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PHYSICA A

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How to discriminate easily between directed-percolation and Manna scaling

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Abstract

Here we compare critical properties of systems in the directed-percolation (DP) universality class with those of absorbing-state phase transitions occurring in the presence of a non-diffusive conserved field, *i.e.*, transitions in the so-called Manna or C-DP class. Even if it is clearly established that these constitute two different universality classes, most of their universal features (exponents, moment ratios, scaling functions,...) are very similar, making it difficult to discriminate numerically between them. Nevertheless, as illustrated here, the two classes behave in a rather different way upon introducing a physical boundary or wall. Taking advantage of this, we propose a simple and fast method to discriminate between these two universality classes. This is particularly helpful in solving some existing discrepancies in self-organized critical systems as sandpiles.

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The concept of universality in equilibrium phase transitions has been successfully extended to the broader realm of non-equilibrium, where rather robust universality classes describing driven diffusive systems, the roughening of non-equilibrium interfaces, or systems with absorbing states, to name but a few, are firmly established. For instance, a huge number of models with absorbing states belong to the directed-percolation (DP) class [1] which constitutes one of the largest and more robust non-equilibrium classes.

One specially fascinating family of non-equilibrium critical phenomena are those appearing in a *self-organized* way, *i.e.*, without apparent tuning of parameters. Since the introduction of *self-organized criticality* (SOC) [2] understanding the universality (or the lack of it) of “self-organized” critical points became a high priority task. Sandpiles [2,3], the archetype example of SOC, are models in which sandgrains are slowly added to a given site in a lattice, and redistributed to neighboring sites whenever certain instability threshold is exceeded. This may generate cascades of topplings (*i.e.*, *avalanches*) which eventually lead to dissipation of grains at the open boundaries (see Refs. [2,3] for model definitions and reviews). This mechanism drives the system to a stationary, critical state with power-law distributed avalanches. For our purpose here, it is

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important to realize that between successive avalanches sandpiles are trapped into one out of many possible quiescent or absorbing states (in which the dynamics is completely frozen). Such a state can be characterized by the distribution of heights below threshold, which is a locally conserved and non-diffusive (grains at sites below threshold do not move) field. After some controversies, it was clearly established that most (isotropic) stochastic sandpiles [3] share the same universal properties [4,5] (the correspondence is best understood using the *fixed-energy ensemble*; see Ref. [4]). Indeed, (isotropic) stochastic sandpiles, as well as many other systems with absorbing states and an additional conserved and non-diffusive field [6], belong to a unique universality class, usually called the Manna class [3] or C-DP class (in analogy with the nomenclature by Hohenberg and Halperin [7]). The essential ingredients (symmetries and conservation laws) of this class are captured by the following set of Langevin equations:

$$\begin{aligned} \partial_t \rho(x, t) &= a\rho - b\rho^2 + D\nabla^2 \rho + \omega\rho\phi + \sigma\sqrt{\rho}\eta(x, t), \\ \partial_t \phi(x, t) &= D_\phi \nabla^2 \rho, \end{aligned} \tag{1}$$

where $\rho(x, t)$ is the activity field, $\phi(x, t)$ the background conserved field, a, b, D, ω, σ and D_ϕ constants, and $\eta(x, t)$ a Gaussian white noise [4,6]. This is to be compared with the standard Langevin equation for generic transitions into absorbing states with no extra symmetry or conservation law, i.e., the celebrated DP Langevin equation [1]

$$\partial_t \rho(x, t) = a\rho - b\rho^2 + D\nabla^2 \rho + \sigma\sqrt{\rho}\eta(x, t), \tag{2}$$

which differs from C-DP just by the absence of the conserved field. The previous two equations represent two distinct universality classes as verified by extensive numerical simulations as well as renormalization group analyses, and other theoretical considerations [1,4]. Given that Langevin equations are mesoscopic coarse-grained descriptions, in which irrelevant terms giving corrections to scaling have been excluded, direct numerical integration of Eqs. (1) and (2) is an excellent way to determine critical exponents with good precision [8]. From such numerical integrations as well as from extensive numerics performed in discrete models, the values of critical exponents for these classes can be obtained (upper rows in Table 1).

Note the remarkable similitude between exponents in both classes. The numerical differences in other critical exponents as β, ν, z etc. are not large either. Moreover, other universal features as scaling functions and moment ratios can also be checked to be very much alike in these two classes [10]. This suggests that Manna/C-DP exponents could be somehow “perturbatively” computable from DP ones, but unluckily this program has not been completed yet.

Given the general lack of working analytical tools to analyze microscopic models, in order to assign a given model to one of these classes one is left mostly at the mercy of numerics which, in the light of the small differences in exponent values, is not a pleasant task. Even worse: for some discrete models (in particular, some sandpiles) corrections to scaling are large and numerics can be plagued with long transients hiding the true asymptotic behavior. This is the reason why some original works trying to relate sandpile criticality with systems with absorbing states concluded, that exponents were “compatible” with DP scaling. Ulterior large

Table 1
Critical exponents for DP and C-DP, without a wall and in the presence of absorbing and reflecting walls

	η	δ	τ	τ_t
DP	0.313(1)	0.159(1)	1.108(1)	1.159(1)
C-DP	0.350(5)	0.170(5)	1.11(2)	1.17(2)
DP _{abs}	0.045(2)	0.426(2)	1.28(3)	1.426(2)
DP _{ref}	0.046(2)	0.425(2)	1.25(3)	1.425(2)
C-DP _{abs}	-0.33(2)	0.85(2)	1.56(2)	1.81(2)
C-DP _{ref}	0.35(3)	0.16(3)	1.11(3)	1.15(3)

θ is the order-parameter decay exponent: $\langle \rho(t) \rangle \sim t^{-\theta}$, η and δ are the usual exponents in spreading experiments (for the growing of the total activity from an initial seed, $N(t) \sim t^\eta$, and the decay of the survival probability, $P(t) \sim t^{-\delta}$), while τ and τ_t are the avalanche exponents of the size and time distributions, respectively, [9]. Values in rows 3 (DP_{abs}) and 4 (DP_{ref}) coincide, also those in 2 (C-DP) and 6 (C-DP_{ref}) are also equal within errorbars.

scale simulations revealed systematic differences with DP, and showed rather unambiguously that sandpiles are generically described by C-DP and its associated set of exponents, as explained above [4,10]. Nevertheless, there remain some controversial cases in which it is not easy to distinguish numerically between these two classes [11] and one has to resort to massive simulations [12]. In this paper we propose a way out to this situation: a simple method to discriminate between these two classes without much computational effort.

The idea is to introduce a wall in the system under scrutiny. It is well known (already from equilibrium statistical mechanics) that the presence of a boundary or wall can induce non-trivial “surface” critical behavior, which is also characteristic and specific of each universality class (or renormalization group fixed point). In the context of systems with absorbing states, the effect of walls in the DP class has been profusely studied from a field theoretical perspective [13,14], using series expansions [15], density matrix renormalization group [16], as well as Monte Carlo simulations [17]. As feedback of activity from behind the wall is impeded (see the nice snapshots in Ref. [14]) the structure of the avalanches is strongly affected by the presence of a wall, be it *absorbing* or *reflecting*. Owing to this, the exponents for spreading or avalanches started nearby a wall differ from the ones without the wall, and are known with good precision [14]. We have simulated a number of (one-dimensional) systems in the DP class by analyzing the evolution of avalanches (or spreading) started nearby the wall and measured the exponent values in Table 1 (rows 3 and 4), in excellent agreement with those in the literature and obeying well-established scaling laws (in particular, the wall induces the appearance of only one independent new exponent, not related to standard bulk ones) [14]. In accordance with what is already known [14], all these exponents take the same values in the DP class for both absorbing and reflecting walls. Also, we have studied a model in the DP class with infinitely many absorbing states, characterized by a *non-conserved* background field [18]. In this case, the critical exponents in the presence of a wall (either absorbing or reflecting) coincide with those reported above for the DP class. Even if the background field becomes inhomogeneous nearby the wall this does not affect critical properties.

On the other hand, to the best of our knowledge, the effect of walls in the Manna/C-DP class has not been studied so far. Analogously to the case before, we have performed Monte Carlo simulations of a family of different one-dimensional models in this class, including: stochastic sandpiles (Oslo, Manna, and different variations of them [3]), reaction diffusion systems with a conserved non-diffusive field [6], as well as the Langevin Eq. (1).

First, we have verified that these types of wall do not alter bulk critical properties: starting avalanches sufficiently far away from the wall one recovers the standard (bulk) spreading exponents with as much precision as wanted.

Our measurements of exponents (summarized in Table 1; rows 5 and 6) lead to the following conclusions (see Fig. 1):

- (1) All models whose bulk properties are in the C-DP class, share the same (universal) surface exponents, which take the values in Table 1. The corresponding exponents are very robust and universal.
- (2) Contrarily to the DP class, in this case absorbing and reflecting walls generate different effects and exponents.
 - Absorbing walls do affect the exponents as happens in DP; surface exponents differ significantly from their bulk counterparts.
 - Reflecting walls do *not* change the exponents with respect to the ones without a wall; even if feedback of activity from “behind” the wall is also impeded in this case, the background field is enhanced nearby the reflecting wall, fostering further creation of activity and compensating the lack of feedback from behind the wall.

Therefore, an efficient and simple way to discriminate between DP and C-DP scaling consists in introducing a wall, either reflecting or absorbing:

- If, upon introducing a *reflecting wall*, exponents are not changed with respect to the original ones then the system is C-DP like. Instead, if they are affected (and take the values in Table 1) it is DP. The difference between the surface exponents in both classes is very large (compare, for instance, the values for η , 0.045(2) versus 0.35(3), 700% larger).

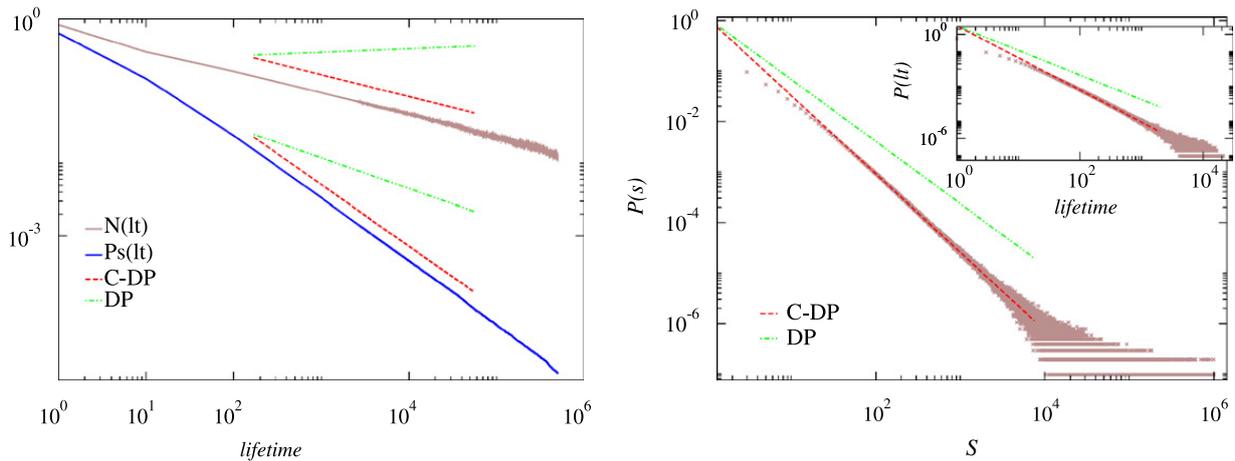


Fig. 1. Avalanche exponents for the Oslo sandpile model in one dimension, averaged over 10^7 runs (system size $L = 2^{15}$) in the presence of an absorbing wall. Left: spreading experiments (see Table 1). Right: avalanche size (main plot) and time (inset) distributions. Green lines mark DP scaling while red ones correspond to the best fit, characterizing C-DP/Manna surface scaling.

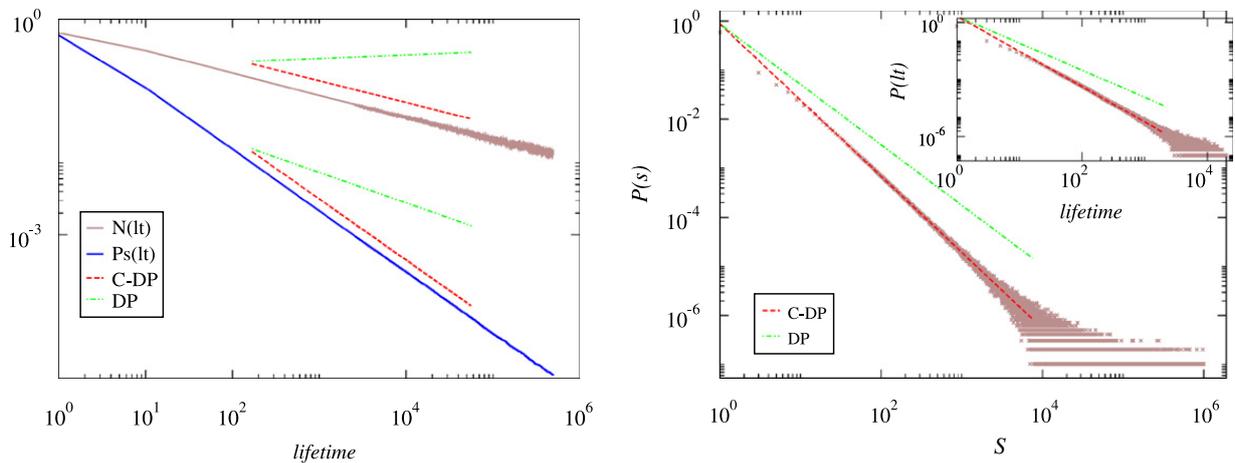


Fig. 2. Avalanche exponents for the MD sandpile in one dimension, averaged over 10^7 runs (system size $L = 2^{15}$) in the presence of an absorbing wall. Left: spreading experiments (see Table 1). Right: avalanche size (main plot) and time (inset) distributions. The four computed magnitudes are in excellent agreement with C-DP values (red lines) and incompatible with DP scaling (green lines).

- In the presence of *absorbing walls* the differences in the exponent values is also very large. For instance, $\eta = 0.045(2)$ for DP while for C-DP the value is $-0.33(2)$ (opposite sign!), and distinguishing them is a trivial matter.

As a straightforward application of these ideas, let us now discuss the sandpile model introduced by Mohanty and Dhar (MD) in its self-organized regime [11]. While the directed version of it is well established to be in the DP class (as happens with other *directed* sandpiles), there has been some controversy about its behavior in the non-directed case (see [11,12]). We have simulated the MD sandpile in the presence of either an absorbing or a reflecting wall. Our main simulation results are plotted in Fig. 2. First, as before, we have verified that bulk critical exponents are actually not affected by the presence of a wall: starting avalanches sufficiently away from the wall one recovers bulk exponents as those reported in Ref. [12] with as much precision as wanted. Then we determine spreading exponents by starting avalanches in a site nearby the wall and measuring their corresponding observables as a function of avalanche time. We also compute the exponents characterizing avalanche size and time distributions. It is clearly observed that exponents with and without a reflecting wall coincide for reflecting walls, while they are different for absorbing walls, in full agreement with what reported for the Manna/C-DP class. Moreover, in both cases (i.e., absorbing and

reflecting walls) all the computed exponents coincide with high accuracy with the ones for Manna/C-DP class in the presence of a wall (see Table 1) and exclude DP behavior.

Let us remark that in similar *directed models* the introduction of a wall leads to a “structured” background field (not very different from the one discussed here) that can induce non-trivial results [19,20]. Indeed, in Ref. [19] (non-DP) critical exponents are analytically derived in an exact way by analyzing the structure of the background and using the directed nature of the process, scaling laws, and DP exponents. It would be extremely interesting to have an analogous calculation for the present case, although owing to the lack of directness this promises to be a much more challenging task.

In summary, by introducing a wall, be it absorbing or reflecting, we have shown that systems in the, otherwise very similar, DP and Manna/C-DP universality classes, behave in a very different way. This provides a simple method to discriminate between these two classes. In a future work we will study analytically the effect of walls on systems in the Manna/C-DP class, complementing the numerics here and providing a more solid theoretical background to the practical strategy presented in this paper.

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