)\psi_2 \\ D_\mu \psi_3 &= (\partial_\mu + \mathrm{i}g' y_3 B_\mu)\psi_3 \end{aligned}\right\} \Rightarrow \quad \mathcal{L}_F$$

where two couplings $g$ and $g'$ have been introduced and

$$\widetilde{W}_\mu(x) \mapsto U_L(x)\widetilde{W}_\mu(x)U_L^\dagger(x) - \frac{\mathrm{i}}{g}(\partial_\mu U_L(x))U_L^\dagger(x)$$

$$B_\mu(x) \mapsto B_\mu(x) + \frac{1}{g'}\partial_\mu \beta(x)$$

⇒ Add gauge invariant kinetic terms for the gauge fields

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}W_{\mu\nu}^i W^{i,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\,, \quad W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk}W_\mu^j W_\nu^k$$

(include self-interactions of the SU(2) gauge fields) and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

## The EWSM with one family | mass terms forbidden

⇒ Note that mass terms are not invariant under $SU(2)_L \otimes U(1)_Y$, since LH and RH components do not transform the same:

$$m\bar{f}f = m(\overline{f_L}f_R + \overline{f_R}f_L)$$

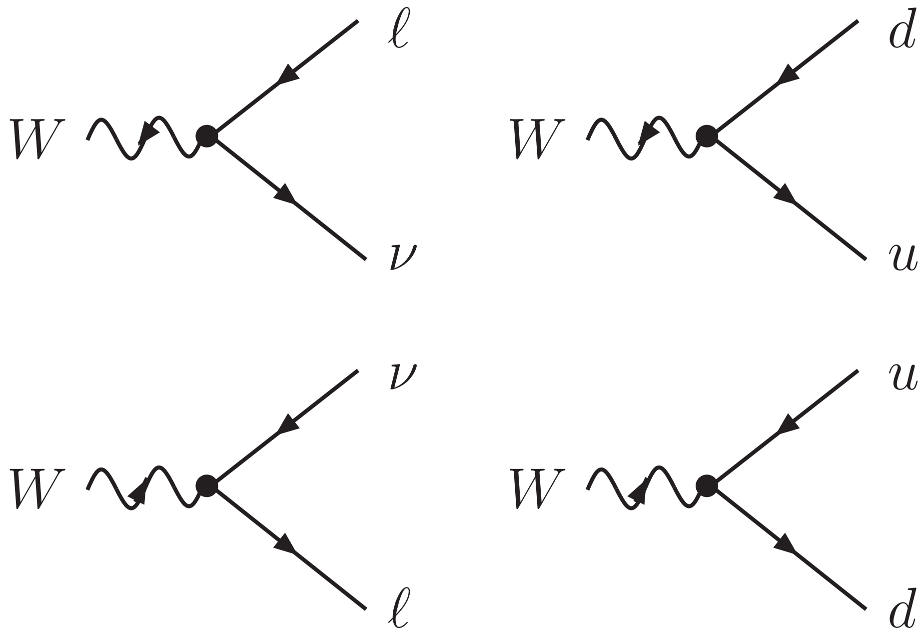
⇒ Mass terms for the gauge bosons are not allowed either

⇒ Next the different types of interactions are analyzed

- $$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu\widetilde{W}_\mu\Psi_1\,, \quad \widetilde{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^\dagger \\ \sqrt{2}W_\mu & -W_\mu^3 \end{pmatrix}$$

⇒ charged current interactions of LH fermions with complex vector boson field $W_\mu$:

$$\mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}}\overline{f}\gamma^\mu(1-\gamma_5)f'W_\mu^\dagger + \text{h.c.}\,, \quad W_\mu \equiv \frac{1}{\sqrt{2}}(W_\mu^1 + \mathrm{i}W_\mu^2)$$



## The EWSM with one family | neutral current interactions

- The diagonal part of

$$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu \widetilde{W}_\mu \Psi_1 - g' B_\mu (y_1 \overline{\Psi}_1 \gamma^\mu \Psi_1 + y_2 \overline{\psi}_2 \gamma^\mu \psi_2 + y_3 \overline{\psi}_3 \gamma^\mu \psi_3)$$

$\Rightarrow$ neutral current interactions with neutral vector boson fields $W^3_\mu$ and $B_\mu$

We would like to identify $B_\mu$ with the photon field $A_\mu$ but that requires:

$$y_1 = y_2 = y_3 \quad \text{and} \quad g' y_j = eQ_j \quad \Rightarrow \quad \text{impossible!}$$

$\Rightarrow$ Since they are both neutral, try a combination:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \qquad \begin{array}{c} s_W \equiv \sin\theta_W \,, \quad c_W \equiv \cos\theta_W \\ \theta_W = \text{weak mixing angle} \end{array}$$

$$\mathcal{L}_{\rm NC} = \sum_{j=1}^{3} \overline{\psi}_j \gamma^\mu \left\{ - \left[ g s_W T^3 + g' c_W y_j \right] A_\mu + \left[ g c_W T^3 - g' s_W y_j \right] Z_\mu \right\} \psi_j$$

with $T^3 = \dfrac{\sigma_3}{2}$ (0) the third weak isospin component of the doublet (singlet)

- To make $A_\mu$ the photon field:

$$\boxed{e = g s_W = g' c_W} \qquad \boxed{Q = T^3 + Y}$$

where the electric charge operator is: $Q_1 = \begin{pmatrix} Q_f & 0 \\ 0 & Q_{f'} \end{pmatrix}$ , $Q_2 = Q_f$ , $Q_3 = Q_{f'}$

$\Rightarrow$ Electroweak unification: $g$ of SU(2) and $g'$ of U(1) are related

$\Rightarrow$ The hyperchages are fixed in terms of electric charges and weak isospin:

$$y_1 = Q_f - \frac{1}{2} = Q_{f'} + \frac{1}{2} , \qquad y_2 = Q_f , \qquad y_3 = Q_{f'}$$

$$\mathcal{L}_{\text{QED}} = -e\, Q_f \overline{f} \gamma^\mu f\, A_\mu \quad + (f \to f')$$

$\Rightarrow$ RH neutrinos are sterile: $y_2 = Q_f = 0$

## The EWSM with one family | neutral current interactions

- The $Z_\mu$ is the neutral weak boson field:

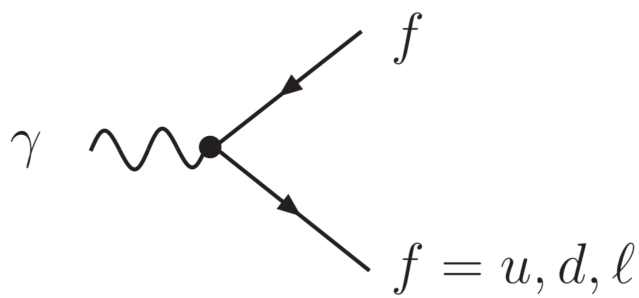
$$\mathcal{L}_{\rm NC}^{Z} = e\,\overline{f}\gamma^\mu(v_f - a_f\gamma_5)f\;Z_\mu \quad + (f \to f')$$

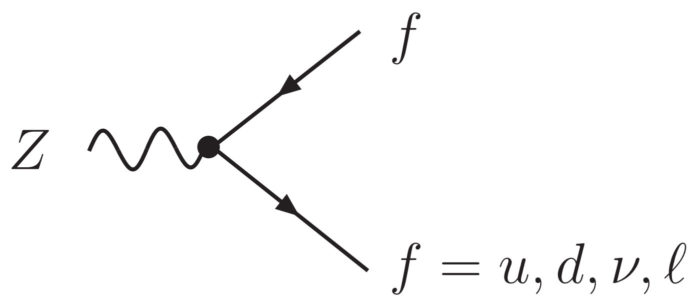
with

$$v_f = \frac{T^3_{f_L} - 2Q_f s_W^2}{2s_W c_W}\,, \quad a_f = \frac{T^3_{f_L}}{2s_W c_W}$$

- The complete neutral current Lagrangian reads:

$$\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm NC}^{Z}$$



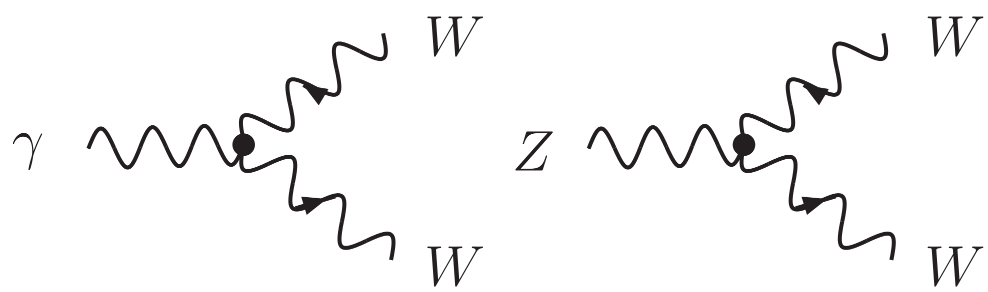



- Cubic:

$$\mathcal{L}_{\text{YM}} \supset \mathcal{L}_3 = -\frac{\text{i}ec_W}{s_W}\left\{W^{\mu\nu}W^\dagger_\mu Z_\nu - W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\dagger_\mu W_\nu Z^{\mu\nu}\right\}$$
$$+\,\text{i}e\left\{W^{\mu\nu}W^\dagger_\mu A_\nu - W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu W_\nu F^{\mu\nu}\right\}$$

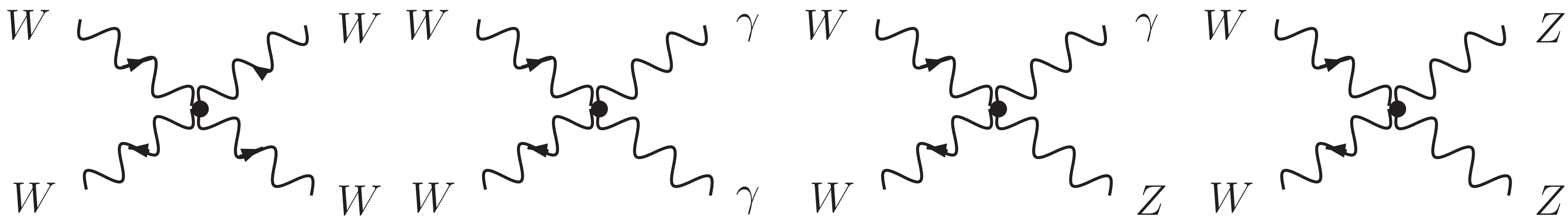
with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \qquad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$



- Quartic:

$$\mathcal{L}_{\text{YM}} \supset \mathcal{L}_4 = -\frac{e^2}{2s_W^2}\left\{\left(W_\mu^\dagger W^\mu\right)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu\right\}$$
$$-\frac{e^2 c_W^2}{s_W^2}\left\{W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu\right\}$$
$$+\frac{e^2 c_W}{s_W}\left\{2W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu\right\}$$
$$-e^2\left\{W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu\right\}$$




Note: even number of $W$ and no vertex with just $\gamma$ or $Z$

# Electroweak symmetry breaking | setup

- Out of the 4 gauge bosons of $SU(2)_L \otimes U(1)_Y$ with generators $T^1,\ T^2,\ T^3,\ Y$ we need all to be broken except the combination $Q = T^3 + Y$ so that $A_\mu$ remains massless and the other three gauge bosons get massive after SSB

  $\Rightarrow$ Introduce a complex SU(2) Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \quad \langle 0|\, \Phi \,|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

with gauge invariant Lagrangian ($\mu^2 = -\lambda v^2$):

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2 , \qquad D_\mu \Phi = (\partial_\mu - \mathrm{i} g \widetilde{W}_\mu + \mathrm{i} g' y_\Phi B_\mu)\Phi$$

$$\text{take } y_\Phi = \frac{1}{2} \quad \Rightarrow \quad (T^3 + Y)\,|0\rangle = Q \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$\{T^1, T^2, T^3 - Y\}\,|0\rangle \neq 0$$

# Electroweak symmetry breaking | gauge boson masses

- Quantum fields in the unitary gauge:

$$\Phi(x) \equiv \exp\left\{\mathrm{i}\frac{\sigma^i}{2v}\theta^i(x)\right\}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$$

$$\Phi(x) \mapsto \exp\left\{-\mathrm{i}\frac{\sigma^i}{2v}\theta^i(x)\right\}\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix} \quad \Rightarrow$$

1 physical Higgs field $H(x)$

3 would-be Goldstones $\theta^i(x)$ gauged away

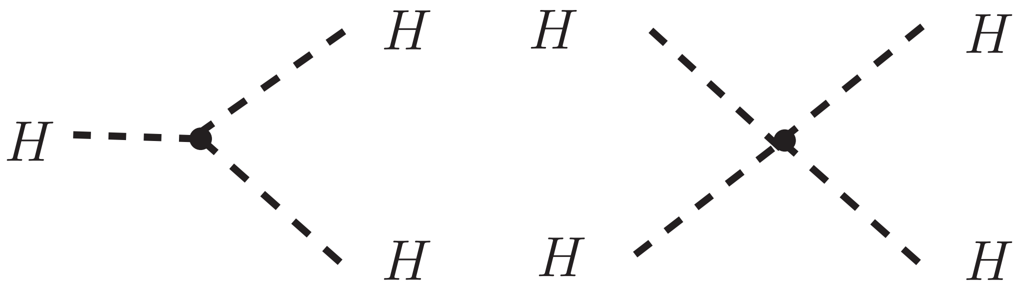
– The 3 dof apparently lost become the longitudinal polarizations of $W^\pm$ and Z that get massive after SSB:

$$\mathcal{L}_\Phi \supset \mathcal{L}_M = \underbrace{\frac{g^2v^2}{4}}_{M_W^2} W_\mu^\dagger W^\mu + \underbrace{\frac{g^2v^2}{8c_W^2}}_{\frac{1}{2}M_Z^2} Z_\mu Z^\mu \quad \Rightarrow \quad M_W = M_Z c_W = \frac{1}{2}gv$$

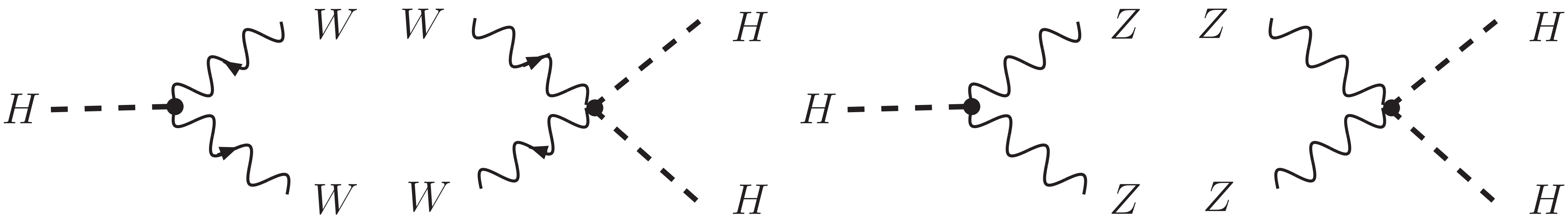
# Electroweak symmetry breaking | Higgs sector

⇒ In the unitary gauge (just physical fields): $\mathcal{L}_\Phi = \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 - \frac{M_H^2}{2v}H^3 - \frac{M_H^2}{8v^2}H^4 \,, \quad M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$$




$$\mathcal{L}_M + \mathcal{L}_{HV^2} = M_W^2 W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}$$



# Electroweak symmetry breaking | Higgs sector

- Quantum fields in the $R_\xi$ gauges:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix} , \quad \phi^-(x) = [\phi^+(x)]^*$$

$$\begin{aligned}
\mathcal{L}_\Phi = \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4 \\
+ (\partial_\mu \phi^+)(\partial^\mu \phi^-) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) \\
+ \mathrm{i}M_W \, (W_\mu \partial^\mu \phi^+ - W_\mu^\dagger \partial^\mu \phi^-) + \; M_Z \, Z_\mu \partial^\mu \chi
\end{aligned}$$

$+$ trilinear interactions [SSS, SSV, SVV]

$+$ quadrilinear interactions [SSSS, SSVV]

## Electroweak symmetry breaking — gauge fixing

- To remove the cross terms $W_\mu \partial^\mu \phi^+$, $W^\dagger_\mu \partial^\mu \phi^-$, $Z_\mu \partial^\mu \chi$ and define propagators add:

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi_\gamma}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z \chi)^2 - \frac{1}{\xi_W}|\partial_\mu W^\mu + \mathrm{i}\xi_W M_W \phi^-|^2$$

$\Rightarrow$ Massive propagators for gauge and (unphysical) would-be Goldstone fields:

$$\widetilde{D}^\gamma_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_\gamma)\frac{k_\mu k_\nu}{k^2}\right]$$

$$\widetilde{D}^Z_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_Z^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_Z)\frac{k_\mu k_\nu}{k^2 - \xi_Z M_Z^2}\right] \quad ; \quad \widetilde{D}^\chi(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\epsilon}$$

$$\widetilde{D}^W_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_W^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right] \quad ; \quad \widetilde{D}^\phi(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\epsilon}$$

('t Hooft-Feynman gauge: $\xi_\gamma = \xi_Z = \xi_W = 1$)

## Electroweak symmetry breaking | Faddeev-Popov ghosts (⋆)

- The SM is a non-Abelian theory ⇒ add Faddeev-Popov ghosts $c_i(x)$ $(i = 1, 2, 3)$

$$c_1 \equiv \frac{1}{\sqrt{2}}(u_+ + u_-)\,, \quad c_2 \equiv \frac{\mathrm{i}}{\sqrt{2}}(u_+ - u_-)\,, \quad c_3 \equiv c_W\, u_Z - s_W\, u_\gamma$$

$$\mathcal{L}_{\rm FP} = \underbrace{(\partial^\mu \bar{c}_i)(\partial_\mu c_i - g\epsilon^{ijk} c_j W^k_\mu)}_{\text{U kinetic + [UUV]}} + \underbrace{\text{interactions with } \Phi}_{\text{U masses + [SUU]}}$$

⇒ Massive propagators for (unphysical) FP ghost fields:

$$\widetilde{D}^{u_\gamma}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\,, \quad \widetilde{D}^{u_Z}(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\epsilon}\,, \quad \widetilde{D}^{u_\pm}(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\epsilon}$$

('t Hooft-Feynman gauge: $\xi_Z = \xi_W = 1$)

## Electroweak symmetry breaking | Faddeev-Popov ghosts (⋆)

$$\mathcal{L}_{\rm FP} = (\partial_\mu \bar{u}_\gamma)(\partial^\mu u_\gamma) + (\partial_\mu \bar{u}_Z)(\partial^\mu u_Z) + (\partial_\mu \bar{u}_+)(\partial^\mu u_+) + (\partial_\mu \bar{u}_-)(\partial^\mu u_-)$$

$$[\mathrm{UUV}] \begin{cases} + \mathrm{i}e[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]A_\mu - \dfrac{\mathrm{i}ec_W}{s_W}[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]Z_\mu \\ - \mathrm{i}e[(\partial^\mu \bar{u}_+)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_-]W^\dagger_\mu + \dfrac{\mathrm{i}ec_W}{s_W}[(\partial^\mu \bar{u}_+)u_Z - (\partial^\mu \bar{u}_Z)u_-]W^\dagger_\mu \\ + \mathrm{i}e[(\partial^\mu \bar{u}_-)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_+]W_\mu - \dfrac{\mathrm{i}ec_W}{s_W}[(\partial^\mu \bar{u}_-)u_Z - (\partial^\mu \bar{u}_Z)u_+]W_\mu \end{cases}$$

$$- \xi_Z M_Z^2\, \bar{u}_Z u_Z - \xi_W M_W^2\, \bar{u}_+ u_+ - \xi_W M_W^2\, \bar{u}_- u_-$$

$$[\mathrm{SUU}] \begin{cases} - e\xi_Z M_Z\, \bar{u}_Z \left[\dfrac{1}{2s_W c_W} H u_Z - \dfrac{1}{2s_W}\left(\phi^+ u_- + \phi^- u_+\right)\right] \\ - e\xi_W M_W\, \bar{u}_+ \left[\dfrac{1}{2s_W}(H + \mathrm{i}\chi)u_+ - \phi^+ \left(u_\gamma - \dfrac{c_W^2 - s_W^2}{2s_W c_W} u_Z\right)\right] \\ - e\xi_W M_W\, \bar{u}_- \left[\dfrac{1}{2s_W}(H - \mathrm{i}\chi)u_- - \phi^- \left(u_\gamma - \dfrac{c_W^2 - s_W^2}{2s_W c_W} u_Z\right)\right] \end{cases}$$

## Electroweak symmetry breaking | fermion masses

- We need masses for quarks and leptons without breaking gauge symmetry

$\Rightarrow$ Introduce Yukawa interactions:

$$\mathcal{L}_\mathrm{Y} = -\lambda_d \begin{pmatrix} \overline{u}_L & \overline{d}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - \lambda_u \begin{pmatrix} \overline{u}_L & \overline{d}_L \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R$$
$$- \lambda_\ell \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \ell_R - \lambda_\nu \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \nu_R \quad + \mathrm{h.c.}$$

where $\Phi^c \equiv i\sigma_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ transforms under SU(2) like $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$\Rightarrow$ After EW SSB, fermions acquire masses:

$$\mathcal{L}_\mathrm{Y} \supset -\frac{1}{\sqrt{2}}(v+H)\left\{\lambda_d\,\overline{d}d + \lambda_u\,\overline{u}u + \lambda_\ell\,\overline{\ell}\ell + \lambda_\nu\,\overline{\nu}\nu\right\} \quad \Rightarrow \quad m_f = \lambda_f \frac{v}{\sqrt{2}}$$

## Additional generations — Yukawa matrices

- There are 3 generations of quarks and leptons in Nature. They are identical copies with the same properties under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ differing only in their masses

$\Rightarrow$ Take a general case of $n_G$ generations and let $u_j^I$, $d_j^I$, $\nu_j^I$, $\ell_j^I$ be the members of family $j$ ($j = 1, \ldots, n_G$). Superindex $I$ (interaction basis) was omitted so far

$\Rightarrow$ General gauge invariant Yukawa Lagrangian:

$$\mathcal{L}_{\mathrm{Y}} = -\sum_{jk} \left\{ \begin{pmatrix} \overline{u}_{jL}^I & \overline{d}_{jL}^I \end{pmatrix} \left[ \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{jk}^{(d)} \, d_{kR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{jk}^{(u)} \, u_{kR}^I \right] + \begin{pmatrix} \overline{\nu}_{jL}^I & \overline{\ell}_{jL}^I \end{pmatrix} \left[ \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{jk}^{(\ell)} \, \ell_{kR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{jk}^{(\nu)} \, \nu_{kR}^I \right] \right\} + \mathrm{h.c.}$$

where $\lambda_{jk}^{(d)}$, $\lambda_{jk}^{(u)}$, $\lambda_{jk}^{(\ell)}$, $\lambda_{jk}^{(\nu)}$ are arbitrary Yukawa matrices

# Additional generations | mass matrices

- After EW SSB, in $n_G$-dimensional matrix form:

$$\mathcal{L}_{\rm Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}^I_L\,\mathbf{M}_d\,\mathbf{d}^I_R + \overline{\mathbf{u}}^I_L\,\mathbf{M}_u\,\mathbf{u}^I_R + \overline{\mathbf{l}}^I_L\,\mathbf{M}_\ell\,\mathbf{l}^I_R + \overline{\nu}^I_L\,\mathbf{M}_\nu\,\nu^I_R + \text{h.c.}\right\}$$

with mass matrices

$$(\mathbf{M}_d)_{ij} \equiv \lambda^{(d)}_{ij}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_u)_{ij} \equiv \lambda^{(u)}_{ij}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\ell)_{ij} \equiv \lambda^{(\ell)}_{ij}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\nu)_{ij} \equiv \lambda^{(\nu)}_{ij}\frac{v}{\sqrt{2}}$$

$\Rightarrow$ Diagonalization determines mass eigenstates $d_j$, $u_j$, $\ell_j$, $\nu_j$ in terms of interaction states $d^I_j$, $u^I_j$, $\ell^I_j$, $\nu^I_j$, respectively

$\Rightarrow$ Each $\mathbf{M}_f$ can be written as

$$\mathbf{M}_f = \mathbf{H}_f\,\mathcal{U}_f = \mathbf{S}^\dagger_f\,\mathcal{M}_f\,\mathbf{S}_f\,\mathcal{U}_f \quad \Longleftrightarrow \quad \mathbf{M}_f\mathbf{M}^\dagger_f = \mathbf{H}^2_f = \mathbf{S}^\dagger_f\,\mathcal{M}^2_f\,\mathbf{S}_f$$

with $\mathbf{H}_f \equiv \sqrt{\mathbf{M}_f\mathbf{M}^\dagger_f}$ a Hermitian positive definite matrix and $\mathcal{U}_f$ unitary

– Every $\mathbf{H}_f$ can be diagonalized by a unitary matrix $\mathbf{S}_f$

– The resulting $\mathcal{M}_f$ is diagonal and positive definite

## Additional generations | fermion masses and mixings

- In terms of diagonal mass matrices (mass eigenstate basis):

$$\mathcal{M}_d = \text{diag}(m_d, m_s, m_b, \ldots)\ , \quad \mathcal{M}_u = \text{diag}(m_u, m_c, m_t, \ldots)$$
$$\mathcal{M}_\ell = \text{diag}(m_e, m_\mu, m_\tau, \ldots)\ , \quad \mathcal{M}_\nu = \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, \ldots)$$

$$\mathcal{L}_\mathrm{Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}\,\mathcal{M}_d\,\mathbf{d} + \overline{\mathbf{u}}\,\mathcal{M}_u\,\mathbf{u} + \overline{\mathbf{l}}\,\mathcal{M}_\ell\,\mathbf{l} + \overline{\vec{\nu}}\,\mathcal{M}_\nu\,\vec{\nu}\right\}$$

where fermion couplings to Higgs are proportional to masses and

$$\mathbf{d}_L \equiv \mathbf{S}_d\,\mathbf{d}_L^I \quad \mathbf{u}_L \equiv \mathbf{S}_u\,\mathbf{u}_L^I \quad \mathbf{l}_L \equiv \mathbf{S}_\ell\,\mathbf{l}_L^I \quad \nu_L \equiv \mathbf{S}_\nu\,\nu_L^I$$
$$\mathbf{d}_R \equiv \mathbf{S}_d\mathcal{U}_d\,\mathbf{d}_R^I \quad \mathbf{u}_R \equiv \mathbf{S}_u\mathcal{U}_u\,\mathbf{u}_R^I \quad \mathbf{l}_R \equiv \mathbf{S}_\ell\mathcal{U}_\ell\,\mathbf{l}_R^I \quad \nu_R \equiv \mathbf{S}_\nu\mathcal{U}_\nu\,\nu_R^I$$

Neutral Currents preserve chirality
$\overline{\mathbf{f}}_L^I\,\mathbf{f}_L^I = \overline{\mathbf{f}}_L\,\mathbf{f}_L$ and $\overline{\mathbf{f}}_R^I\,\mathbf{f}_R^I = \overline{\mathbf{f}}_R\,\mathbf{f}_R$
$\Rightarrow \mathcal{L}_\mathrm{NC}$ does not change flavor

$\Rightarrow$ GIM mechanism

[Glashow, Iliopoulos, Maiani '70]

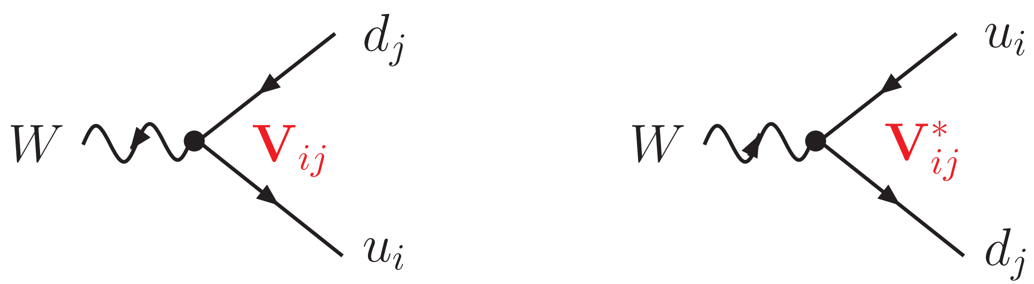
# Additional generations | quark sector

- However, in Charged Currents (also chirality preserving and only LH):

$$\overline{\mathbf{u}}_L^I\,\mathbf{d}_L^I = \overline{\mathbf{u}}_L\,\mathbf{S}_u\,\mathbf{S}_d^\dagger\,\mathbf{d}_L = \overline{\mathbf{u}}_L\mathbf{V}\mathbf{d}_L$$

with $\mathbf{V} \equiv \mathbf{S}_u\,\mathbf{S}_d^\dagger$ the (unitary) CKM mixing matrix [Cabibbo '63; Kobayashi, Maskawa '73]

$$\Rightarrow \quad \mathcal{L}_{CC} = \frac{g}{2\sqrt{2}}\sum_{ij}\overline{u}_i\gamma^\mu(1-\gamma_5)\,\mathbf{V}_{ij}\,d_j\,W_\mu^\dagger + \text{h.c.}$$




$\Rightarrow$ If $u_i$ or $d_j$ had degenerate masses one could choose $\mathbf{S}_u = \mathbf{S}_d$ (field redefinition) and flavor would be conserved in the quark sector. But they are not degenerate

$\Rightarrow$ $\mathbf{S}_u$ and $\mathbf{S}_d$ are not observable. Just masses and CKM mixings are observable

## Additional generations | quark sector

- How many physical parameters in this sector?

  - Quark masses and CKM mixings determined by mass (or Yukawa) matrices

  - A general $n_G \times n_G$ unitary matrix, like the CKM, is given by

$$n_G^2 \text{ real parameters} = n_G(n_G-1)/2 \text{ moduli} + n_G(n_G+1)/2 \text{ phases}$$

Some phases are unphysical since they can be absorbed by field redefinitions:

$$u_i \to \mathrm{e}^{\mathrm{i}\phi_i}\, u_i \,, \quad d_j \to \mathrm{e}^{\mathrm{i}\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to \mathbf{V}_{ij}\, \mathrm{e}^{\mathrm{i}(\theta_j-\phi_i)}$$

Therefore $2n_G - 1$ unphysical phases and the physical parameters are:

$$\boxed{(n_G-1)^2 = n_G(n_G-1)/2 \text{ moduli } + (n_G-1)(n_G-2)/2 \text{ phases}}$$

# Additional generations | quark sector

⇒ Case of $n_G = 2$ generations: 1 parameter, the Cabibbo angle $\theta_C$:

$$\mathbf{V} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

⇒ Case of $n_G = 3$ generations: 3 angles + 1 phase. In the standard parameterization:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ub} \\ \mathbf{V}_{cd} & \mathbf{V}_{cs} & \mathbf{V}_{cb} \\ \mathbf{V}_{td} & \mathbf{V}_{ts} & \mathbf{V}_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \Rightarrow$$

$\delta_{13}$ only source of CP violation in the SM !

with $c_{ij} \equiv \cos\theta_{ij} \geq 0, \quad s_{ij} \equiv \sin\theta_{ij} \geq 0 \quad (i < j = 1, 2, 3) \quad$ and $0 \leq \delta_{13} \leq 2\pi$

# Complete SM Lagrangian — fields and interactions

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{\text{YM}} + \mathcal{L}_{\Phi} + \mathcal{L}_{\text{Y}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}$$

- Fields: [F] fermions [S] scalars
  [V] vector bosons [U] unphysical ghosts

- Interactions: [FFV] [FFS] [SSV] [SVV] [SSVV]
  [VVV] [VVVV] [SSS] [SSSS]
  [SUU] [UUVV]

## Complete SM Lagrangian | Feynman rules

- Feynman rules for generic couplings normalized to $e$ (all momenta incoming):

| | | |
|---|---|---|
| (i$\mathcal{L}$) [FFV$_\mu$] | $\mathrm{i}e\gamma^\mu(g_V - g_A\gamma_5) = \mathrm{i}e\gamma^\mu(g_L P_L + g_R P_R)$ | |
| [FFS] | $\mathrm{i}e(g_S - g_P\gamma_5) = \mathrm{i}e(c_L P_L + c_R P_R)$ | |
| [SV$_\mu$V$_\nu$] | $\mathrm{i}eKg_{\mu\nu}$ | |
| [S$(p_1)$S$(p_2)$V$_\mu$] | $\mathrm{i}eG(p_1 - p_2)_\mu$ | |
| [V$_\mu(k_1)$V$_\nu(k_2)$V$_\rho(k_3)$] | $\mathrm{i}eJ\left[g_{\mu\nu}(k_2-k_1)_\rho + g_{\nu\rho}(k_3-k_2)_\mu + g_{\mu\rho}(k_1-k_3)_\nu\right]$ | |
| [V$_\mu(k_1)$V$_\nu(k_2)$V$_\rho(k_3)$V$_\sigma(k_4)$] | $\mathrm{i}e^2C\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right]$ | |
| [SSV$_\mu$V$_\nu$] | $\mathrm{i}e^2C_2g_{\mu\nu}$ | also [UUVV] |
| [SSS] | $\mathrm{i}eC_3$ | also [SUU] |
| [SSSS] | $\mathrm{i}e^2C_4$ | |

Note: $g_{L,R} = g_V \pm g_A$

$c_{L,R} = g_S \pm g_P$

Attention to symmetry factors!

http://www.ugr.es/local/jillana/SM/FeynmanRulesSM.pdf

- Parameters:

| $17+9$ | $=$ | 1 | 1 | 1 | 1 | $9+3$ | 4 | 6 |
|---|---|---|---|---|---|---|---|---|
| formal: | | $g$ | $g'$ | $v$ | $\lambda$ | $\lambda_f$ | $\mathbf{V}_{\text{CKM}}$ | $\mathbf{U}_{\text{PMNS}}$ |
| practical: | | $\alpha$ | $M_W$ | $M_Z$ | $M_H$ | $m_f$ | | |

where $e = g s_W = g' c_W$ and

$$\underbrace{\alpha = \frac{e^2}{4\pi} \qquad M_W = \frac{1}{2} g v \qquad M_Z = \frac{M_W}{c_W}}_{g,\ g',\ v} \qquad M_H = \sqrt{2\lambda}\, v \qquad m_f = \frac{v}{\sqrt{2}} \lambda_f$$

$\Rightarrow$ Many (more) experiments

$\Rightarrow$ After Higgs discovery, for the first time *all* parameters measured!

- Experimental values [Particle Data Group '15]

  - Fine structure constant:

    $\alpha^{-1} = 137.035\,999\,074\,(44)$ from Harvard cyclotron $(g_e)$

  - The SM predicts $M_W < M_Z$ in agreement with measurements:

    $M_Z = (91.1876 \pm 0.021)$ GeV from LEP1/SLD

    $M_W = (80.387 \pm 0.016)$ GeV from LEP2/Tevatron/LHC

  - Top quark mass:

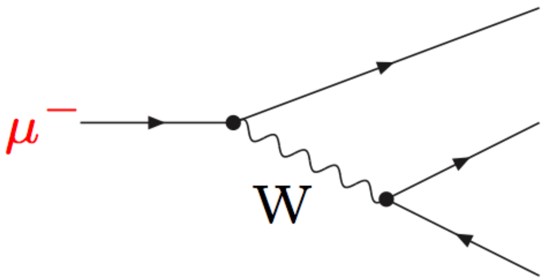
    $m_t = (173.24 \pm 0.95)$ GeV from Tevatron/LHC

  - Higgs boson mass:

    $M_H = (125.6 \pm 0.4)$ GeV from LHC

  - …

- Low energy observables
  - $\nu$-nucleon (NuTeV) and $\nu e$ (CERN) scattering: asymmetries CC/NC and $\nu/\bar{\nu}$ $\Rightarrow$ $\boxed{s_W^2}$
  - atomic parity violation (Ce, Tl, Pb): asymmetries $e_{R,L}N \to eX$ due to Z-exchange between e and nucleus $\Rightarrow$ $\boxed{s_W^2}$
  - muon decay (PSI): lifetime




$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f(m_e^2/m_\mu^2) \quad \Rightarrow \boxed{G_F}$$

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$\mathrm{i}\mathcal{M} = \left(\frac{\mathrm{i}e}{\sqrt{2}s_W}\right)^2 \bar{e}\gamma^\rho \nu_L \frac{-\mathrm{i}g_{\rho\delta}}{q^2 - M_W^2} \overline{\nu_L}\gamma^\delta \mu \equiv \overbrace{\mathrm{i}\frac{4G_F}{\sqrt{2}} (\bar{e}\gamma^\rho \nu_L)(\overline{\nu_L}\gamma_\rho \mu)}^{\text{Fermi theory } (-q^2 \ll M_W^2)} \; ; \quad \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2}$$

- Low energy observables

⇒ Fermi constant provides the Higgs VEV (electroweak scale):

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \approx 246\,\text{GeV}$$

⇒ Consistency checks: e.g.

From muon lifetime:

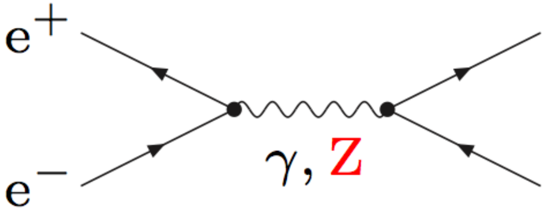
$$G_F = 1.166\,378\,7(6) \times 10^{-5}\ \text{GeV}^{-2}$$

If one compares with (tree level result)

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}$$

using measurements of $M_W$, $M_Z$ and $\alpha$ there is a discrepancy that disappears when quantum corrections are included

- $e^+e^- \to \bar{f}f$




$$\frac{d\sigma}{d\Omega} = N_c^f \frac{\alpha^2}{4s}\beta_f \Big\{ \Big[1+\cos^2\theta + (1-\beta_f^2)\sin^2\theta\Big] G_1(s) + 2(\beta_f^2-1)\, G_2(s) + 2\beta_f \cos\theta\, G_3(s)\Big\}$$

$$G_1(s) = Q_e^2 Q_f^2 + 2Q_e Q_f v_e v_f \mathrm{Re}\chi_Z(s) + (v_e^2+a_e^2)(v_f^2+a_f^2)|\chi_Z(s)|^2$$

$$G_2(s) = (v_e^2+a_e^2)a_f^2|\chi_Z(s)|^2$$

$$G_3(s) = 2Q_e Q_f a_e a_f \mathrm{Re}\chi_Z(s) + 4v_e v_f a_e a_f|\chi_Z(s)|^2 \qquad \Leftarrow A_{FB}$$

with $\chi_Z(s) \equiv \dfrac{s}{s - M_Z^2 + iM_Z\Gamma_Z}$, $N_c^f = 1$ (3) for $f$ = lepton (quark), $\beta_f$ = velocity

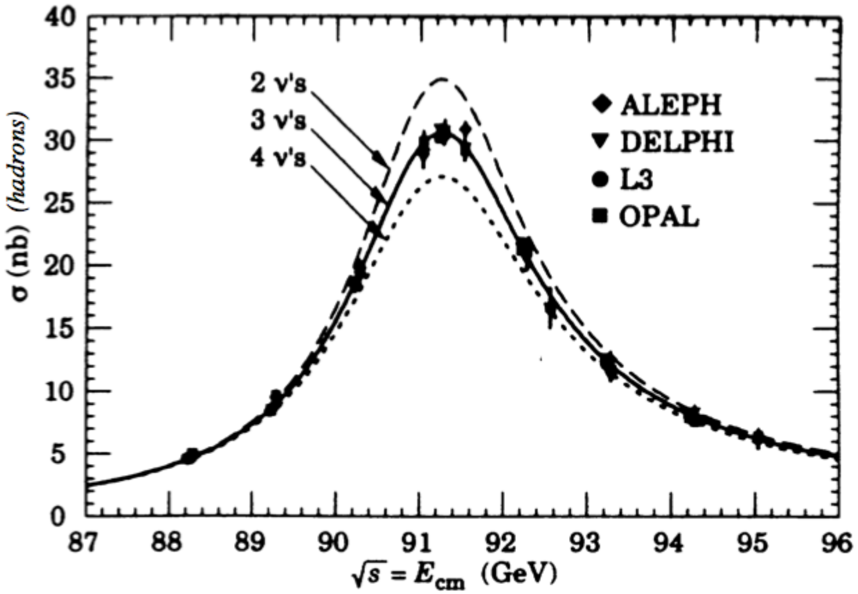
$$\sigma(s) = N_c^f \frac{2\pi\alpha^2}{3s}\beta_f \Big[(3-\beta_f^2)G_1(s) - 3(1-\beta_f^2)G_2(s)\Big]\,, \quad \beta_f = \sqrt{1-4m_f^2/s}$$

# Phenomenology | Observables and experiments

- Z production (LEP1/SLD)

  $M_Z,\ \Gamma_Z,\ \sigma_{\rm had},\ A_{FB},\ A_{LR},\ R_b,\ R_c,\ R_\ell \quad \Rightarrow \boxed{M_Z,\ s_W^2}$

  from $e^+e^- \to \bar{f}f$ at the Z pole ($\gamma - Z$ interference vanishes). Neglecting $m_f$:




$$\sigma_{\rm had} = 12\pi \frac{\Gamma(e^+e^-)\Gamma({\rm had})}{M_Z^2\Gamma_Z^2}$$

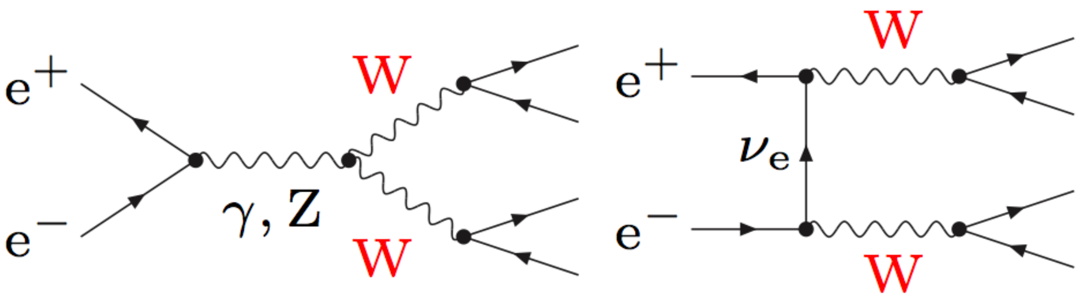
$$R_b = \frac{\Gamma(b\bar{b})}{\Gamma({\rm had})} \qquad R_c = \frac{\Gamma(c\bar{c})}{\Gamma({\rm had})} \qquad R_\ell = \frac{\Gamma({\rm had})}{\Gamma(\ell^+\ell^-)}$$

$$\left[\Gamma(Z \to f\bar{f}) \equiv \Gamma(f\bar{f}) = N_c^f \frac{\alpha M_Z}{3}(v_f^2 + a_f^2)\right]$$

Forward-Backward and (if polarized $e^-$) Left-Right asymmetries due to Z:

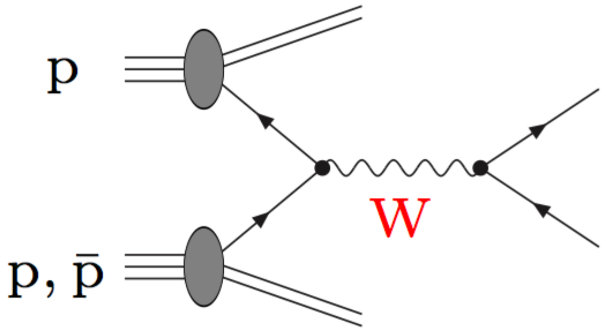
$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_f \frac{A_e + P_e}{1 + P_e A_e} \qquad A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e P_e \quad \text{with } A_f \equiv \frac{2 v_f a_f}{v_f^2 + a_f^2}$$

- W-pair production (LEP2)
  $e^+e^- \to WW \to 4\,f\ (+\gamma)$



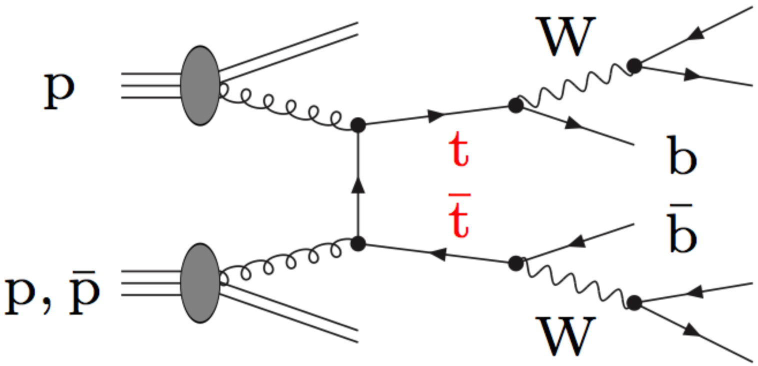

$\Rightarrow$ $\boxed{M_W}$

- W production (Tevatron/LHC)
  $p\bar{p}/pp \to W \to \ell\nu_\ell\ (+\gamma)$




$\Rightarrow$ $\boxed{M_W}$

- Top-quark production (Tevatron/LHC)
  $p\bar{p}/pp \to t\bar{t} \to 6\,f$

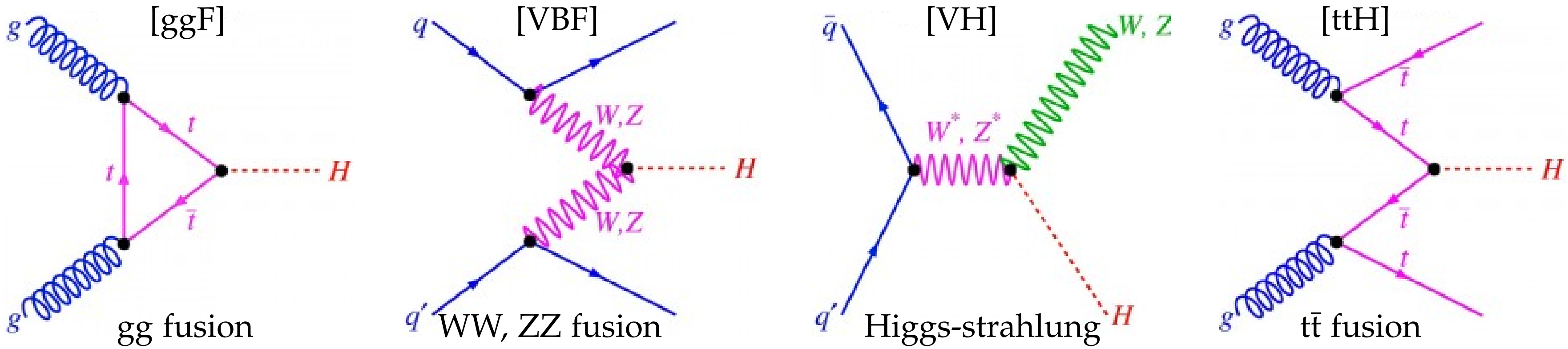


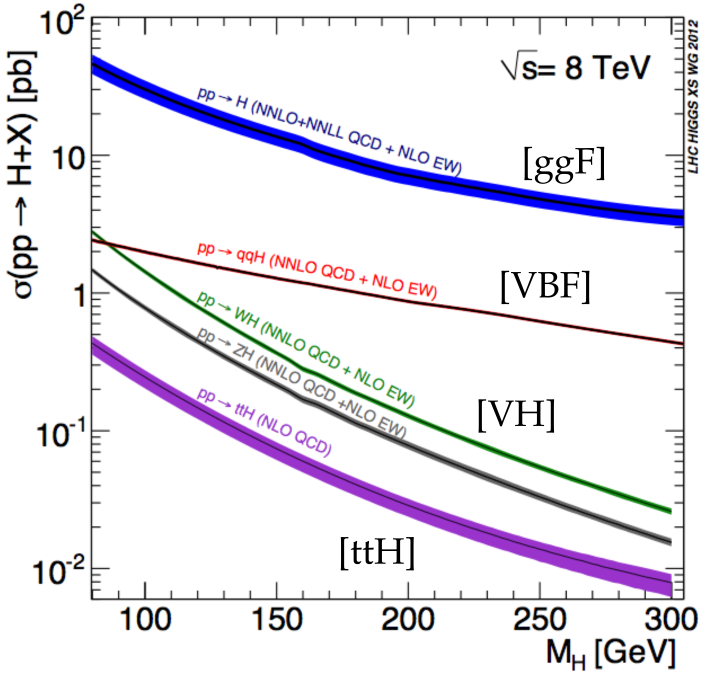

$\Rightarrow$ $\boxed{m_t}$

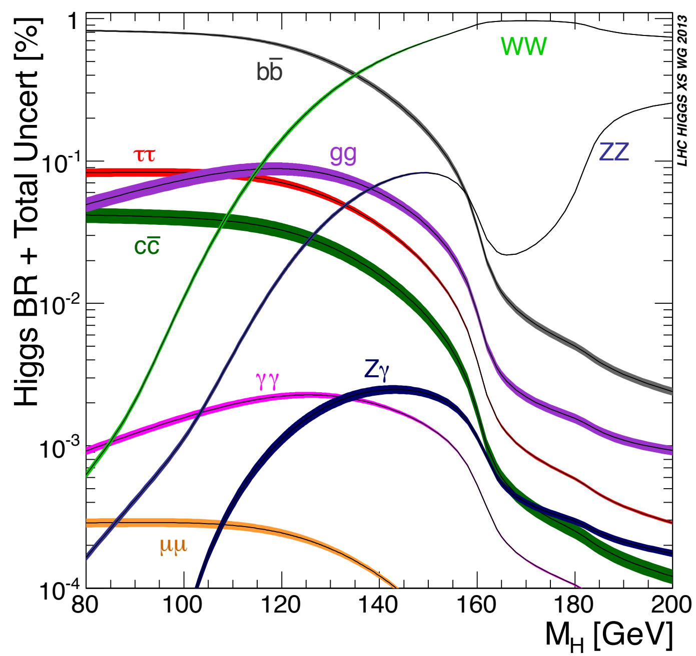
**Phenomenology** | **Observables and experiments**

- Higgs production (LHC)

  pp $\to H + X$ and $H$ decays to different channels $\Rightarrow$ $\boxed{M_H}$



# 5. Strong interactions

# Strong interactions | Properties

- Quantum Chromodynamics (QCD) is *the* theory of strong interactions
- *Quarks* and *gluons* are the fundamental *dof* but they never show up as free states: they are bound in hadrons (confinement):

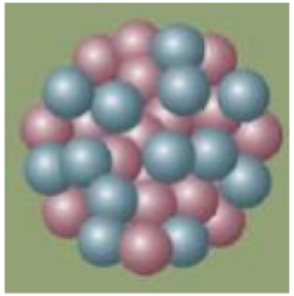
| **Baryons** ($q_1q_2q_3$ or $\bar{q}_1\bar{q}_2\bar{q}_3$) | | | | |
|---|---|---|---|---|
| name | | content | $Q$ [$e$] | $m$ [GeV] |
| p | proton | uud | +1 | 0,938 |
| $\bar{\mathrm{p}}$ | antiproton | $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}$ | −1 | |
| n | neutron | ddu | 0 | 0,939 |
| $\bar{\mathrm{n}}$ | antineutron | $\bar{\mathrm{d}}\bar{\mathrm{d}}\bar{\mathrm{u}}$ | | |
| $\Lambda$ | lambda | uds | 0 | 1,116 |
| $\bar{\Lambda}$ | antilambda | $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{s}}$ | | |
| … ∼ 120 … | | | | |

| **Mesons** ($q_1\bar{q}_2$) | | | | |
|---|---|---|---|---|
| name | | content | $Q$ [$e$] | $m$ [GeV] |
| $\pi^0$ | neutral pion | $\mathrm{u}\bar{\mathrm{u}}$, $\mathrm{d}\bar{\mathrm{d}}$ | 0 | 0,135 |
| $\pi^+$ | charged pion | $\mathrm{u}\bar{\mathrm{d}}$ | +1 | 0,140 |
| $\pi^-$ | | $\mathrm{d}\bar{\mathrm{u}}$ | −1 | |
| $K^+$ | charged kaon | $\mathrm{u}\bar{\mathrm{s}}$ | +1 | 0,494 |
| $K^-$ | | $\mathrm{s}\bar{\mathrm{u}}$ | −1 | |
| $K^0$ | neutral kaon | $\mathrm{d}\bar{\mathrm{s}}$ | 0 | 0,498 |
| $\bar{K}^0$ | | $\mathrm{s}\bar{\mathrm{d}}$ | | |
| … ∼ 140 … | | | | |

and **exotics** (glueballs, tetraquarks, pentaquarks, …)

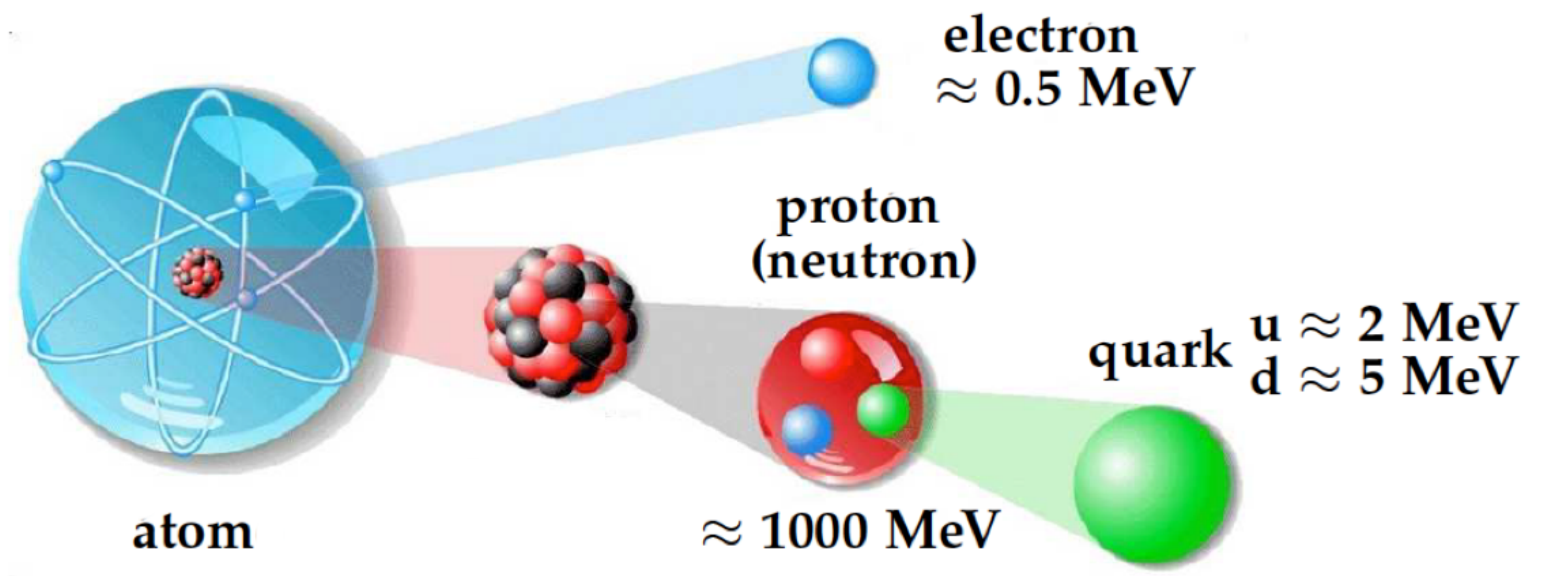
# Strong interactions | Properties

- Strong interactions are responsible for:
  - Stability of nuclei (nucleon-nucleon interaction is a residual strong force)



*strong* attraction is greater than *electric* repulsion

  - $\sim 99\,\%$ of nucleon mass is binding energy, i.e. most of the mass in everything!



# QCD | Lagrangian | SU(3) gauge symmetry

$$\mathcal{L}_{\mathrm{QCD}} = \underbrace{\overline{\psi}_{fi}\left(\mathrm{i}\not{D}_{ij} - m\delta_{ij}\right)\psi_{fj}}_{\text{quarks}} - \underbrace{\frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu}}_{\text{gluons}} \qquad \text{(flavor diagonal)}$$

$$F^{a}_{\mu\nu} = \partial_\mu \mathcal{A}^{a}_{\nu} - \partial_\nu \mathcal{A}^{a}_{\mu} + g_s f^{abc}\mathcal{A}^{b}_{\mu}\mathcal{A}^{c}_{\nu}$$

- (Anti–)quarks $\psi_f$ come in $N_c = 3$ colors (anticolors) and there are $n_f = 6$ flavors:

$$\psi_{fi} \qquad \begin{cases} f = \mathrm{u, d, s, c, b, t} & \text{(flavor index)} \\ i = 1, \ldots, N_c = 3 & \text{(color index)} \end{cases} \qquad \text{fundamental irrep } \mathbf{3}\ (\bar{\mathbf{3}})$$

- Gluons $\mathcal{A}^{a}_{\mu}$ come in $N_c^2 - 1 = 8$ combinations of color and anticolor:

$$\mathcal{A}^{a}_{\mu} \qquad a = 1, \ldots, N_c^2 - 1 = 8 \quad \text{(color index)} \qquad \text{adjoint irrep } \mathbf{8}$$

**QCD** **Lagrangian** **SU(3) gauge symmetry**

$$\mathcal{L}_{\text{QCD}} = \underbrace{\overline{\psi}_{fi}\left(\mathrm{i}\not{D}_{ij} - m\delta_{ij}\right)\psi_{fj}}_{\text{quarks}} - \underbrace{\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}}_{\text{gluons}} \qquad \text{(flavor diagonal)}$$

$$F^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_s f^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu$$

- Quark kinetic terms and quark-gluon interactions come from covariant derivative:

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - \mathrm{i}g_s t^a_{ij}\mathcal{A}^a_\mu\,, \quad t^a_{ij} = \frac{1}{2}\lambda^a_{ij} \quad \text{(8 Gell-Mann matrices } 3\times 3\text{)}$$

- Gluon kinetic terms and self-interactions fixed by SU(3) structure constants $f^{abc}$:

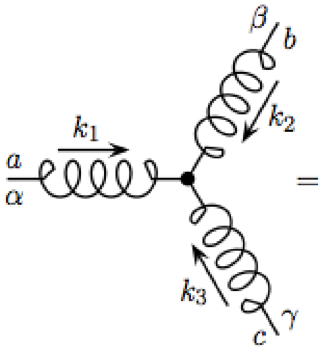
$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}(\partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu)(\partial^\mu \mathcal{A}^{a,\nu} - \partial^\nu \mathcal{A}^{a,\mu})$$

$$\mathcal{L}_{\text{cubic}} = -\frac{1}{2}g_s f^{abc}\,(\partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu)\mathcal{A}^{b,\mu}\mathcal{A}^{c,\nu}$$

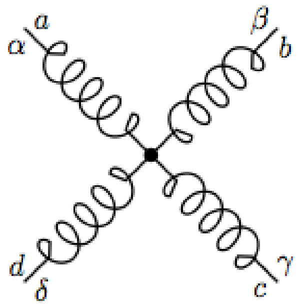
$$\mathcal{L}_{\text{quartic}} = -\frac{1}{4}g_s^2 f^{abe}f^{cde}\,\mathcal{A}^a_\mu \mathcal{A}^b_\nu \mathcal{A}^{c,\mu}\mathcal{A}^{d,\nu}$$

# QCD | Lagrangian | Feynman rules

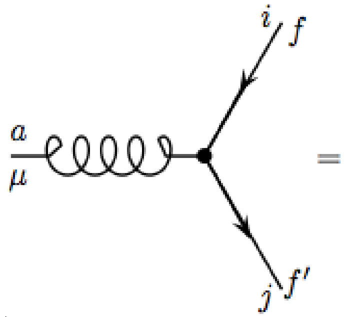
- Quark and gluon external legs and propagators are as usual
- Vertices:




$$= -g f^{abc}\left[g^{\alpha\beta}(k_1-k_2)^\gamma + g^{\beta\gamma}(k_2-k_3)^\alpha + g^{\gamma\alpha}(k_3-k_1)^\beta\right]$$




$$= -ig^2\left[\begin{array}{l} \quad f^{abe}f^{cde}(g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}) \\ + f^{ace}f^{bde}(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\gamma\beta}) \\ + f^{ade}f^{bce}(g^{\alpha\beta}g^{\delta\gamma}-g^{\alpha\gamma}g^{\delta\beta}) \end{array}\right]$$



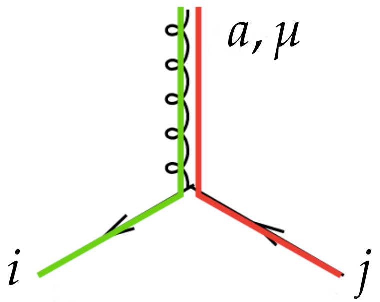

$$= ig t^a_{ij}\gamma^\mu \delta_f^{f'}$$

(interactions with would-be Goldstones and Faddeev-Popov ghosts omitted here)

# About color charges

- Quarks carry color charge: $\psi = \psi(x) \otimes \begin{pmatrix} R \\ G \\ B \end{pmatrix}$
- Antiquarks carry anticolor charge: $\overline{\psi} = \overline{\psi}(x) \otimes \begin{pmatrix} \overline{R} & \overline{G} & \overline{B} \end{pmatrix}$
- Gluons carry color and anticolor. A gluon emission *repaints* the quark:

e.g.



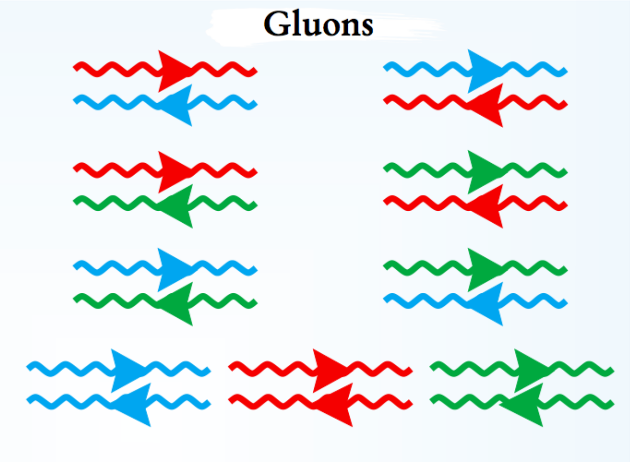

$$\overline{\psi}_i t^1_{ij} \psi_j \sim \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix},$$

# QCD | About color charges

- 8 gluons:




$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

(1 in 9 combinations is color-neutral)

$$\begin{cases} R\overline{R} - G\overline{G} \\ R\overline{R} + G\overline{G} - 2B\overline{B} \end{cases}$$

If the color-singlet massless gluon state existed:

$$R\overline{R} + G\overline{G} + B\overline{B}$$

it would give rise to a strong force of infinite range!

- Likewise, only color-singlet states can exist as free particles:

| | | |
|---|---|---|
| $q_1\bar{q}_2$ | $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$ | (here color-singlets are **mesons**) |
| $q_1q_2q_3$ | $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ | (here color-singlets are **baryons**) |

but $q_1q_2$ color singlets do not exist, since $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$

# QCD | About color charges

- Color algebra (useful identities): $t^a = \frac{1}{2}\lambda^a$

$$\mathrm{Tr}(t^a t^b) = T_R \delta_{ab}\,, \quad T_R = \frac{1}{2} \qquad \text{(convention)}$$

$$t^a_{ik} t^a_{kj} = C_F \delta_{ij}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c}$$

$$f^{acd} f^{bcd} = C_A \delta_{ab}\,, \quad C_A = N_c$$

$$t^a_{ij} t^a_{kl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ij}\delta_{kl} \qquad \text{(Fierz)}$$

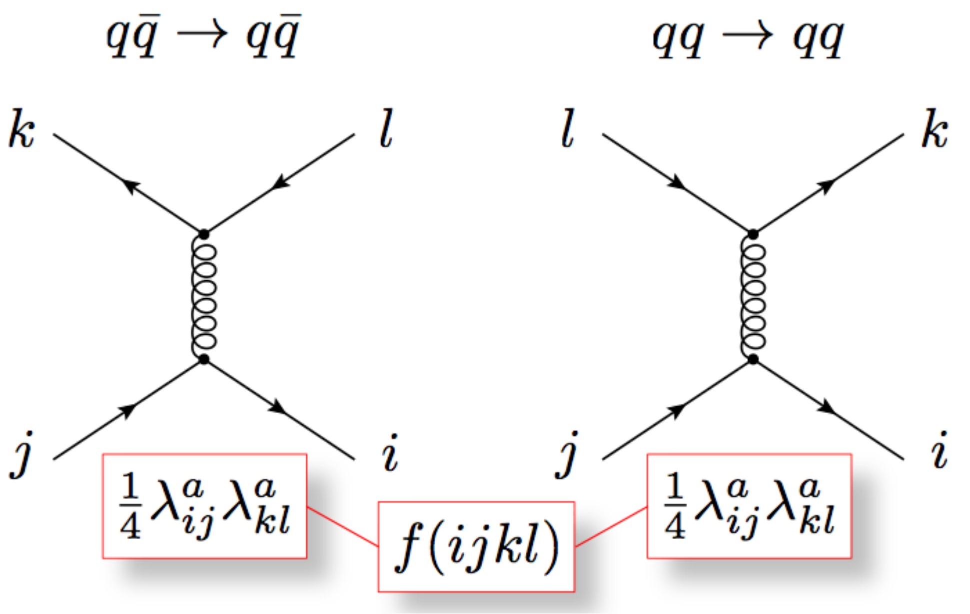
In QCD: $N_c = 3\,, \quad C_F = \frac{4}{3}\,, \quad C_A = 3$

$\Rightarrow$ Since $C_A > C_F$, gluons have a larger color charge than quarks and therefore gluons interact more strongly

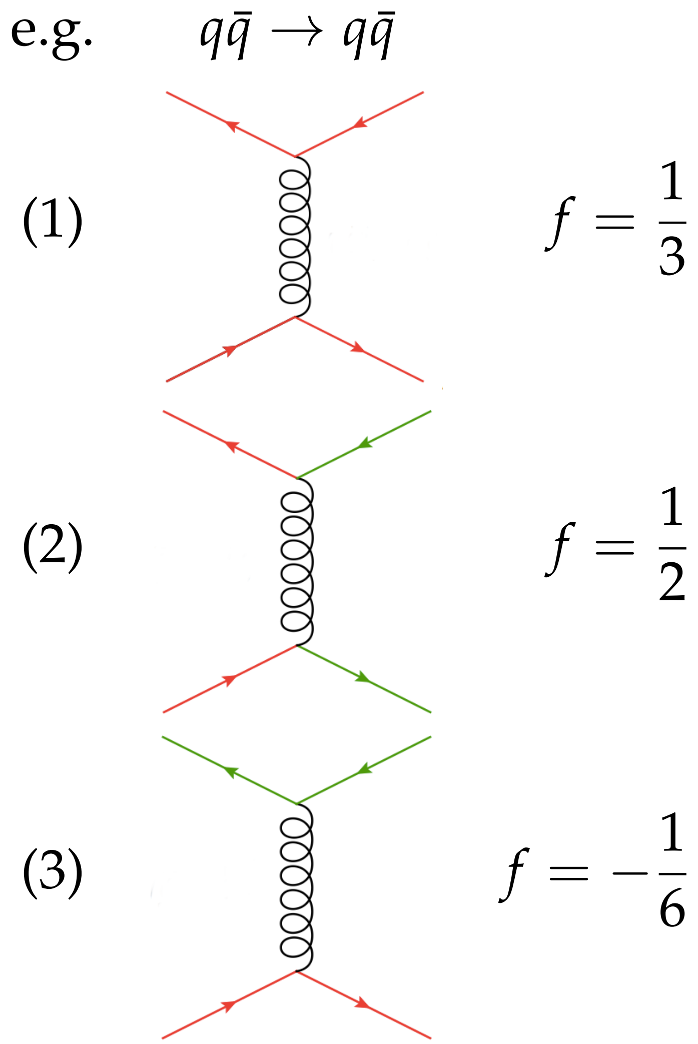
QCD

# About color charges

- Color flow:




| | $q\bar{q} \to q\bar{q}$<br>$jk \to il$ | $qq \to qq$<br>$jl \to ik$ | $f(ijkl)$ |
|---|---|---|---|
| (1) | $xx \to xx$ | $xx \to xx$ | $f(xxxx) = \frac{1}{3}$ |
| (2) | $xx \to yy$ | $xy \to yx$ | $f(yxxy) = \frac{1}{2}$ |
| (3) | $xy \to xy$ | $xy \to xy$ | $f(xxyy) = -\frac{1}{6}$ |
| | $xy \to yx$ | $xx \to yy$ | $f(yxyx) = 0$ |



**QED vs QCD** **running coupling**

- All coupling constants *run*:

$\alpha \equiv \dfrac{g^2}{4\pi} = \alpha(Q^2)$, where $Q$ is the momentum scale of the process

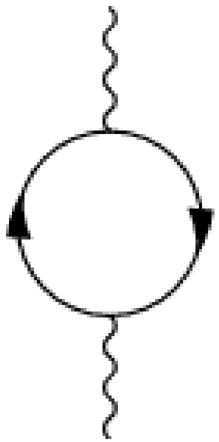
$$Q^2 \frac{\partial \alpha}{\partial Q^2} = \beta(\alpha)\,, \quad \beta(\alpha) \equiv -\beta_0 \alpha^2 (1 + \beta_1 \alpha + \beta_2 \alpha^2 + \dots)$$

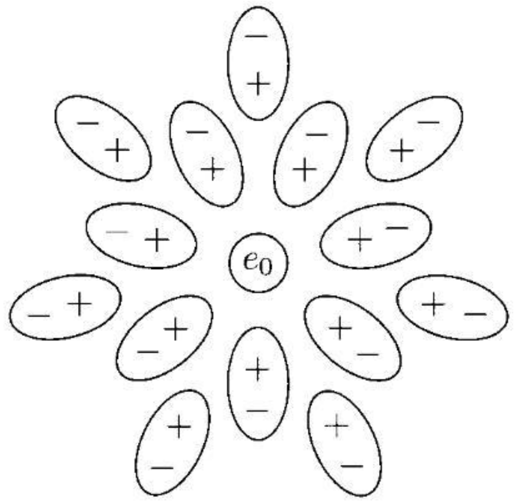
$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 + \beta_0 \alpha(Q_0^2) \ln \dfrac{Q^2}{Q_0^2}} \qquad \text{(Leading Order)}$$

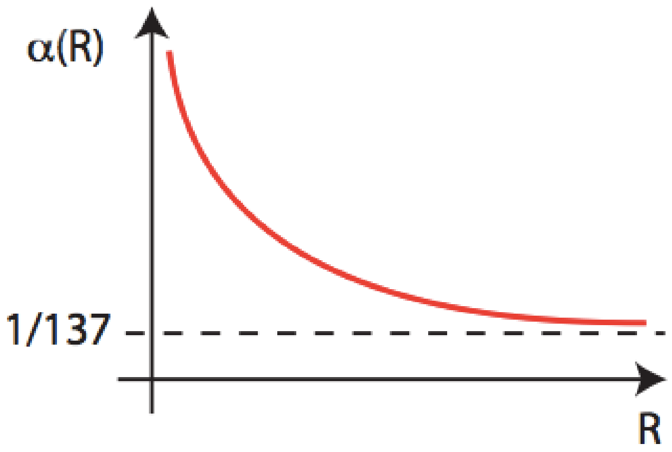
- Physically, this is related to the (anti-)screening of the fundamental charges by quantum fluctuations, depending on the sign of $\beta_0$:
  - In QED: $\alpha_{\rm em} = \dfrac{e^2}{4\pi}\,, \quad \beta_{0,\rm QED}(\alpha_{\rm em}) = -\dfrac{1}{3\pi} \quad (< 0)$
  - In QCD: $\alpha_s = \dfrac{g_s^2}{4\pi}\,, \quad \beta_{0,\rm QCD}(\alpha_s) = \dfrac{33 - 2n_f}{12\pi} \quad (> 0 \text{ for } n_f \leq 16)$

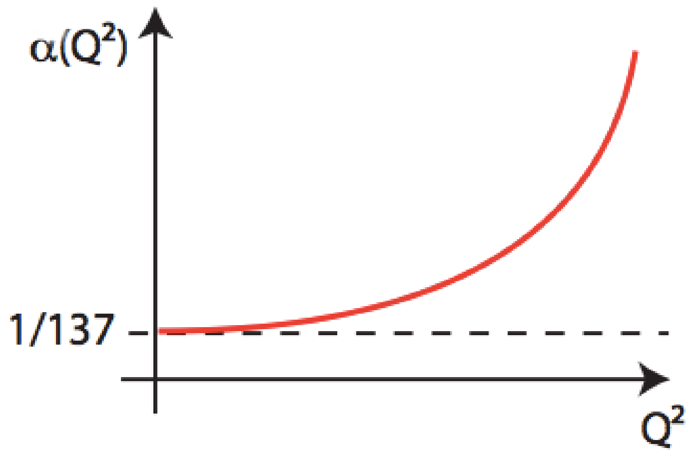
# QED running coupling

- In QED, the fluctuating vacuum behaves like a dielectric medium, screening the bare electric charge $e_0$ at increasing distances $R \sim 1/Q$:





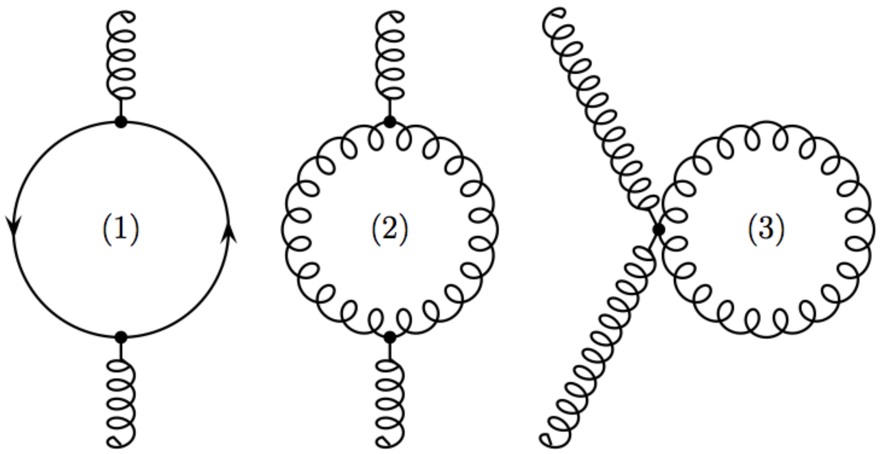




e.g. $\alpha(M_Z^2) \approx 1/128$

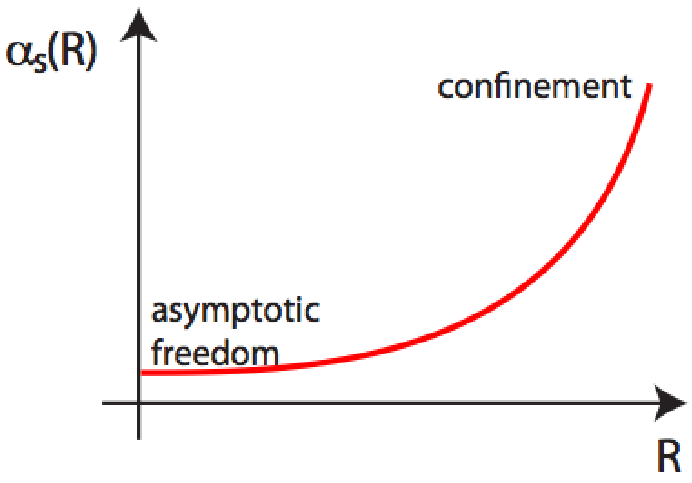
# QCD running coupling

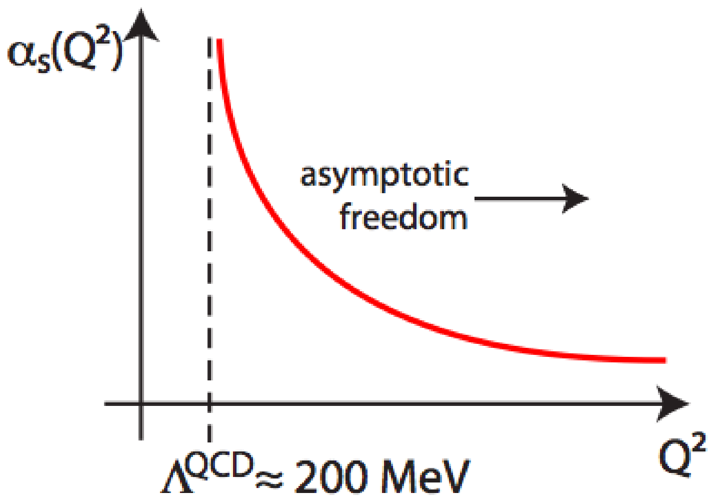
- Contributions to the QCD beta function $\beta(\alpha_s)$ (from QCD vacuum polarization):




(1) screening

(2), (3) anti-screening (non-abelian!)



# QCD running coupling

⇒ There is a scale $\Lambda_{\rm QCD}$ where $\alpha_s \to \infty$ (dimensional transmutation) given at LO by

$$\Lambda^2_{\rm QCD} = Q^2 \exp\left\{-\frac{1}{\beta_0 \alpha_s(Q^2)}\right\} \quad \Leftrightarrow \quad \alpha_s(Q^2) = \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2_{\rm QCD}}}$$

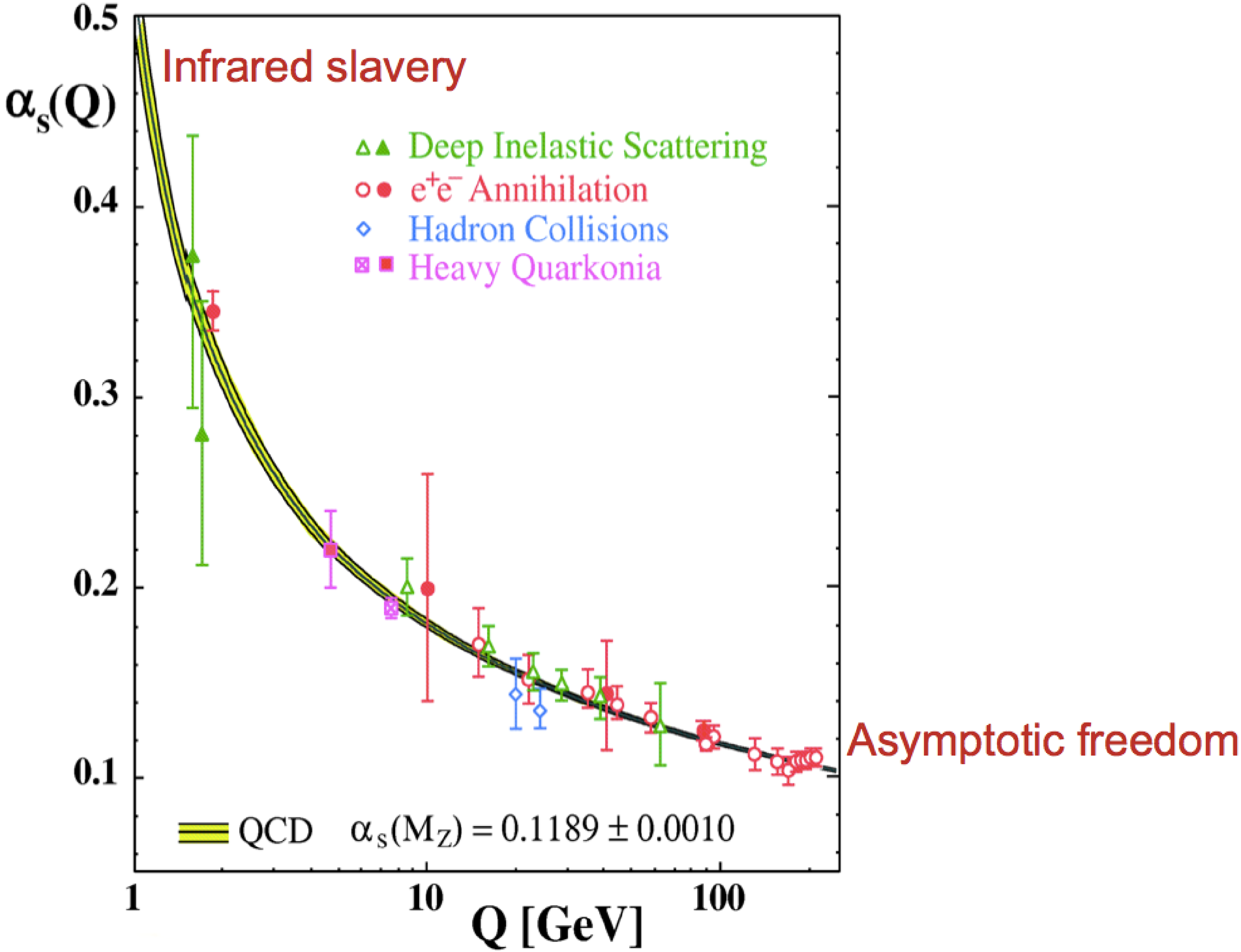
$\Lambda_{\rm QCD} \approx 200$ MeV, that is $R \sim 1/Q \approx 1$ fm (the size of a proton!)

- Asymptotic freedom:
  At short distances ($Q \gg \Lambda_{\rm QCD}$) quarks and gluons are almost free, they interact *weakly*: perturbative regime

- Infrared slavery:
  At long distances ($Q \sim \Lambda_{\rm QCD}$) the coupling diverges (Landau pole), quarks and gluons interact very *strongly* (**confinement into hadrons**): non-perturbative regime

⇒ Strong interactions are short-range, despite of gluon being massless

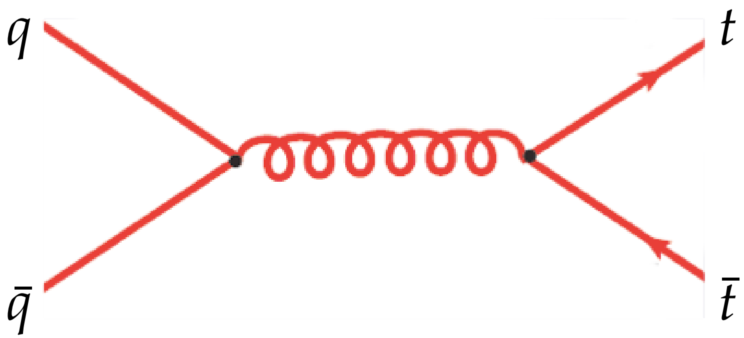
# QCD running coupling



# QCD — Perturbative methods

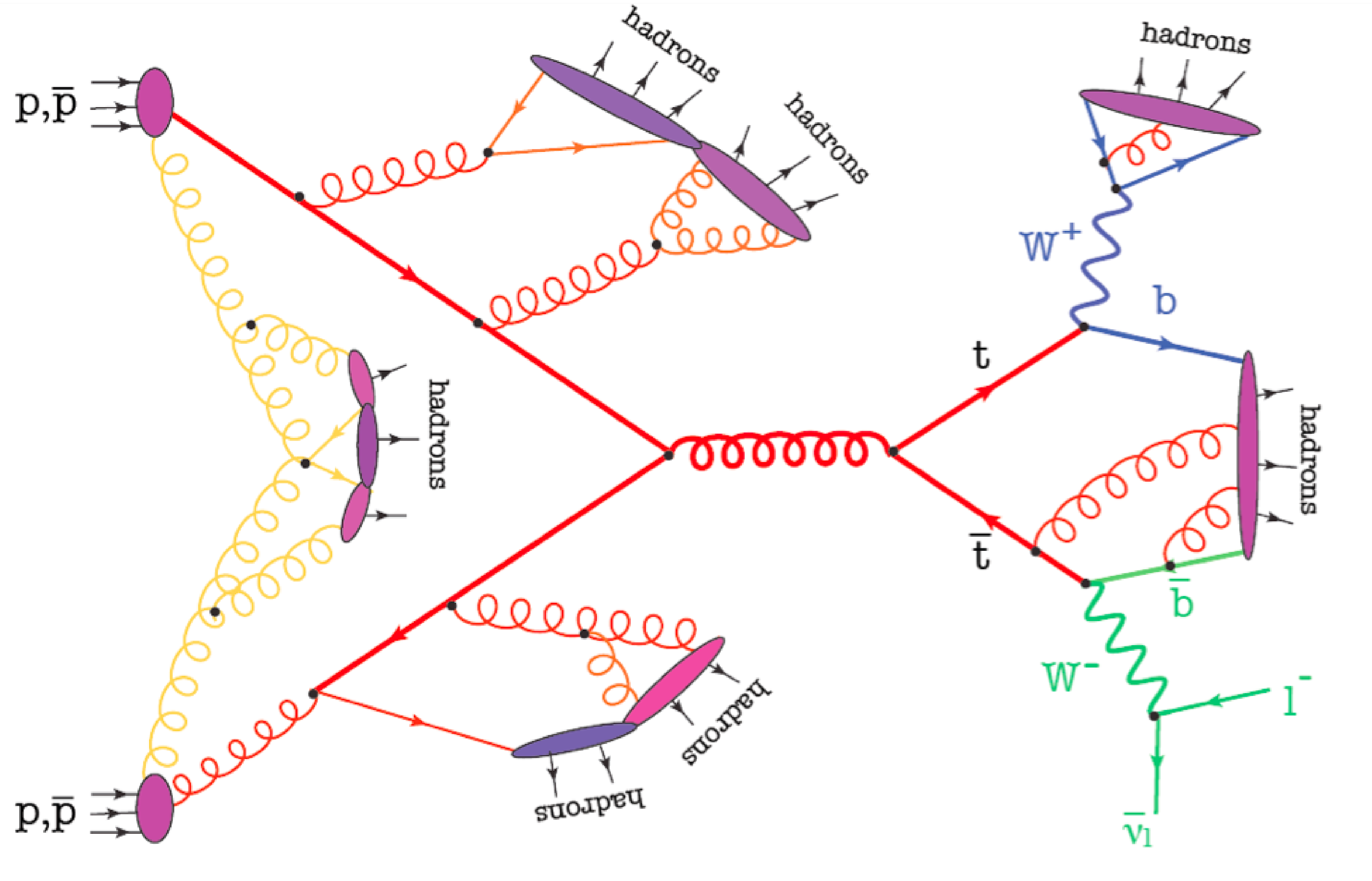
- Only when $\alpha_s(Q^2) \ll 1$, i.e. when $Q \gg \Lambda_{\rm QCD}$
- Starting point: diagrams involving quarks and gluons at a given order

  e.g.

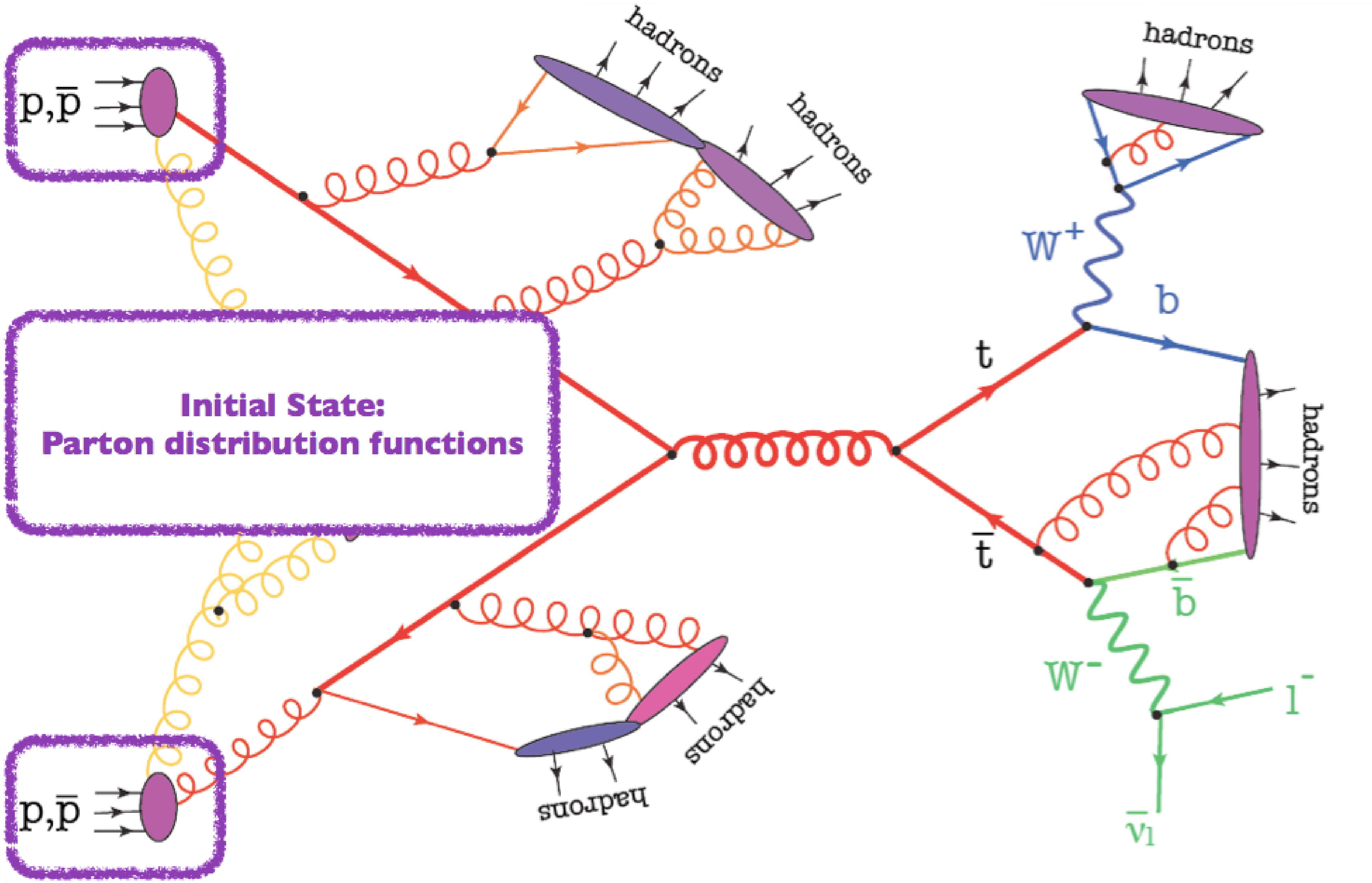



- Then: many sophisticated techniques to match real life …

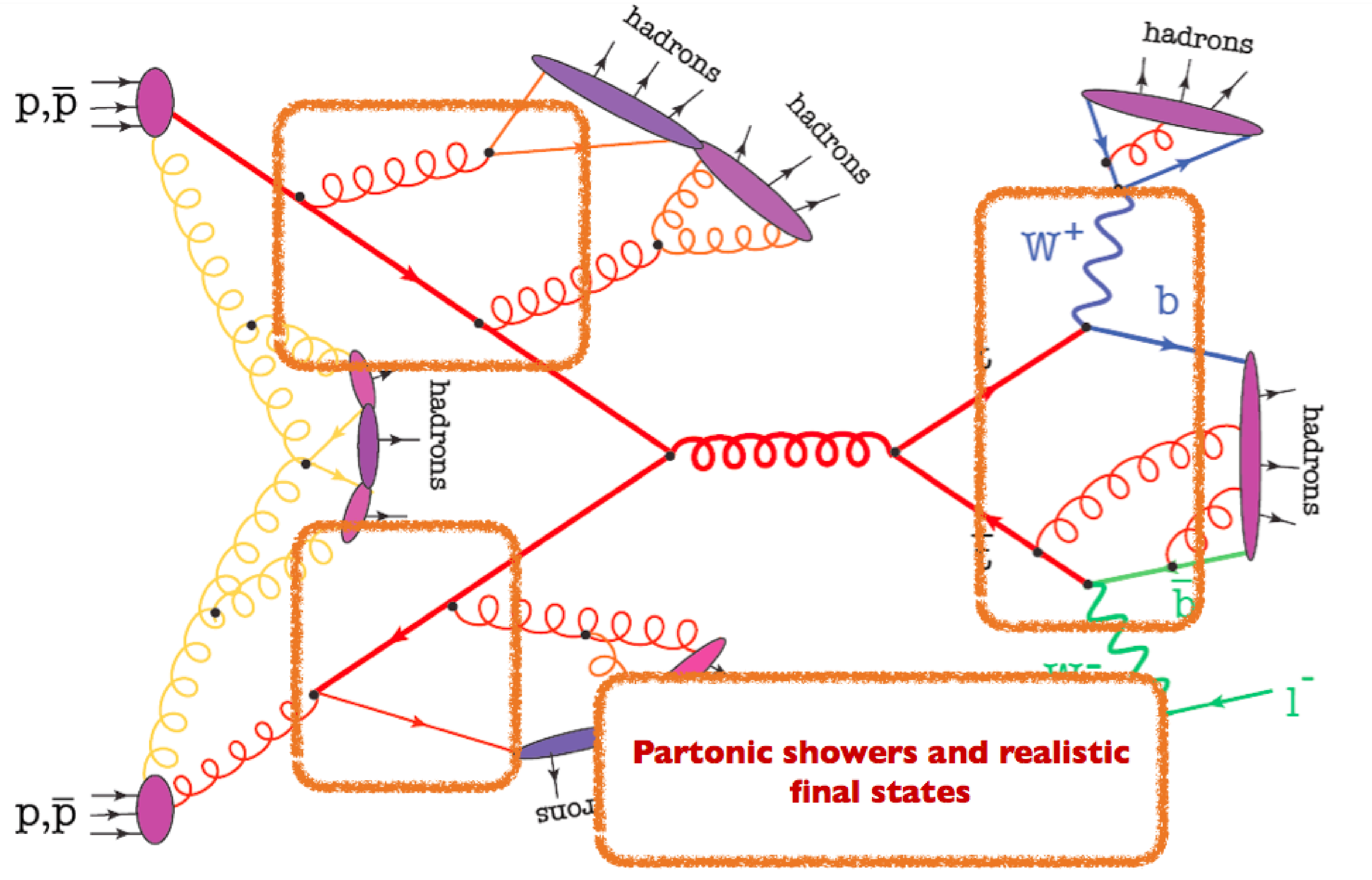
**QCD** **real life** (e.g. at LHC)



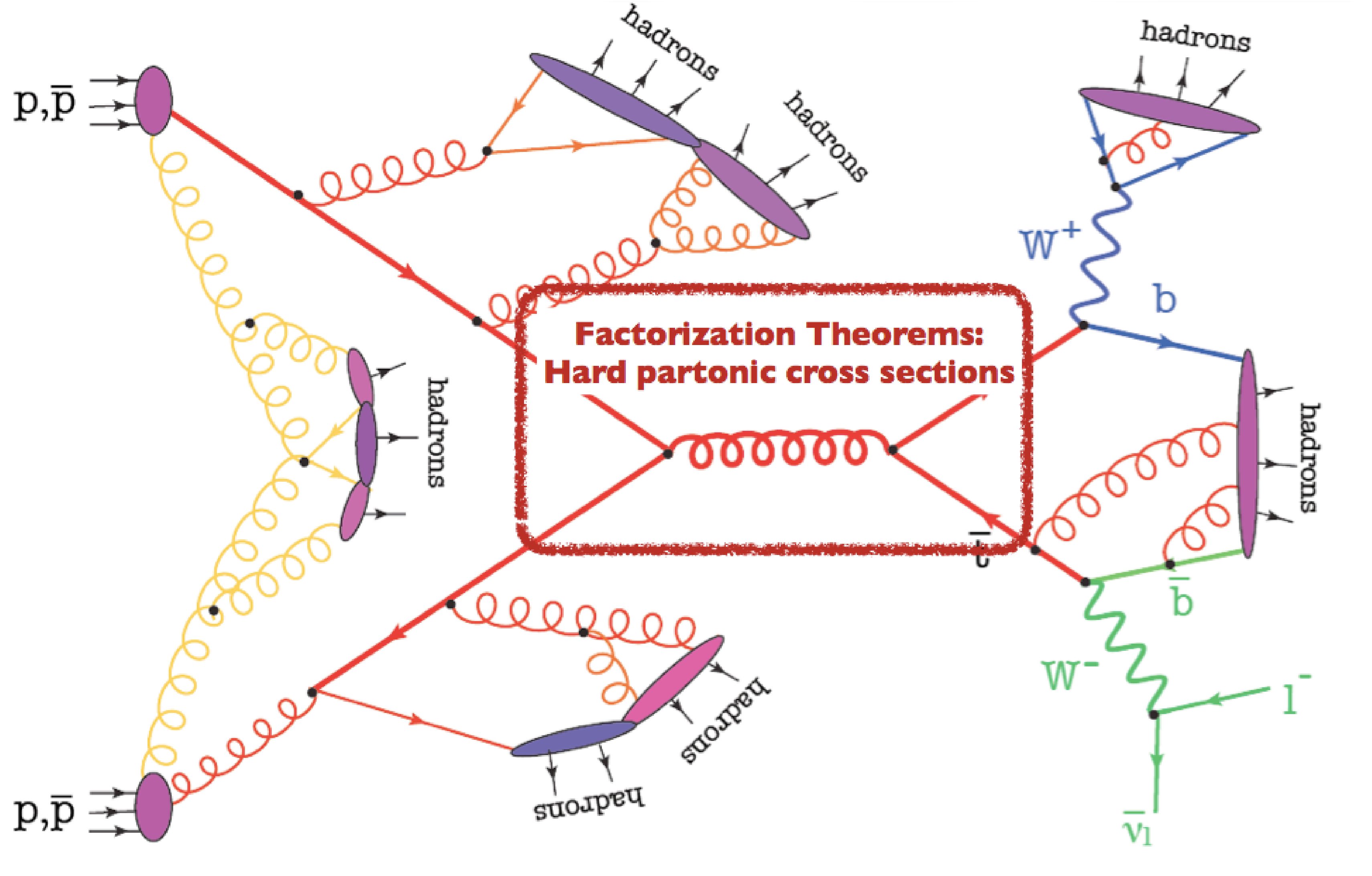
**QCD** **real life** (e.g. at LHC)



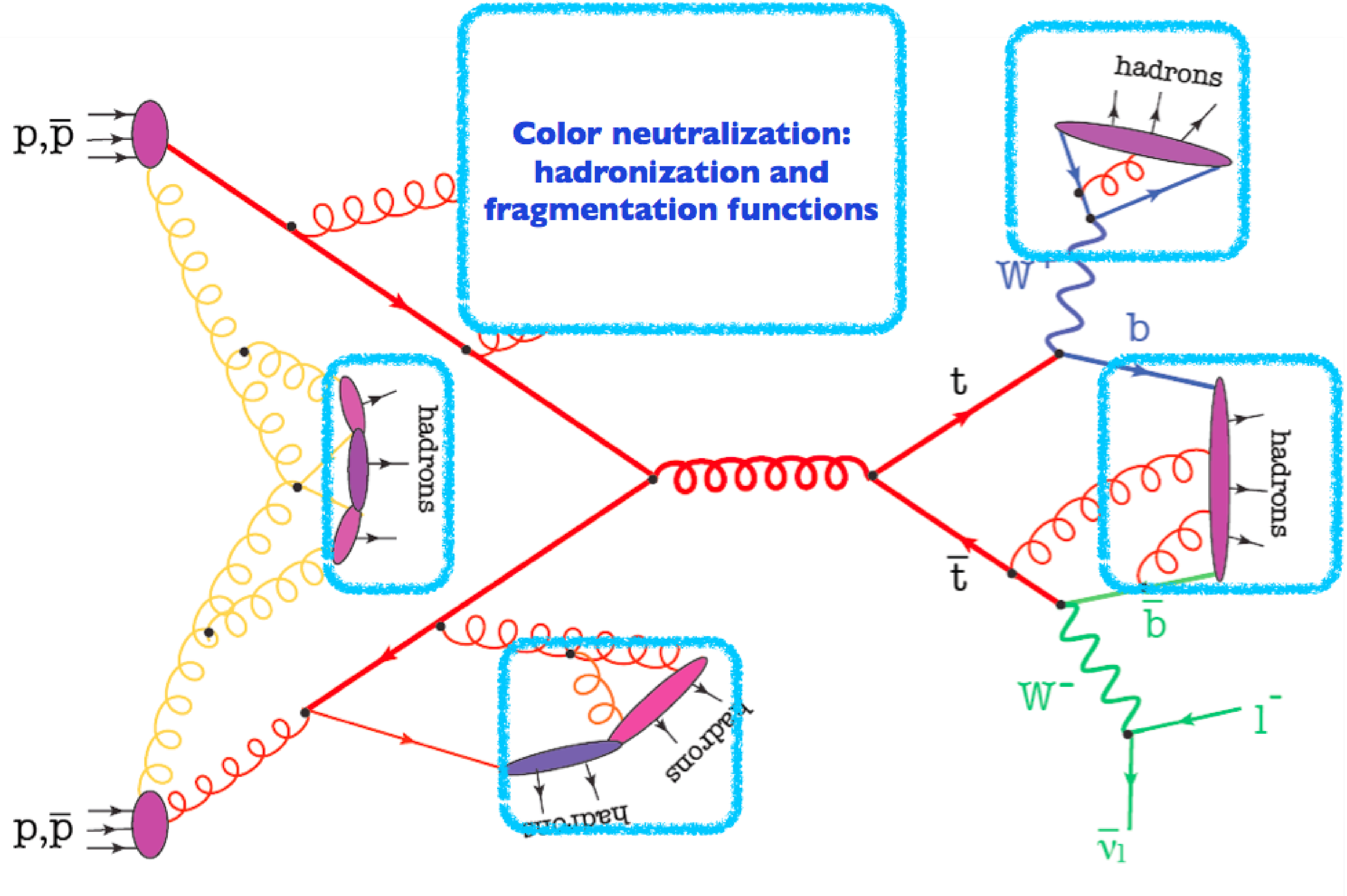
**QCD** **real life** (e.g. at LHC)



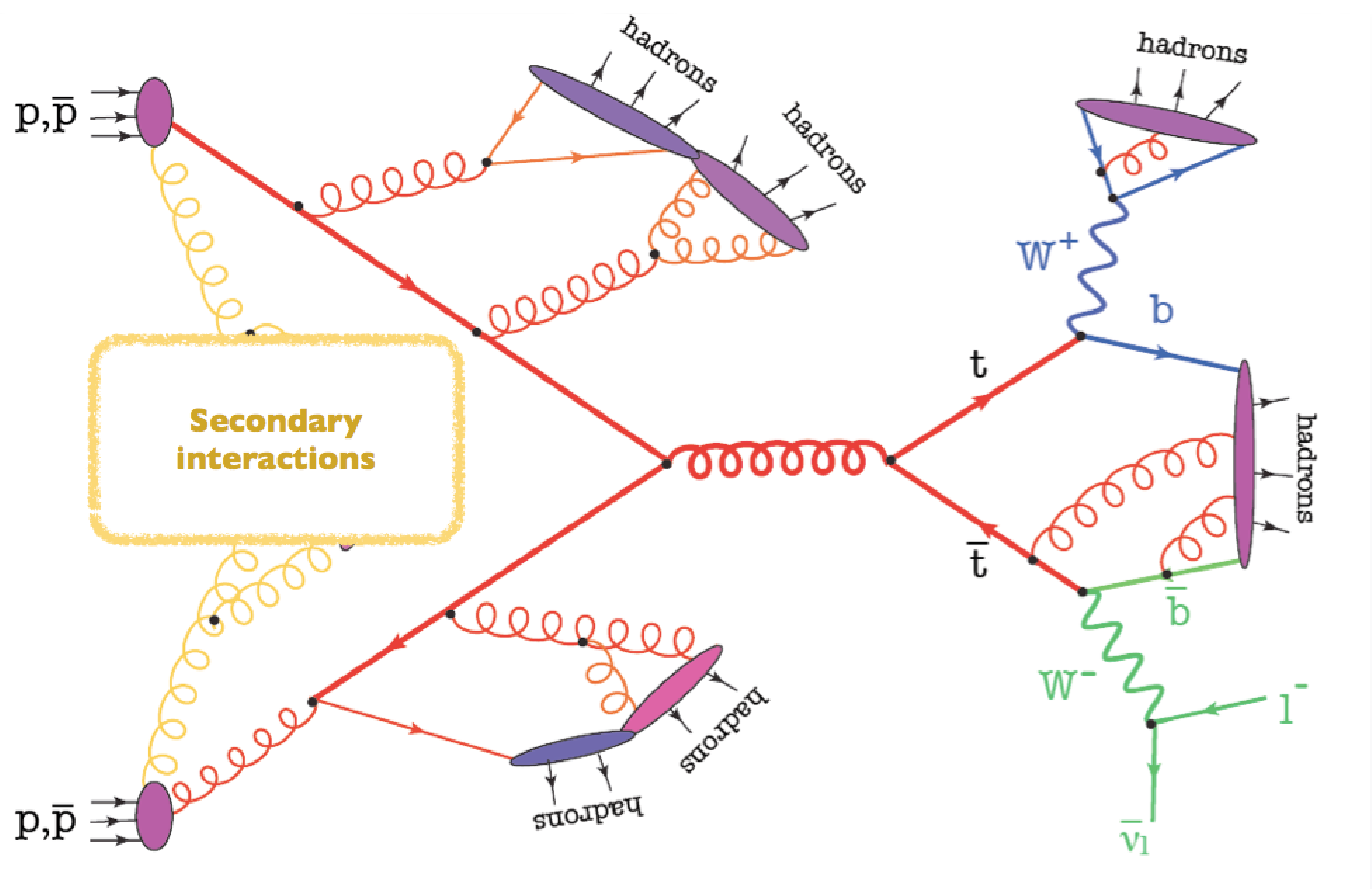
**QCD** **real life** (e.g. at LHC)



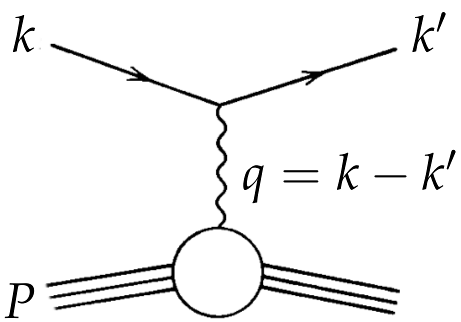
**QCD** **real life** (e.g. at LHC)



**QCD** **real life** (e.g. at LHC)



- Consider the following hadron scattering, described by the invariant quantities:




$$Q^2 = -q^2 \qquad \text{(momentum transfer squared)}$$

$$x = \frac{Q^2}{2(Pq)} \qquad \text{(hadron's momentum fraction carried by the struck quark, as shown later)}$$

$$y = \frac{(Pq)}{(Pk)} \qquad \text{(inelasticity)}$$

Some have an easier interpretation in the lab frame, where the hadron is at rest:

$$\left.\begin{aligned} P &= (M,0,0,0) \\ k &= (E,0,0,E) \\ k' &= (E', E'\sin\theta, 0, E'\cos\theta) \end{aligned}\right\} \quad \Rightarrow \quad y = \frac{E-E'}{E}$$

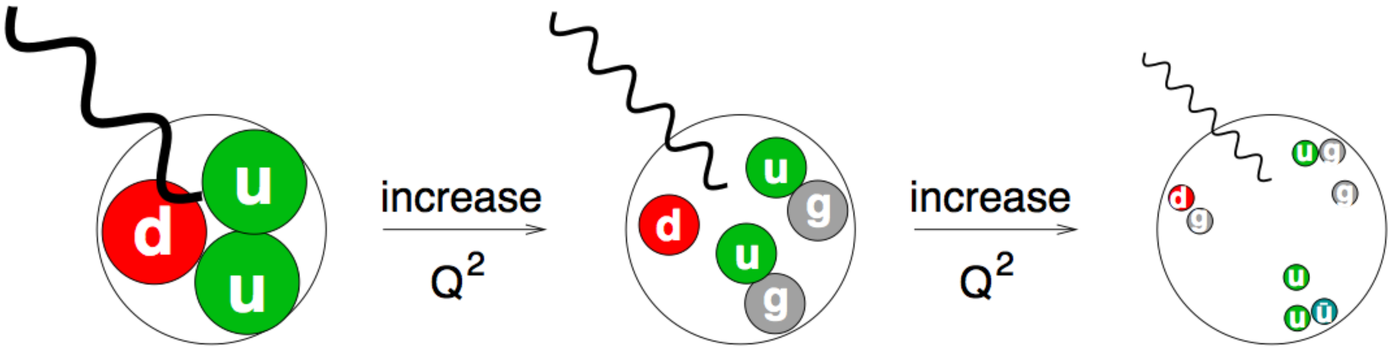
with $s = (P+k)^2 = 2ME + M^2$ the CM energy squared.

Notice that

$$Q^2 = 2(Pk)xy = xy(s - M^2) \qquad (\text{usually } s \gg M^2)$$

Hadron structure | Deep Inelastic Scattering

- Varying $Q^2$ changes the resolution of our microscope:

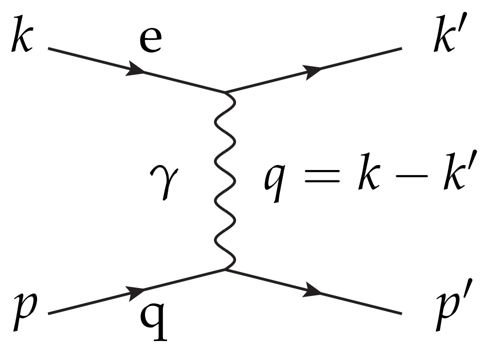



- Al low $Q^2$ (long wavelength) one *sees* a pointlike hadron (unresolved structure)

- At high $Q^2$ (short wavelength) the probe interacts with a hadron constituent resolving its structure

  ⇒ This is the deep inelastic scattering (DIS)

- To understand the underlying processes, consider first the case of eq $\to$ eq mediated by a photon exchange:




$$i\mathcal{M} = j^{\mu}_{e,r_1,r'_1}(k,k')\frac{-ig_{\mu\nu}}{q^2}j^{\nu}_{q,r_2,r'_2}(p,p')$$

$$j^{\mu}_{e,r_1,r'_1}(k,k') = \overline{u}^{(r'_1)}(k')(ie\gamma^{\mu})u^{(r_1)}(k)$$

$$j^{\nu}_{q,r_2,r'_2}(p,p') = \overline{u}^{(r'_2)}(p')(-iee_q\gamma^{\nu})u^{(r_2)}(p)$$

The (unpolarized) differential cross section is:

$$\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2}\frac{1}{4}\sum_{r_1,r_2,r'_1,r'_2}|\mathcal{M}|^2 = 2\pi\alpha^2_{em}e^2_q\frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2\hat{t}^2}$$

or $$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2_{em}}{Q^4}\left[1+(1-y)^2\right]\frac{1}{2}e^2_q$$

where $\hat{s} = (p+k)^2$ , $\hat{t} = (k-k')^2 = -Q^2$ , $\hat{u} = (p-k')^2 \Rightarrow y = \overset{massless}{\frac{pk - pk'}{pk}} = 1 + \frac{\hat{u}}{\hat{s}}$

- Assuming that the struck quark carries a fraction $\xi$ of the hadron momentum $P$,

$$p^\mu = \xi P^\mu$$

taking an on-shell (massless) quark,

$$p'^2 = (p+q)^2 = q^2 + 2(pq) = -2(Pq)(x-\xi) = 0 \quad \Rightarrow \quad x = \xi$$

[as promised: $x$ is the hadron's momentum fraction carried by the struck quark]

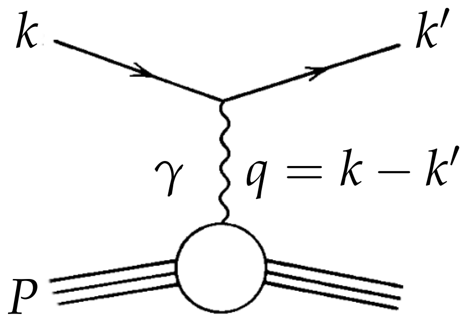
and introducing

$$\int_0^1 \mathrm{d}x\, \delta(x-\xi) = 1$$

we have:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{Q^4}\left[1+(1-y)^2\right]\frac{1}{2}e_{\mathrm{q}}^2\delta(x-\xi) \tag{1}$$

- Consider now the case of eh $\to$ eh:




- Notice that before, in the case of eq $\to$ eq, the cross section can be written as the contraction of a leptonic tensor ($L^{\mu\nu}$) and a hadronic tensor ($W_{\mathrm{q}}^{\mu\nu}$):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{1}{16\pi\hat{s}^2}\frac{1}{4}\sum_{r_1,r_2,r'_1,r'_2}|\mathcal{M}|^2 \equiv \frac{4\pi\alpha_{\mathrm{em}}^2}{Q^4}L_{\mu\nu}(k,k')W_{\mathrm{q}}^{\mu\nu}(p,p')$$

$$L^{\mu\nu}(k,k') \equiv [k^\mu k'^\nu + k^\nu k'^\mu - (kk')g^{\mu\nu}]\frac{1}{\hat{s}}$$

$$W_{\mathrm{q}}^{\mu\nu}(p,p') \equiv [p^\mu p'^\nu + p^\nu p'^\mu - (pp')g^{\mu\nu}]\frac{e_{\mathrm{q}}^2}{\hat{s}}$$

where an appropriate normalization has been chosen

- Now, replace $W_{\mathrm{q}}^{\mu\nu}$ by the most general tensor built from momenta $P$ and $q$:

$$W^{\mu\nu} \equiv -W_1 g^{\mu\nu} + \frac{W_2}{M^2} P^\mu P^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}(P^\mu q^\nu + q^\mu P^\nu)$$

compatible with parity conservation. For weak interactions ($W^\pm$,$Z$) add:

$$\mathrm{i}W_3 \frac{\epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta}{2M^2} \qquad \text{(to be contracted with an extra } \mathrm{i}\epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \text{ in } L^{\mu\nu}\text{)}$$

- The form factors $W_i = W_i(x, Q^2)$ depend on two (scalar) variables at a given $s$. Since $L^{\mu\nu}$ is symmetric, antisymmetric contributions are not introduced in $W^{\mu\nu}$. Furthermore, current conservation $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$ implies that:

$$W_5 = -\frac{(Pq)}{q^2} W_2\,, \quad W_4 = \left(\frac{(Pq)}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1$$

- The resulting hadronic tensor is:

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2}{M^2}\left(P^\mu - \frac{(Pq)}{q^2} q^\mu\right)\left(P^\nu - \frac{(Pq)}{q^2} q^\nu\right)$$

- Contracting the leptonic tensor with our generalized hadronic tensor one gets:

$$\hat{s}\, L_{\mu\nu} W^{\mu\nu} = 2(kk')W_1 + [2(Pk)(Pk') - (kk')M^2]\frac{W_2}{M^2} - [(Pk)(qk') - (Pk')(qk)]\frac{W_3}{M^2}$$

- It is customary to redefine the form factors:

$$F_1 = \frac{(Pk)}{(Pq)}W_1\,, \quad F_2 = \frac{(Pk)}{M^2}W_2\,, \quad F_3 = \frac{(Pk)}{M^2}W_3$$

so that, introducing $x$ and $y$ with $p^\mu = xP^\mu$ and neglecting $M^2 \ll s$ (then $\hat{s} = xs$),

$$L_{\mu\nu}W^{\mu\nu} = \frac{1}{x}\left[xy^2F_1 + (1-y)F_2 + xy\left(1-\frac{y}{2}\right)F_3\right]$$

where we have used

$$(Pq) = y(Pk) = \frac{(kk')}{x}\,, \quad (Pk') = (Pk) - (Pq) = \frac{(kk')}{xy}(1-y)\,,$$

$$(qk') = -(qk) = (kk')\,, \quad (kk') = \frac{\hat{s}}{2}y$$

- The differential cross section eh $\to$ eh (photon exchange) then reads:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{Q^4}\left\{[1+(1-y)^2]F_1 + \left(\frac{1-y}{x}\right)(F_2 - 2xF_1)\right\}$$

that can be compared to the cross section eq $\to$ eq in (1):

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{Q^4}\left[1+(1-y)^2\right]\frac{1}{2}e_q^2\delta(x-\xi)$$

$\Rightarrow$ If hadron constituents were free quarks (spin $\frac{1}{2}$ particles with charge $e_q$), then:

– Callan-Gross relation: $F_2 = 2xF_1$ $\quad [F_2 = e_q^2 x\delta(x-\xi)]$

– Bjorken scaling. The structure functions would not depend on $Q^2$:

$$F_i(x, Q^2) = F_i(x)$$

(the constituents are pointlike particles)

Both properties are modified by gluon corrections

- From previous expressions one can easily obtain cross sections for $\nu N$ scattering, CC ($W$-exchange) and NC ($Z$-exchange), replacing the photon propagator by

$$-\frac{1}{Q^2} \to -\frac{1}{Q^2 + M_V^2}, \quad M_V = M_{W,Z}$$

and extracting/absorbing some constants from/in the form factors, as

$$G_F^2 = \frac{\pi^2 \alpha_{\mathrm{em}}^2}{2 s_W^4 M_W^4} = \frac{g^4}{32 M_W^4}$$

The $\nu N$ cross sections are then:

$$\frac{\mathrm{d}^2\sigma_{\nu N}^{\mathrm{CC,NC}}}{\mathrm{d}x\mathrm{d}y} = xs \frac{\mathrm{d}^2\sigma_{\nu N}^{\mathrm{CC,NC}}}{\mathrm{d}x\mathrm{d}Q^2}$$

$$= \frac{G_F^2 s}{\pi} \left( \frac{M_V^2}{Q^2 + M_V^2} \right)^2 \left[ xy^2 F_1^{\mathrm{CC,NC}} + (1-y) F_2^{\mathrm{CC,NC}} + xy \left(1 - \frac{y}{2}\right) F_3^{\mathrm{CC,NC}} \right]$$

- The hadron is composed of partons: quarks and/or antiquarks, in principle. Then, one can introduce the Parton Distribution Functions (PDF)

$$F_2(x) = 2xF_1(x) = \sum_{q,\bar{q}} e_q^2\, x f_{q/h}(x) \qquad\qquad f_{q/h}(x) \equiv q^h(x)$$

where $f_{q/h}\mathrm{d}x$ expresses the probability to find a quark q inside hadron h carrying a fraction of the hadron longitudinal momentum in $[x, x+\mathrm{d}x]$.

In e-proton and e-neutron scattering one probes the nucleon structure functions

$$\frac{1}{x}F_2^{ep} = \left(\frac{2}{3}\right)^2 (u^p + \bar{u}^p) + \left(-\frac{1}{3}\right)^2 (d^p + \bar{d}^p) + \left(-\frac{1}{3}\right)^2 (s^p + \bar{s}^p) + \ldots$$

$$\frac{1}{x}F_2^{en} = \left(\frac{2}{3}\right)^2 (u^n + \bar{u}^n) + \left(-\frac{1}{3}\right)^2 (d^n + \bar{d}^n) + \left(-\frac{1}{3}\right)^2 (s^n + \bar{s}^n) + \ldots$$

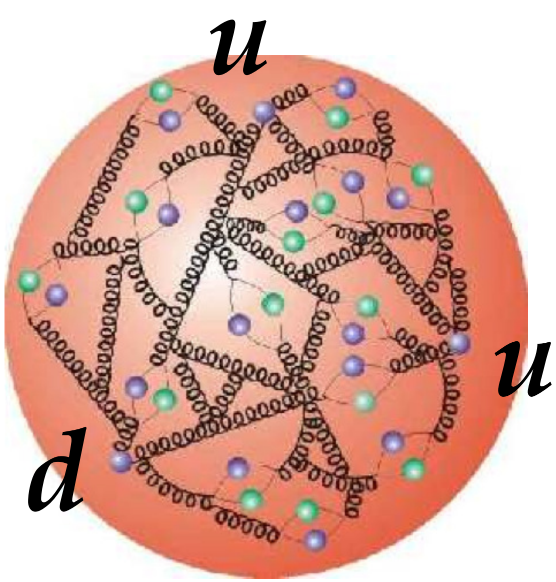
where the PDFs ($f_{u/p}(x) \equiv u^p(x)$, etc.) are related by isospin symmetry:

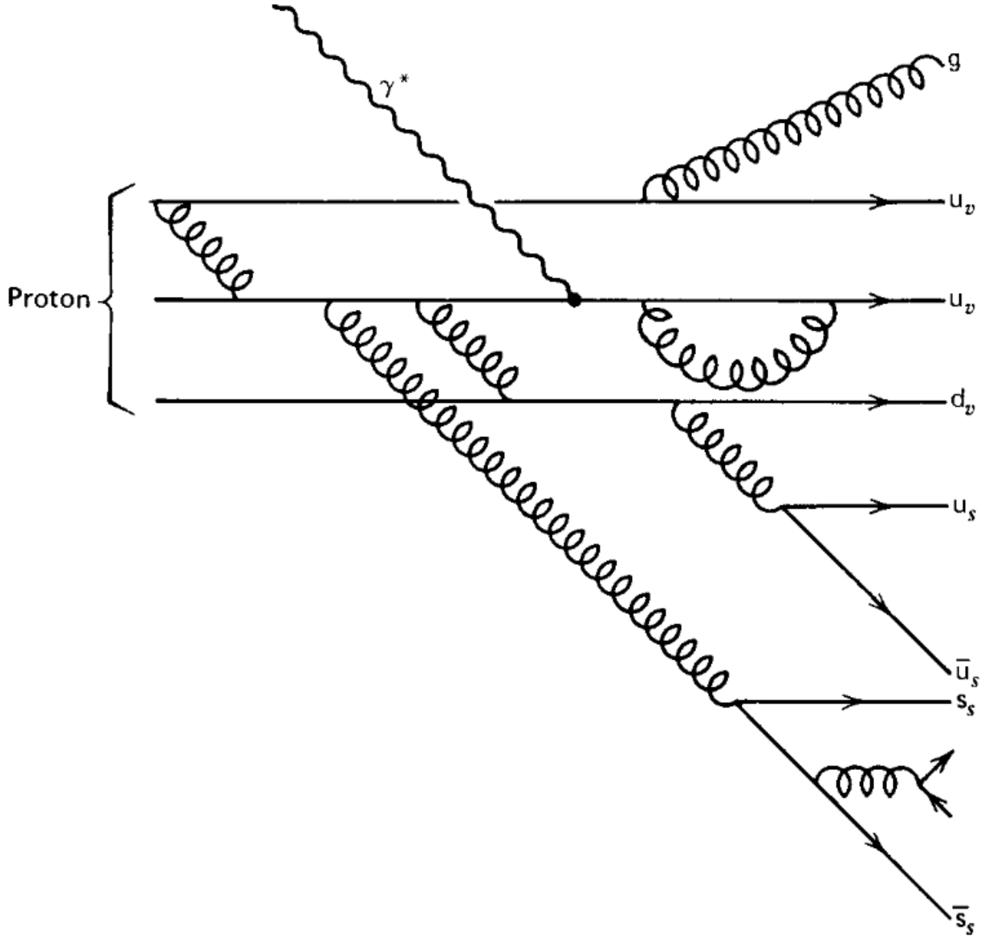
$$u^p = d^n \equiv u\ , \qquad d^p = u^n \equiv d\ , \qquad s^p = s^n \equiv s\ , \qquad \ldots$$

# Hadron structure | DIS | Parton Model

- The quantum numbers of a proton must be those of a uud combination of valence quarks ($f_v$). The rest are sea quarks ($f_s$):

$$u = u_v + u_s \;, \quad d = d_v + d_s \;, \quad \overline{u} = u_s = \overline{u}_s \;, \quad \overline{d} = d_s = \overline{d}_s \;, \quad s = s_s = \overline{s}_s \;, \quad \ldots$$



- Then the following sum rules apply:

$$\int_0^1 \mathrm{d}x\, u_v(x) = \int_0^1 \mathrm{d}x\, [u(x) - \overline{u}(x)] = 2$$
$$\int_0^1 \mathrm{d}x\, d_v(x) = \int_0^1 \mathrm{d}x\, [d(x) - \overline{d}(x)] = 1$$

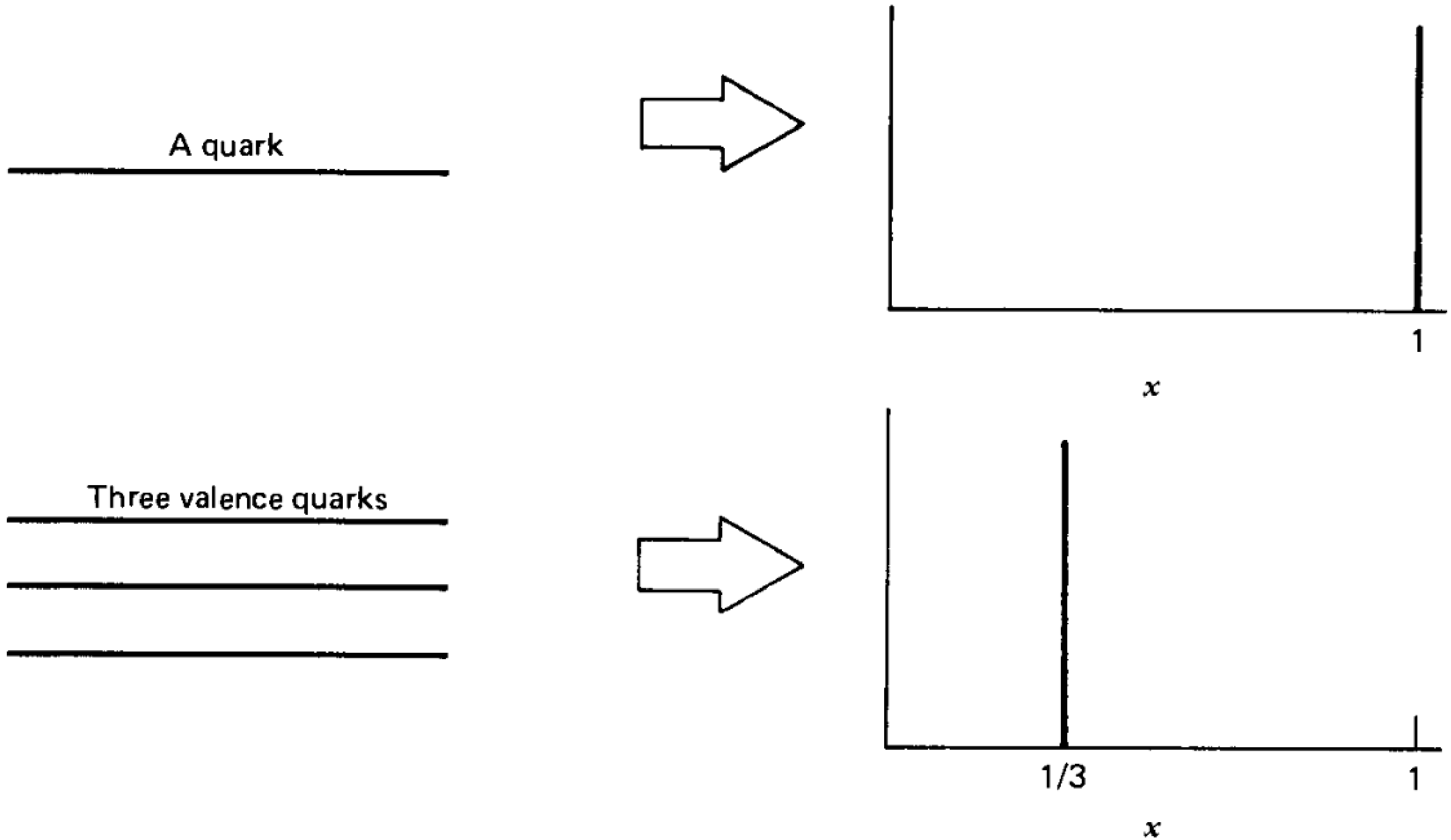
- And, assumimg just u, d and s and taking $S$ as the total sea contribution:

$$\frac{1}{x} F_2^{\text{ep}} = \frac{1}{9}(4u_v + d_v) + \frac{4}{3} S$$
$$\frac{1}{x} F_2^{\text{en}} = \frac{1}{9}(u_v + 4d_v) + \frac{4}{3} S$$

- One can now guess what the proton structure function $F_2^{\text{ep}}$ looks like …

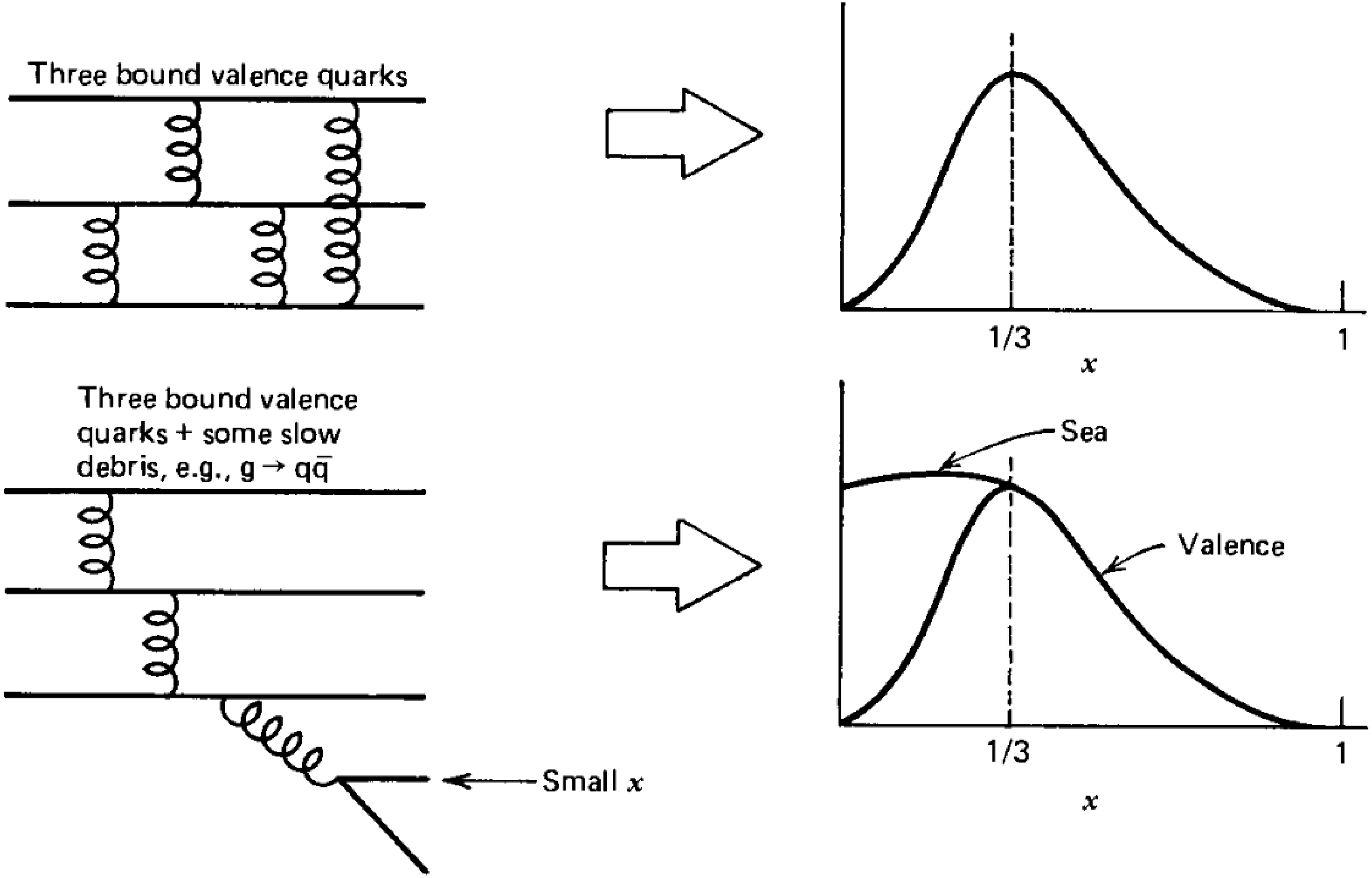
## Hadron structure | DIS | Parton Model

If the proton is … then $F_2^{ep}(x)$ is



**Hadron structure** | **DIS** | **Parton Model**

If the proton is … then $F_2^{ep}(x)$ is



**Hadron structure** **DIS** **Parton Model** (Where are the gluons?)

- Another sum rule: sum of partons must carry *all* hadron momentum,

$$\int_0^1 \mathrm{d}x\, x[u(x) + \overline{u}(x) + d(x) + \overline{d}(x) + s(x) + \overline{s}(x) + \ldots] = 1$$

Actually gluons carry about 50 % of hadron momentum: $\int_0^1 \mathrm{d}x\, xg(x) \approx 0.5$